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Symmetry-Compatible Information in the Chrysene Formalism

Axiomatic Realization, Coding Obstructions, Constructive Repair, and Falsifiable Molecular Models

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Abstract

The Chrysene Formalism organizes molecular structure, assembly, information, and operator response through a typed axiomatic framework. We develop a compatibility theorem for information architectures whose states are identified by a structural symmetry. An invariant code with unique bounded-distance decoding induces a unique decoder on the orbit space; the corresponding statement may fail if an incompatible code is first closed under the symmetry. On a five-contact quaternary model for an open six-unit carrier, an explicit $[5, 3, 3]_4$ Hamming code loses single-error correction under naive reversal closure. A coordinate-equivalent code satisfies a syndrome covariance identity, has sixteen reversal-fixed words and forty codeword orbits, and gives perfect radius-one decoding on all 544 ambient orbits. In contrast, reversal composed with Frobenius is incompatible with every linear code of these parameters: the induced permutation on the projective line would have both even and odd parity. These finite results are connected to the twelve-axiom framework by explicit alphabet, state-preparation, symmetry-action, disturbance, and readout hypotheses. We derive conditional implementation and robust failure bounds, finite feasibility certificates, and comparative tests for molecular realization. The resulting research program combines deterministic mathematical constraints with stochastic preparation and measured physical response. We place this contribution within the journal's mathematical, biogenesis, molecular-machine, and abstract-application programs, with versioned verification and AI-augmented human review. Exact finite checks and a scoped Lean 4 companion accompany the manuscript; physical realization and historical attribution remain empirical questions.

Keywords: Chrysene Formalism; equivariant decoding; quaternary code; molecular symmetry; biogenesis; falsification; formal verification.

1 Introduction

A molecular information architecture has two obligations. Its physical states must satisfy the structural conditions of the proposed assembly, and its encoded states must satisfy the conditions of the proposed information operation. These obligations concern one system. An assembly that realizes a symmetry in one operating regime and a code that corrects errors in a different representation do not yet constitute a symmetry-compatible molecular code. The passage between them requires a map, an action, and a disturbance model.

The Chrysene Formalism gives a setting in which to state this passage. Its controlling twelve-axiom formulation begins with an embedded labeled benzenoid node, distinguishes several stabilizers, specifies directional connectors and translational closure, and introduces a discrete network, local tensor algebras, structural ciphers, spectral geometry, electronic transfer, polarization, and admissibility [Sch26i]. The purpose is not to rename each of these established mathematical subjects. It is to study their joint realization when the molecular reduct and its attachment data are fixed. The mathematical program and the molecular program then share a vocabulary while retaining different criteria of success.

The present article contributes a concrete obstruction and construction at that interface. A quaternary code can be perfect in an oriented coordinate system but fail to preserve its error-correction distance after the coordinates are identified by reversal. We exhibit such a failure for a particular parity-check matrix, repair it by a coordinate-equivalent matrix, and prove that decoding descends to a space of orientation classes. We also prove that the repair cannot be transferred unchanged to an action that combines reversal with the nontrivial field automorphism. Thus the representation of the molecular operation is a

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mathematical input with an observable consequence, rather than a naming convention.

1.1 Principal results

Let $K = \mathbb{F}_4$ and let $V = K^5$ be endowed with Hamming distance. The coordinates model five consecutive contacts of an open six-unit carrier. This is a mathematical contact model; a physical realization is specified separately in Section 3. Let R reverse the five coordinates and let σ square each field entry. The distinction between R and $R\sigma$ is central.

Theorem 1.1 (Principal finite conclusions)**.** *There are parity-check matrices H, H_* of rank two over K with the following properties.*

(i) $C = \ker H$ and $C_* = \ker H_*$ are perfect $[5, 3, 3]_4$ codes. The reversal closure $C \cup RC$ has minimum distance one.

(ii) C_* is R-invariant. Its 64 words have sixteen fixed points and forty reversal orbits. The orbit code has minimum distance three and a perfect radius-one decoder on 544 ambient orbits.

(iii) No K-linear $[5, 3, 3]_4$ code is invariant under $R\sigma$. The same projective obstruction applies to semilinear monomial actions with coordinate permutation R, even if coordinate scalings are allowed.

The explicit matrices and proofs appear in Sections 5–7. The result is an integration of finite coding and symmetry constraints. Reversible codes and projective descriptions of Hamming codes belong to established coding theory [Mas64, MS77, HP03]. We do not claim novelty for the existence of reversible codes as such. The contribution advanced here is the explicit obstruction–repair–representation test in the chosen contact model, together with a typed route from the molecular formalism to that test.

1.2 A compatibility result rather than a counting analogy

An equality between two cardinalities does not define an encoding. A number of molecular configurations equal to a number of codewords is insufficient unless preparation, identification, errors, and readout have compatible meanings. Conversely, unequal cardinalities do not necessarily obstruct a useful implementation: a physical state set may have many microstates per encoded symbol. The correct object is a structured map with a declared image and symmetry action.

This distinction prevents two conflations. First, the forty reversal classes in this paper are not the registry of twenty developed elsewhere in the corpus. A map between those classifications would be another theorem. Second, the classical K-symbol code is not automatically a quantum code. A quantum encoding requires a Hilbert space, a noise channel, and recovery conditions that are not supplied by classical Hamming distance alone. Section 10 explains how the axiomatic operator framework can support, but does not replace, those specifications.

1.3 Two objectives of the research program

The first objective of the journal is a mathematical discipline in which the substituted-chrysene molecular reduct is instantiated in an axiom system and then used to study constructions across different domains. The second is to investigate substituted chrysene as a candidate common structural and informational material for biogenesis. The strong hypothesis that it is the unique such material is a research question. It must be divided into a specified comparison class, an admissibility criterion, and a test of each candidate.

The canonical classification in Article 40 is a precise example: within its seven-tetrihex comparison and stated signature, the canonical carbon reduct is chrysene. It is not a theorem that every possible chemical scaffold, every substitution pattern, or the historical origin of life must be chrysene. Mathematical uniqueness becomes informative by making its quantifiers explicit. Enlarging the comparison class changes the claim and may change the result.

The patent US 12,195,423 B1 provides design and intellectual-property provenance for substituted-chrysene constructions [Sch25]. It is cited here for that purpose. Its legal status does not decide the realization hypotheses of the present theorem or the history of biogenesis. Those questions are addressed by measured structural, kinetic, and functional evidence.

1.4 Deterministic implications and stochastic sources

A deterministic theorem states that every object satisfying its hypotheses has a specified property. It does not require that the preparation of such objects be deterministic. An endogenous source can have a probability law, a controlled preparation can have yield and defect distributions, and a mathematically deterministic decoder can act on random errors. The source/placed-source distinction developed in Article 99 separates preparation mechanisms from downstream structural constraints [Sch26f].

Accordingly, the alternative offered by the formalism is a molecule-anchored constructive program: structural choices and compatibility constraints are specified before asking which distributions can produce them. This is not a claim that contemporary origins-of-life research treats chemical assembly as unconstrained chance. Mechanistic precursor chemistry and environmental selection already provide important alternatives

[PPR+15, MAS+24]. Comparative evaluation requires matched questions and data, not a contrast between rigorous mathematics and a caricature of probability.

1.5 Organization and evidentiary status

Sections 2 and 3 set the types and hypotheses. Sections 4–7 prove the finite core. Sections 8 and 9 derive probabilistic and observable consequences without assuming independence where it is unnecessary. Sections 10 and 11 define physical and empirical interfaces. The final sections place the contribution within the corpus and state the verification and editorial requirements.

The theorem proofs are mathematical arguments, supported by exact finite checks and the scoped Lean 4 companion described in Section 13. A complete Lean formalization of all statements is not claimed. No new chemical experiment is reported here. The article's mathematical center can therefore be evaluated independently of whether a particular substituted-chrysene specimen has yet been shown to implement it.

2 Axiomatic foundation and permitted extensions

2.1 Models, reducts, and interfaces

Definition 2.1 (Controlling model class)**.** Write $\mathfrak{M}_{40}$ for the class of models of the twelve-axiom system of Article 40 [Sch26i], with the quantifiers, labels, domains, and analytic admissibility conditions of that source. A *molecular reduct* is the embedded carbon graph together with the specified locant and attachment data obtained by forgetting the additional network and operator structures.

This convention imports the source system rather than replacing it by a shorter list. The following account identifies the dependencies used in this paper. It is not a reaxiomatization, and no summary sentence removes a hypothesis from the source.

Convention 2.2 (Typed inference)**.** A conclusion about a reduct is not automatically a conclusion about every extension of that reduct. A conclusion about a mathematical extension is not automatically an empirical assertion about a specimen. Every map crossing these levels is named, and every extra condition used below is stated separately.

This rule is particularly important for symmetry. A core graph can possess an inversion that a decorated assembly lacks. A symbol model can possess an involution that does not correspond to a spatial motion. A measured reversal can change a readout basis as well as a contact order. The three operations have different domains.

2.2 Structural axioms

Axiom A_1 specifies an embedded labeled benzenoid molecular node. It includes a graph in the four-ring polyhex class, an injective spatial embedding, chemical and locant labels, and distinguished attachment data. The primary sites are $3, 6, 9, 12$; the distinguished dual sites are $2, 5, 8, 11$. The graph and its labels must be fixed before a claim concerning an attachment symmetry is made.

Axiom A_2 distinguishes the fully labeled Euclidean stabilizer, the embedded carbon-framework stabilizer, and the canonical reference stabilizer. The canonical C_{2h} classification is conjugacy inside $O(3)$, not just an abstract group isomorphism. The fully labeled stabilizer can be smaller. This prevents a point-group classification of an ideal core from being used as a proof that a substituted, solvated, or electrode-bound specimen has the same action on all states.

Axiom A_3 supplies ordered directional attachment data, with $x = (3, 9)$ and $y = (12, 6)$. Their displacement vectors are nonzero and independent, and the corresponding data are inequivalent in the stated senses. Independence does not mean Euclidean orthogonality. Inequivalence does not by itself give a numerical inequality between conductances, polarizabilities, or decoherence rates.

Axioms A_4 and A_5 describe directional bridge families and translational closure. If d_α is the core contribution, c_α a terminal contribution, and $B_\alpha(N)$ a bridge displacement, the closure residual has the form

$$r_\alpha(N) = a_\alpha - d_\alpha - c_\alpha - B_\alpha(N). \tag{1}$$

The declared exact closure counts make this residual zero. Their minimality, energetic preference, or realization by a particular linker chemistry is not a consequence of writing the equation. In particular, a relation of the form $N_y = N_x + 2$ is a separate realization target whose offset and equal-repeat assumptions must be established.

Axiom A_6 places the nodes in a discrete $\mathbb{Z}^3$-fibrated tensor network, with three independent translations, connector relations in two directions, and binary stacking-edge registry in the third. A translation acts on the underlying network and transports registry labels. It preserves a fixed decorated network only when the decoration is invariant. Binary registry is not identified with molecular chirality.

Lemma 2.3 (Forgetting does not enlarge a premise)**.** *Let X be a structured object and let $U(X)$ be its reduct. Any automorphism of X induces an automorphism of $U(X)$ when*

the forgetful map preserves the defining relations. The converse requires a lifting condition and can fail.

Proof. An automorphism of X preserves every relation retained by U, hence induces the required map on the reduct. For failure of the converse, take a two-point graph with an exchange automorphism and assign distinct labels to its two vertices. The exchange preserves the unlabelled graph but not the labelled graph. Therefore a reduct automorphism need not lift. □

The lemma is elementary, but it identifies a recurrent source of unsupported molecular inference. The lifting condition is not a technical nuisance: it is precisely what an encoding experiment may have to establish.

2.3 Algebraic and informational axioms

Axiom A_7 introduces a quasi-local tensor algebra and a translation action, giving a crossed product of the schematic form

$$\mathfrak{A} = \mathfrak{Q} \rtimes_{\alpha} \mathbb{Z}^3.$$

The translation unitaries satisfy covariance with localized operations. The resulting noncommutativity is an algebraic property of sequence actions; it is not established merely by drawing two attachment directions at right angles.

Axiom A_8 supplies a primary/dual cipher relation, an additive four-state alphabet

$$\Sigma \cong \mathbb{Z}_2^{\mathrm{dc}} \times \mathbb{Z}_2^{z},$$

and an antiunitary involution implementing the stated commutant duality. The cipher involution is not assumed to be a Euclidean molecular symmetry. Nor does the additive group by itself select a field multiplication. We use $K = \mathbb{F}_4$ as an external coding alphabet after choosing an additive identification with Σ. The stronger goal of deriving multiplication from a molecular composition law remains a different task.

Proposition 2.4 (Nonuniqueness of the field presentation)**.** *Let S be a set with the additive group structure of $\mathbb{F}_2^2$. An additive identification $\iota : S \to (K, +)$ transports multiplication to S, but the additive structure alone does not distinguish an identity element for that multiplication.*

Proof. Transporting the field operations along ι produces a field, with multiplicative identity $\iota^{-1}(1)$. The additive automorphism group acts transitively on the three nonzero elements of S: extend any chosen nonzero vector to a basis and map it to a basis beginning with any other nonzero vector. Consequently no nonzero element is distinguished by the additive structure alone. A field identity must be additional structure or arise from additional relations. □

This proposition does not prevent using a finite field in an abstract code. It states what must be supplied before the multiplication is attributed to a molecular implementation. An arbitrary relabelling is enough for a mathematical alphabet, but not for a mechanistic claim.

2.4 Geometric and physical operators

Axiom A_9 concerns a faithful spectral representation, a selfadjoint operator D, invariant dense algebraic domains, bounded commutators, and compact resolvent. These analytic conditions are part of the hypothesis. Neither a finite matrix example nor a successful finite program automatically establishes an infinite spectral triple. Additional real-structure and order-one conditions require their own proofs.

Axiom A_{10} introduces a selected physical Hamiltonian and effective transfer model. In a terminal/bridge decomposition, the effective transfer may be written

$$T(z) = J + W_L^{\dagger}(zI - H_B)^{-1}W_R, \tag{2}$$

where the direct term J cannot be dropped without a model-specific reason. This electronic Hamiltonian is not identified by definition with the geometric operator D.

Axiom A_{11} introduces bounded selfadjoint dipole observables and causal linear response under specified perturbations and states. Axiom A_{12} supplies admissible structural states, an embedding into algebra states, a weighted transition graph, and constraints. The induced path distance and spectral distance are distinct constructions. A compatibility relation between them is an extra theorem or hypothesis.

Remark 2.5 (A useful separation)**.** The finite coding theorem below uses only a finite alphabet, metric, action, and parity-check map. It does not require the analytic conditions of A_9. A physical application can use A_{10} and A_{11} to model the preparation and readout of those symbols, but it must then verify the corresponding parameters and regimes. This organization makes the finite result portable without claiming that the operator model is physically vacuous or automatically realized.

2.5 Characterization, consistency, and realization

Article 40 proves a canonical carbon-framework classification within seven free tetrihexes and establishes nonemptiness of its full model class. These are different results. The canonical reduct may be constrained without the full model class being categorical. Its explicit consistency witness permits choices such as zero response, constant registry, and a singleton admis-

sibility set. That witness discharges a mathematical consistency obligation; it is not offered as a chemically realistic system.

Definition 2.6 (Additional interface hypotheses)**.** An *interface hypothesis* is a condition on an extension of a model in $\mathfrak{M}_{40}$ that identifies a physical or modeled state with an information-theoretic object. The alphabet map, contact ordering, reversal action, state-preparation map, disturbance class, and readout instrument in Section 3 are interface hypotheses.

The new theorem can therefore be stated without enlarging the twelve axioms silently. It says what follows for every extension meeting these hypotheses. Whether such an extension occurs in an endogenous environment, can be engineered in a placed source, or has a competing realization on another scaffold is a subsequent question with its own evidence.

2.6 Dependency and concordance map

The source dependencies are deliberately separated. Article 40 supplies the controlling twelve-axiom model class; Article 92 studies an additional register identification and its Frobenius action, which is not silently identified with pure contact reversal [Sch26e]. Article 98 supplies the five-contact reference parity-check matrix and its classical labeled-code parameters [Sch26g]. Article 99 distinguishes endogenous and placed-source reachability from their preparation distributions and dynamics [Sch26f]. Classical coding theory supplies the Hamming-code and reversible-code background [MS77, Mas64].

On top of those sources, Section 3 explicitly adds the alphabet, preparation, action, disturbance and readout interface. Sections 5 and 6 then establish the representation-sensitive obstruction and repair; the quotient theorem in Section 4 explains their decoding consequences. These arrows are logical dependencies, not assertions that an inherited axiom alone establishes a physical implementation.

3 Typed molecular information realizations

Figure 1 summarizes the distinct roles of the axiomatic model and its added realization interface.

3.1 State sets and alphabets

Definition 3.1 (Realization datum)**.** A realization datum consists of a model $M \in \mathfrak{M}_{40}$, an admitted assembly A, a state set Ω, a finite alphabet S, a symbol space $V \subseteq S^n$, and a map $q : \Omega \to V$. It also includes a finite group G, actions

$$\alpha : G \longrightarrow \mathrm{Bij}(\Omega), \qquad \rho : G \longrightarrow \mathrm{Isom}(V, d),$$

a code $C \subseteq V$, a preparation rule, an admitted disturbance relation, and a readout rule. The domains of these data are part of the specification.

An element of Ω can represent a resolved microscopic state or an explicitly coarse-grained state. The choice determines which distinctions are retained. The symbol map may be many-to-one. A codeword is a target of preparation, not necessarily a unique microscopic state. If a state falls outside the calibrated symbol domain, it is reported as leakage rather than assigned an arbitrary symbol to preserve a desired theorem.

Hypothesis 3.2 (Encoding and action)**.** The code is G-invariant. Every codeword has at least one admitted preparation state. For all $g \in G$ and $x \in \Omega$,

$$q(\alpha_g x) = \rho_g q(x). \tag{3}$$

The state transformations preserve the admitted domain. The symbol action is fixed by the calibration, not selected after comparing code performance.

The first preparation requirement is not implied by equivariance. A constant map can be equivariant while omitting almost every codeword. Likewise, a surjective symbol map does not identify an experimentally feasible preparation procedure. When preparation is stochastic, a codeword may instead be supported by a prescribed distribution over its preimage, with failure included in the error budget.

Definition 3.3 (Orientation identification)**.** Two physical states are orientation-equivalent if they belong to the same G-orbit under α. Two symbols are orientation-equivalent if they belong to the same orbit under ρ. Under Hypothesis 3.2, q induces

$$\bar{q} : \Omega/G \longrightarrow V/G, \qquad [x] \longmapsto [q(x)].$$

Well-definedness. If $x' = \alpha_g x$, then $q(x') = \rho_g q(x)$, so the two image symbols have the same orbit. No injectivity or surjectivity is asserted. □

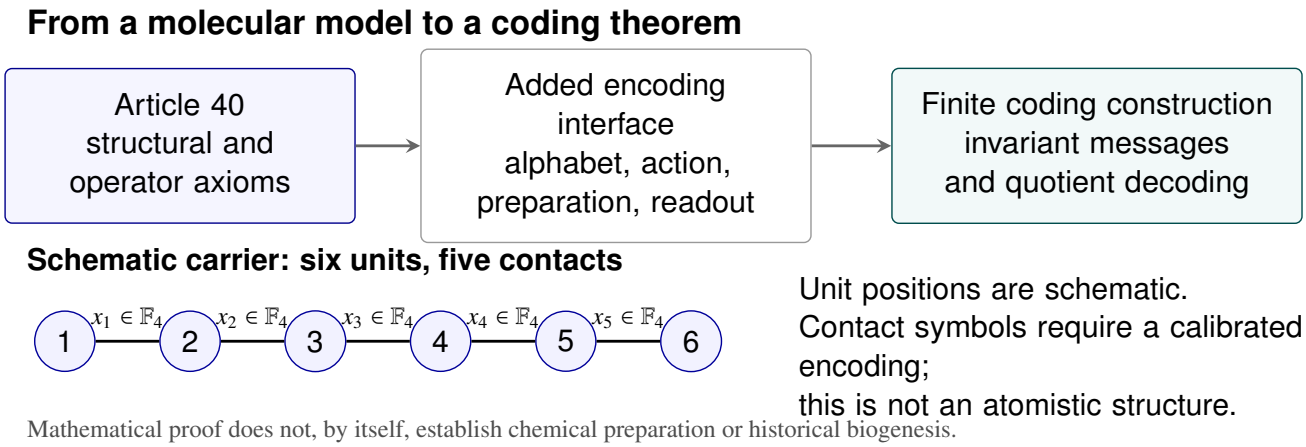


Figure 1: The axiomatic and physical interfaces of the coding construction. Six numbered units and five contact symbols are schematic positions, not a chemical drawing. The field identification, physical action and preparation/readout correspondence are added realization hypotheses; the diagram does not establish them.

3.2 Five contacts of a six-unit carrier

Fix six ordered units $u_0, \dots, u_5$. A contact is the ordered adjacent pair (u_{i-1}, u_i) for $1 \le i \le 5$. A pure reversal exchanges u_i with u_{5-i} and reverses contact order. To obtain the action R on K^5, the symbol assigned to an individual contact must remain unchanged under its local reversal. If the local symbol changes, that change must be included in ρ.

Hypothesis 3.4 (Pure-reversal branch)**.** There is a calibrated identification $S = K = \mathbb{F}_4$ such that reversal of the admitted six-unit assembly induces

$$\rho_r(x_1, x_2, x_3, x_4, x_5) = (x_5, x_4, x_3, x_2, x_1).$$

The contact-to-coordinate assignment of H_* is physically admissible for preparation and readout.

Hypothesis 3.5 (Frobenius-reversal branch)**.** The corresponding calibrated reversal instead induces

$$\rho_r(x_1, \dots, x_5) = (x_5^2, \dots, x_1^2).$$

The desired code is K-linear with parameters $[5, 3, 3]_4$.

Both are model branches. Neither follows merely from the C_{2h} canonical core. Article 92 uses a semilinear action in a different register [Sch26e]; that motivates checking the action here, but does not identify the five contacts with its paired register. Article 99's binary agreement field is also a different object [Sch26f]. A quaternary contact word is not obtained by changing the name of a binary word.

Because adjacent contacts share a carrier unit, their admissible symbols need not vary independently. Four calibrated states at every isolated contact do not establish joint codeword realizability. The preparation hypothesis concerns the full assembly and every required codeword, including shared-unit steric, charge, conformational and electronic compatibility. Correlated disturbances remain admissible outside the independent-channel model.

3.3 Preparation, disturbances, and observation

Definition 3.6 (Bounded symbol disturbance)**.** For a prepared codeword c, a received symbol y is a radius-t disturbance if $d(y, c) \le t$. An orientation-free radius-t disturbance satisfies

$$\min_{g \in G} d(y, \rho_g c) \le t.$$

The model permits a set of errors without assigning a distribution to them.

A physical disturbance may be small in an analog norm but large after thresholding into symbols. Conversely, a large motion within a calibrated state basin may cause no symbol error. The relation between these two descriptions must therefore be proved from margins or measured. The phrase "small perturbation" is insufficient to invoke a Hamming-distance theorem.

Hypothesis 3.7 (Readout adequacy)**.** For every intended received state in the calibrated domain, the readout returns its symbol orbit, or returns a flagged failure. The successful

branch does not conflate two distinct symbol orbits needed by the decoder. Its probability of failure is evaluated on the same preparation and disturbance experiment as the coding claim.

This hypothesis does not assume nondemolition. A destructive final readout can support a classical communication task. If repeated nondemolition observation is desired, the instrument and its state-update behavior must be specified, not inferred from geometric channel separation.

3.4 Conditional implementation theorem

Theorem 3.8 (Implementation from a common datum)**.** *Let t be a nonnegative integer and let a realization datum satisfy Hypotheses 3.2 and 3.7. Suppose the code has minimum distance at least $2t + 1$. For any successful preparation of a codeword, any admitted orientation-free radius-t symbol disturbance, and any adequate readout, the decoded orbit equals the prepared codeword orbit.*

Proof. Preparation provides a state with symbol $c \in C$. An orientation-free disturbance gives a received orbit within quotient distance t of $[c]$. The quotient decoding theorem proved in Section 4 gives uniqueness of the code orbit at that distance. Adequate readout returns the correct received orbit. Therefore the decoder returns $[c]$, regardless of the chosen representatives. □

Remark 3.9 (Logical content)**.** The theorem is an implication with experimentally meaningful premises. It neither proves that the premises hold for a specimen nor assumes the desired conclusion as a premise. Equivariance is testable on transformed states; preparation coverage is testable on the intended message set; disturbance and readout can be independently challenged by controls. A failed premise identifies which implementation claim has been rejected while leaving the finite theorem intact.

3.5 A physical compatibility diagram

The required agreement can be written as a commutative square:

$$\begin{array}{ccc} \Omega & \xrightarrow{\alpha_g} & \Omega \\ q\downarrow & & \downarrow q \\ V & \xrightarrow{\rho_g} & V. \end{array}$$

A second square relates physical readout to symbol-orbit observation. These diagrams are not decorative analogies. An experimentally detected failure of commutativity invalidates the corresponding implementation hypothesis. For a finite set of calibration states, the discrepancy can be tabulated. For a continuum of physical states, coverage requires a regularity argument or a justified interpolation regime.

Example 3.10 (Separate feasibility is insufficient)**.** Let a device family have two operating settings. In the first, all codeword states are preparable but reversal swaps the readout labels incorrectly. In the second, reversal is equivariant but only the zero word is stable. Each requirement has a witness in the family, but no common setting satisfies both. A valid realization must belong to the intersection of the requirement sets.

The example motivates the joint feasible-set analysis in Section 9. It also distinguishes the present construction from a collection of mutually incompatible demonstrations on different molecules, environments, or timescales.

Convention 3.11 (Measurable realization)**.** Whenever probabilistic claims are invoked, the admitted physical states form a measurable space $(\Omega, \mathcal{F})$, the finite symbol space carries its discrete σ-algebra, and the symbol map q is measurable. Preparation, action, disturbance and readout maps used to form events are measurable on the declared protocol probability space. No probability-preserving action is assumed. These requirements are unnecessary for the purely set-theoretic finite coding results.

4 Codes on metric orbit spaces

The decoding principle is stated independently of the molecular setting. This separates its mathematical proof from the hypotheses needed to apply it. Throughout this section, G is a finite group acting isometrically on a metric space (V, d).

Definition 4.1 (Orbit distance)**.** For $x, y \in V$, define

$$\bar{d}([x], [y]) = \min_{g \in G} d(x, gy).$$

The minimum exists because G is finite. Write $\pi : V \to V/G$ for the orbit projection.

Lemma 4.2 (Metric descent)**.** *The function $\bar{d}$ is independent of the representatives and is a metric. Moreover, π is nonexpansive: $\bar{d}(\pi x, \pi y) \le d(x, y)$.*

Proof. If the representatives are replaced by hx and ky, then

$$d(hx, gky) = d(x, h^{-1}gky).$$

As g ranges through G, so does $h^{-1}gk$, proving independence. Symmetry follows from $d(x, gy) = d(g^{-1}x, y)$. Nonnegativity is inherited from d. If the minimum is zero, some g satisfies $x = gy$, which means the orbits agree; the converse is immediate. Choose g, h attaining the minima for $([x], [y])$ and $([y], [z])$. Then

$$\begin{aligned}\bar{d}([x], [z]) &\le d(x, ghz) \\ &\le d(x, gy) + d(gy, ghz) \\ &= \bar{d}([x], [y]) + \bar{d}([y], [z]).\end{aligned}$$

Finally the identity element is among the candidates in the minimum, giving the nonexpansive inequality. □

For infinite groups, replacing the minimum by an infimum may give only a pseudometric when distinct orbits have intersecting closures. That complication is absent here. The finite hypothesis has a specific role and is not silently discarded.

Definition 4.3 (Code distance and decoding neighborhood)**.** A code is a nonempty subset $C \subseteq V$. Its minimum distance is

$$d(C) = \inf\{d(c, c') : c, c' \in C,\ c \neq c'\},$$

All distance infima are taken in the extended nonnegative reals, with $\inf \varnothing = +\infty$; in particular this gives $+\infty$ for a singleton code or a singleton orbit image. For a nonnegative integer t, let

$$N_t(C) = \{y \in V : d(y, c) \le t \text{ for some } c \in C\}.$$

When the witnessing c is unique, write $D_C(y) = c$.

Lemma 4.4 (Unique bounded-distance decoding)**.** *If $d(C) \ge 2t + 1$, the map $D_C : N_t(C) \to C$ is defined and unique.*

Proof. Existence is the definition of the neighborhood. If two distinct words c, c' were each within t of y, the triangle inequality would give $d(c, c') \le 2t$, a contradiction. □

For a nonintegral metric the same proof works under $d(C) > 2t$ with real $t \ge 0$. The integer version is natural for Hamming distance.

Theorem 4.5 (Equivariant decoding)**.** *Suppose $gC = C$ for all $g \in G$ and $d(C) \ge 2t + 1$. Then:*

(i) $N_t(C)$ is invariant and $D_C(gy) = gD_C(y)$;

(ii) distinct code orbits in C/G have distance at least $2t + 1$;

(iii) D_C induces a unique radius-t decoder on $N_t(C)/G$;

(iv) if the radius-t balls about C partition V, the induced decoder is defined on all of V/G.

Proof. If $d(y, c) \le t$, then $d(gy, gc) \le t$ and $gc \in C$, giving invariance of the neighborhood. Uniqueness of the ordinary decoder implies $D_C(gy) = gc = gD_C(y)$. For distinct code orbits $[c], [c']$, every gc' belongs to C and differs from c, so every distance in the orbit minimum is at least $2t + 1$. Hence the same bounded-distance argument applies in the metric quotient. The induced map sends $[y]$ to $[D_C(y)]$, and equivariance proves that this definition is independent of the representative. If the ordinary balls partition V, every representative lies in the neighborhood, proving the final assertion. □

Corollary 4.6 (Orbit neighborhoods)**.** *Under the theorem's hypotheses,*

$$\pi(N_t(C)) = N_t(C/G).$$

For a fixed code orbit $[c]$, its radius-t quotient ball is the image of the union of the ordinary balls centered at the words in $[c]$.

Proof. If y is within t of a codeword, its orbit is within t of the corresponding code orbit. Conversely, if $\bar{d}([y], [c]) \le t$, some gc is within t of y. The same equivalence for one orbit proves the second statement. □

4.1 Why invariance is a substantive condition

Definition 4.7 (Saturation and compatibility defect)**.** The saturation of a code is $GC = \bigcup_{g \in G} gC$. Define

$$\delta_G(C) = \inf\{d(c, gc') : [c] \neq [c'],\ c, c' \in C,\ g \in G\}.$$

This is the minimum distance of $\pi(C)$ when that set has at least two elements.

The saturated code may acquire nearby words from different original coordinate systems. If those words lie in distinct orbits, the quotient itself loses distance. A nearby pair in the same orbit would not prove that stronger statement, so witnesses must be typed carefully.

Proposition 4.8 (Direct quotient criterion)**.** *Whether or not C is invariant, the orbit image $\pi(C)$ corrects every quotient error of radius t whenever $\delta_G(C) \ge 2t + 1$. If C is invariant, this lower bound follows from $d(C) \ge 2t + 1$.*

Proof. The first claim is the unique-decoding lemma in the quotient metric. The second is Theorem 4.5. No converse claiming that invariance is necessary for every quotient code is made. □

An invariant code is therefore a convenient sufficient design condition, not the only possible coding architecture. A designer could select fewer messages from a noninvariant family and verify their orbit distance directly. Such a selection changes the message set and should be included in the payload comparison.

4.2 Orbit counting and perfect codes

Proposition 4.9 (Burnside count)**.** *If a finite group acts on a finite set X, then*

$$|X/G| = \frac{1}{|G|}\sum_{g\in G} |\operatorname{Fix}_X(g)|.$$

Proof. Count pairs (g, x) satisfying $gx = x$. Counting by g gives the numerator. For an orbit O, its contribution when counting by x is $\sum_{x\in O} |\operatorname{Stab}(x)| = |O||G|/|O| = |G|$. Dividing by $|G|$ gives one per orbit. □

Proposition 4.10 (Quotient ball count)**.** *Suppose a finite code is invariant and is radius-t perfect, meaning that its radius-t ordinary balls partition V, for a nonnegative integer t. Let $B_t(c)$ be its ordinary decoding ball. The number of quotient received states decoded to $[c]$ is*

$$|B_t(c)/\operatorname{Stab}(c)|.$$

Proof. Every orbit in the union of balls with centers in $[c]$ has a representative in $B_t(c)$. If $y, y' \in B_t(c)$ and $y' = gy$, equivariance of decoding gives $c = D_C(y') = gD_C(y) = gc$, so $g \in \operatorname{Stab}(c)$. Thus exactly the stabilizer orbits inside $B_t(c)$ are identified. This gives the asserted bijection. □

The formula explains why orbit balls need not have a common cardinality even when ordinary balls do. A fixed codeword has a larger stabilizer than a generic codeword. Ignoring this difference can produce an incorrect orbit-space packing count.

4.3 Encoding an orbit message

The quotient code is a message set, not necessarily a vector space. An encoder may choose a representative of each orbit and prepare that representative. A different choice of representatives describes the same orbit messages if the physical realization respects the group action. Uniform sampling of codewords and uniform sampling of code orbits are generally different distributions because orbit sizes vary. Section 8 keeps those distributions separate.

5 A quaternary reference code and its obstruction

Figure 2 previews the explicit compatibility defect and the algebraic reassignment that repairs it.

5.1 Field and matrix conventions

Notation 5.1. Let $K = \mathbb{F}_2[\omega]/(\omega^2+\omega+1)$. Thus $\omega^2 = \omega+1$, $\omega^3 = 1$, and the characteristic is two. Write $V = K^5$. Vectors are columns for parity checks. Let R denote both the reversal map on V and its permutation matrix.

The arithmetic is exact. In particular, a plus sign is not a real-valued sum when it appears inside a field matrix. Appendix A gives the operation tables and computational conventions used by the checker.

The following reference matrix is the five-contact H_5 construction already used in Article 98 [Sch26g]. Its classical Hamming-code parameters are inherited, not claimed as a new code family. The contribution here is the representation-sensitive reversal obstruction, its compatible coordinate reassignment, and the induced orbit decoder. Define

$$H = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & \omega & \omega^2 \end{pmatrix}, \qquad C = \ker H. \tag{4}$$

Its columns are representatives of the five points of the projective line over K. The first two columns are independent. No column is zero and no two columns are proportional.

Proposition 5.2 (Parameters and perfection)**.** *The code C has dimension three, cardinality 64, and minimum distance three. Its radius-one balls partition K^5.*

Proof. Rank–nullity gives dimension $5-2 = 3$. A weight-one kernel vector would give a zero column after multiplication by a nonzero scalar, impossible. A weight-two kernel vector would make two columns proportional, also impossible. Three columns in a two-dimensional vector space are dependent; taking the first, second, and third gives the explicit relation $h_1 + h_2 + h_3 = 0$. Hence $(1, 1, 1, 0, 0)$ is a weight-three word and the minimum distance is exactly three.
Each radius-one ball contains its center and $5(4-1) = 15$ one-coordinate substitutions. Different balls are disjoint by the distance bound. There are $64 \cdot 16 = 1024 = 4^5$ words in their union, so the union is all of V. □

Remark 5.3 (Unique syndrome representatives)**.** Every nonzero syndrome $s \in K^2$ can be written in a unique form $s = ah_i$ with $a \in K^\times$ and $1 \le i \le 5$. Projective complete-

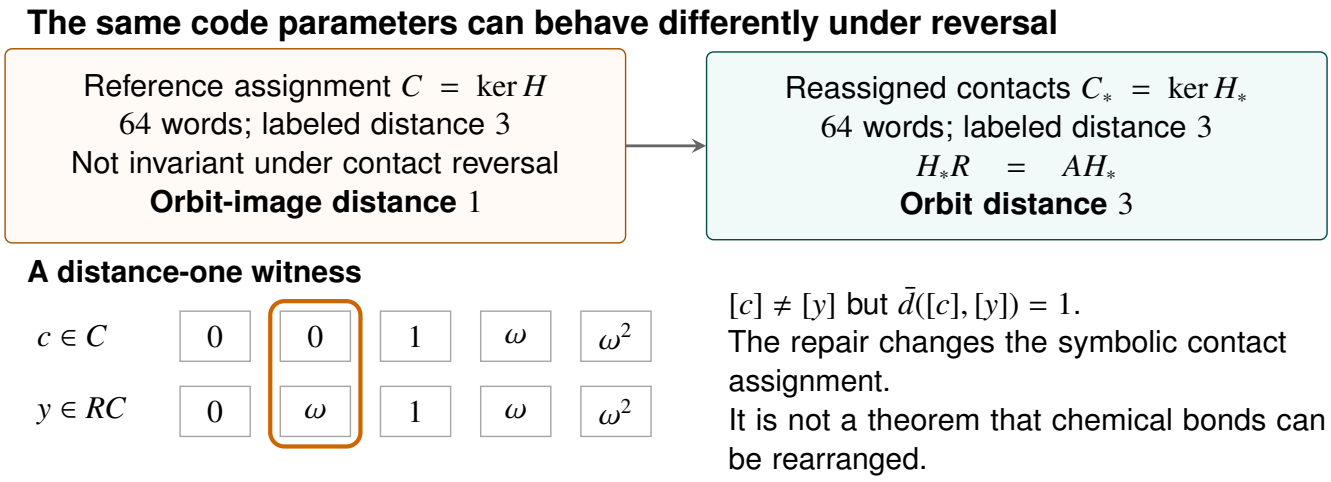


Figure 2: A reversal incompatibility and its algebraic repair. The highlighted coordinate gives the explicit distance-one witness of Proposition 5.4. Exchanging the second and third parity-check columns yields the invariant code of Theorem 6.1; implementation requires an admissible physical contact assignment.

ness gives existence, and pairwise nonproportionality gives uniqueness. This is the algebraic basis of the one-symbol decoder.

For $y \in V$, define an error representative $e(Hy)$ to be zero when $Hy = 0$, and otherwise the weight-one vector ae_i for the unique pair above. Then

$$D_C(y) = y - e(Hy).$$

Subtraction equals addition in characteristic two. The formula is defined on every word, but its operational correctness for a transmitted word is guaranteed only within the one-symbol radius. At larger error weights it may return a different codeword.

5.2 The explicit compatibility defect

Proposition 5.4 (Failure after reversal identification)**.** *The code C is not invariant under R. Both its saturation $C \cup RC$ and its orbit image in $V/\langle R\rangle$ have minimum distance one.*

Proof. Set

$$c = (0, 0, 1, \omega, \omega^2), \qquad y = (0, \omega, 1, \omega, \omega^2).$$

Direct field arithmetic gives $Hc = 0$, $HRy = 0$, and $Hy = (0, \omega) \neq 0$. Hence $c \in C$, $y \in RC \setminus C$, and $d_H(c, y) = 1$. This proves noninvariance and the distance-one assertion for the saturation.
For the quotient, $[y] = [Ry]$ is the orbit of a codeword. It is distinct from $[c]$: $y \neq c$, and $Ry = (\omega^2, \omega, 1, \omega, 0)$ is not c. Thus $\bar{d}([c], [Ry]) = 1$. Distinct orbits have positive integral distance, so the quotient minimum is exactly one. □

The proposition is not merely an observation that some particular decoder fails. Distance one means that there are distinct intended orbit messages which cannot enjoy a uniform one-symbol correction guarantee. The obstruction belongs to the code and the identification, independently of the implementation of a decoding algorithm.

Lemma 5.5 (Intersection dimension)**.** $|C \cap RC| = 4$.

Proof. A word $x = (x_1, \ldots, x_5)$ lies in the intersection precisely when $Hx = 0$ and $HRx = 0$. The first equations in the two systems imply $x_2 = x_4$. Write this common value as b. The second equation of $Hx = 0$ becomes $x_3 = \omega^2(b + x_5)$; the second equation of $HRx = 0$ becomes $x_3 = \omega^2(x_1 + b)$. Thus $x_1 = x_5 = a$. The first equation then gives $x_3 = b$. Combining with the expression for x_3 gives $b = \omega^2(b + a)$, or $b = \omega a$. Hence

$$x = (a, \omega a, \omega a, \omega a, a), \qquad a \in K.$$

There are exactly four such vectors. □

The intersection count is useful for regression testing, but it is not a substitute for the distinct-orbit witness in Proposition 5.4. The witness proves the performance obstruction; the rank or count test confirms additional structure.

5.3 What the obstruction does and does not say

The oriented code remains a valid perfect code. The problem arises only when a particular reversal action is imposed as an identification of messages. A device with a stable orientation reference could continue to use the original oriented code. A device that cannot retain that reference must instead examine the orbit messages or change the encoding. A physical orientation marker therefore has an informational resource cost which must be counted in a fair comparison.

The obstruction also does not assert that molecular reversal is necessarily R. If calibration gives $R\sigma$, the relevant theorem is different. The representation must be determined before the code is judged. Selecting the representation that makes a favored construction work would reverse the evidentiary order.

Finally, the coordinate permutation used below is an algebraic change of address assignment. It does not rearrange chemical bonds by theorem. If the five contacts have fixed roles that cannot be reassigned, the repair may be unavailable in that implementation. That possibility is a physical rejection of the repair hypothesis, not a contradiction in the finite construction.

6 An invariant construction and perfect orbit decoding

Figure 3 displays the orbit decomposition established below.

6.1 The repaired parity check

Exchange the second and third columns of H and set

$$H_* = \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & \omega & \omega^2 \end{pmatrix}, \qquad C_* = \ker H_*. \tag{5}$$

Let

$$A = \begin{pmatrix} 1 & 0 \\ \omega^2 & 1 \end{pmatrix}.$$

The field identity $\omega^2 + \omega = 1$ gives the covariance relation

$$H_* R = A H_*, \qquad A^2 = I. \tag{6}$$

This relation is checked column by column. It is stronger than a numerical equality of dimensions: it identifies the action on the syndrome space.

Theorem 6.1 (Invariant perfect code)**.** *C_* is a perfect $[5, 3, 3]_4$ code, is invariant under reversal, and its unique one-symbol decoder satisfies*

$$D_*(Ry) = R D_*(y) \qquad (y \in V).$$

Proof. A coordinate permutation is a linear Hamming isometry, so the parameters and perfection follow from Proposition 5.2. If $H_* c = 0$, then $H_* R c = A H_* c = 0$, proving invariance. Reversal is an isometry; therefore it maps the unique codeword within one of y to a codeword within one of Ry. Uniqueness gives the displayed decoder identity. Since the code is perfect, the argument applies to every received word. □

Proposition 6.2 (Syndrome covariance and correction)**.** *Let $s(y) = H_* y$ and let $e(s)$ be the unique weight-at-most-one error with syndrome s. Then*

$$s(Ry) = A s(y), \qquad e(As) = R e(s).$$

Consequently the syndrome decoder $y \mapsto y - e(s(y))$ is reversal equivariant.

Proof. The first identity is Equation (6). The vector $Re(s)$ has weight at most one and syndrome As. Uniqueness of the weight-at-most-one representative gives the second identity. Substituting both identities in the decoder formula proves equivariance. □

A readout of a syndrome need not identify an orientation-specific representative if the decoder is designed at the orbit level. However, it must still distinguish the syndrome information needed by the chosen instrument. Algebraic covariance does not supply an experimental measurement channel.

The coordinate permutation defining the repaired code is a choice of encoding assignment, not a theorem that chemically unlike contacts can be exchanged. Passive relabeling, rigid reorientation, and chemical rearrangement are distinct operations. The realization must specify the applicable action, including terminal groups and measurement geometry, and verify the stated local-symbol transformation. Terminal or readout asymmetry can invalidate a pure-reversal identification without invalidating the conditional finite theorem.

6.2 Parametrization and fixed words

The parity equations can be solved without enumeration. For free variables $a = x_3$, $b = x_4$, $c = x_5$, one obtains

$$(x_1, x_2, x_3, x_4, x_5) = (a + \omega^2 b + \omega c,\ a + \omega b + \omega^2 c,\ a, b, c). \tag{7}$$

This yields an explicit linear encoder from K^3. It is a systematic encoder with information coordinates $3, 4, 5$, but those coordinate choices are part of the model rather than a claim about a synthesis sequence.

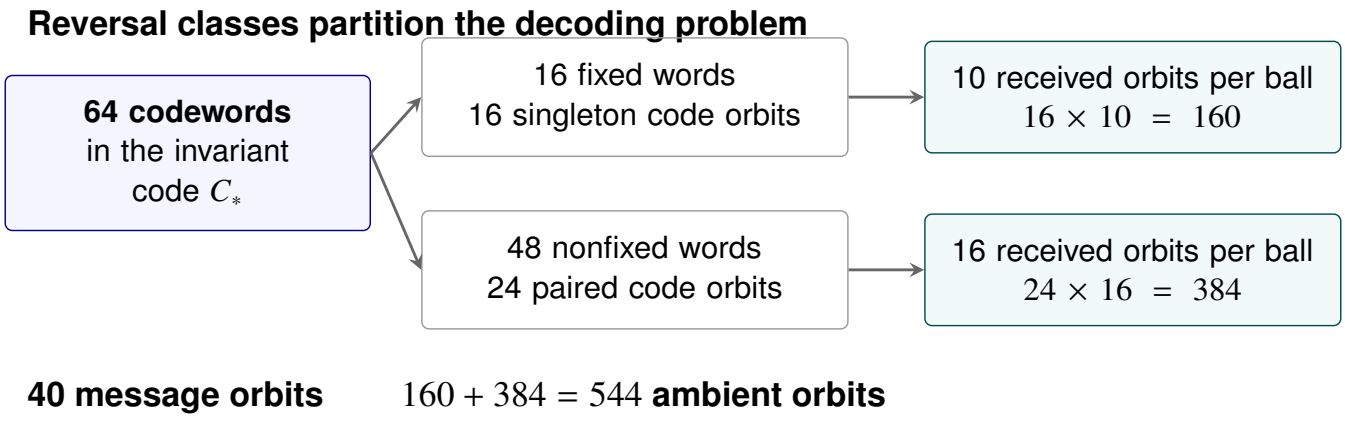


Figure 3: Orbit counts and the perfect quotient partition. The two kinds of code orbit have different radius-one ball sizes, as proved in Corollary 6.5. Counts of admissible messages do not specify their source probabilities.

Proposition 6.3 (Fixed subcode and orbit count)**.** *The reversal-fixed subcode is*

$$\mathrm{Fix}_{C_*}(R) = \{(a, b, \omega^2(a+b), b, a) : a, b \in K\}.$$

It has sixteen words. The code has forty reversal orbits: sixteen singletons and twenty-four two-element orbits.

Proof. A fixed word has the form (a, b, c, b, a). The first parity equation is $a + b + b + a = 0$, which is automatic. The second is $b + c + \omega b + \omega^2 a = 0$, hence $c = \omega^2(a + b)$. The free pair (a, b) gives sixteen words. The other 48 codewords form pairs under the involution, giving $16 + 48/2 = 40$ orbits. □

Theorem 6.4 (Perfect quotient decoding)**.** *The quotient code $C_*/\langle R\rangle$ has forty messages and minimum orbit distance three. It corrects every radius-one quotient error. Its decoding balls partition all 544 orbits of K^5.*

Proof. The distance is at least three by Theorem 4.5. The zero word is fixed and the code contains a weight-three word; its orbit distance to zero is three, so the minimum is exactly three. Perfection descends by the same theorem. The ambient fixed words have form (a, b, c, b, a), giving $4^3 = 64$ fixed words. Burnside's formula gives

$$|K^5/\langle R\rangle| = (4^5 + 4^3)/2 = 544.$$

□

Corollary 6.5 (Nonuniform orbit spheres)**.** *Each fixed codeword gives a quotient radius-one ball of size ten. Each nonfixed codeword orbit gives such a ball of size sixteen. Thus*

$$16 \cdot 10 + 24 \cdot 16 = 544.$$

Proof. An ordinary radius-one ball has sixteen elements. If its center is fixed, the fixed received words in that ball are its center and the three changes of its middle coordinate. Any change of an off-center coordinate alone destroys reversal symmetry. Burnside therefore gives $(16 + 4)/2 = 10$. If the center is not fixed, its two ordinary decoding balls are disjoint and exchanged by reversal. Their union has 32 elements and no fixed points: a fixed received word would decode equivariantly to a fixed codeword, contradicting the center's orbit. Hence their quotient has sixteen elements. □

6.3 A worked decoding example

Take the zero codeword and an error of value ω at coordinate two. Then

$$y = (0, \omega, 0, 0, 0), \qquad s(y) = (\omega, \omega).$$

Reversal moves the error to coordinate four. Its syndrome is $\omega(1, \omega) = (\omega, \omega^2)$. The action on the syndrome agrees:

$$A(\omega, \omega)^{\mathsf{T}} = (\omega, \omega^2\omega + \omega)^{\mathsf{T}} = (\omega, \omega^2)^{\mathsf{T}}.$$

The two received words are in the same orbit and both decode to zero. More generally, adding a codeword shifts the example to that codeword, and reversal simultaneously transforms the center and the error.

The example illustrates why a covariance identity is operationally preferable to a statement that "both orientations have the same error rate." Covariance specifies how the measured syndrome changes. Equal rates alone would leave the decoder's interpretation unresolved.

6.4 Payload and symmetry

Proposition 6.6 (Uniform words are not uniform orbit messages)**.** *If a word is chosen uniformly from C_* and only its orbit is retained, each fixed orbit has probability* $1/64$ *and each nonfixed orbit has probability* $1/32$*. The resulting orbit entropy in bits is* $21/4$*.*

Proof. A singleton has one preimage and a two-element orbit has two. Thus the entropy is

$$-16\frac{1}{64}\log_2\frac{1}{64} - 24\frac{1}{32}\log_2\frac{1}{32} = \frac{16\cdot 6}{64} + \frac{24\cdot 5}{32} = \frac{21}{4}.$$

□

Uniformly chosen orbit messages instead have entropy $\log_2 40$. They require a different preparation distribution: choose the orbit uniformly, then choose any admitted representative according to a declared rule. Neither distribution is automatically produced by an endogenous source. The count forty describes possible messages, not their preparation frequencies.

A fair engineering comparison must state whether orientation is a stored reference, an uncontrolled nuisance, or a gauge identification. It must also count any marker, readout, or control resource used to preserve orientation. The quotient construction removes a need for an orientation-specific message label within its assumptions; it does not prove a net energy, speed, or material advantage for a physical device.

The distinct message conventions are compared in Table 1. In particular, the reference orbit image is not the same object as a group quotient of an invariant code.

7 A semilinear obstruction

7.1 The changed action

The field automorphism $\sigma : K \to K$, $a \mapsto a^2$, fixes $0, 1$ and exchanges ω, ω^2. Apply it coordinatewise to V and define $T = R\sigma$. Since reversal commutes with coordinatewise squaring and $\sigma^2 = \mathrm{id}$, the map T is an involutive Hamming isometry. It is additive over $\mathbb{F}_2$ but not K-linear.

The fact that both R and T are involutions does not make them interchangeable. They have different actions on the projective columns of a parity-check matrix. The obstruction below is a consequence of that representation difference.

Definition 7.1 (Semilinear monomial map)**.** A semilinear monomial map with coordinate permutation p has the form

$$T_{D,p}(x) = DP_p\sigma(x),$$

where P_p is a permutation matrix and D is an invertible diagonal matrix over K. Coordinate scalings are permitted, but no coordinate is deleted or duplicated.

7.2 Projective columns

Lemma 7.2 (Projective completeness)**.** *If a K-linear code has parameters $[5, 3, 3]_4$, any rank-two parity-check matrix has five nonzero pairwise nonproportional columns. These columns represent every point of $\mathbb{P}^1(K)$ exactly once.*

Proof. A zero column gives a weight-one codeword. Two proportional columns give a nonzero linear relation supported on those two positions, hence a weight-two codeword. Both contradict distance three. The number of one-dimensional subspaces of K^2 is $(4^2 - 1)/(4 - 1) = 5$, so the five distinct projective columns exhaust the line. □

We identify the projective line with $K \cup \{\infty\}$ by the usual affine coordinate. An invertible 2×2 matrix acts by a fractional linear transformation. Scalar matrices act trivially, giving the projective linear group $\mathrm{PGL}(2, 4)$.

Lemma 7.3 (Parity of projective linear maps)**.** *Every element of* $\mathrm{PGL}(2, 4)$ *induces an even permutation of the five points of $\mathbb{P}^1(K)$. The coordinatewise Frobenius map induces an odd permutation. Therefore every projective semilinear map with nontrivial field automorphism induces an odd permutation.*

Proof. Fractional linear transformations are generated by translations $x \mapsto x + b$, nonzero scalings $x \mapsto ax$, and inversion $x \mapsto 1/x$. Indeed, a transformation with denominator constant is affine. If its denominator is nonconstant, division expresses it as an affine constant plus a nonzero multiple of the inverse of an affine function, so it belongs to the generated group.

A nonzero translation of K has no fixed affine point and order two. It fixes ∞, and therefore acts as two disjoint transpositions on the four affine points. A nonidentity scaling fixes 0 and ∞ and cycles the three nonzero elements. Inversion exchanges 0 with ∞, fixes 1, and exchanges ω with ω^2. These permutations are all even. Hence every product of them is even.

Frobenius fixes $0, 1, \infty$ and exchanges ω, ω^2, so it is odd. Every nontrivial semilinear projective map is a projective linear

Table 1: Parameters under labeled and reversal-identified message conventions. All distances are symbolic Hamming or orbit distances.

Message construction	Words in code	Message labels	Minimum distance	Corrects one symbol
Reference code C, with orientation retained	64	64	3	Yes
Orbit image $\{[c] : c \in C\}$	64	64	1	No
Repaired code C_*, with orientation retained	64	64	3	Yes
Invariant quotient $C_*/\langle R\rangle$	64	40	3	Yes

The reference orbit-image count is 64 because its only codewords with reversed partners in C are the four palindromic words of $C \cap RC$. The repaired quotient has 16 singleton and 24 paired messages. "Corrects" denotes the uniform radius-one guarantee in the specified message model.

map composed with Frobenius. Its sign is the product of an even sign and an odd sign, hence odd. □

For reference, $|\mathrm{GL}(2,4)| = (16-1)(16-4) = 180$, and division by the three nonzero scalar matrices gives $|\mathrm{PGL}(2,4)| = 60$. Its faithful action is the alternating group on five points, but the parity proof above does not require importing that group identification.

7.3 The no-go theorem

Theorem 7.4 (Frobenius-reversal obstruction)**.** *No K-linear $[5,3,3]_4$ code is invariant under $R\sigma$. More generally, no such code is invariant under a semilinear monomial map whose coordinate permutation is the reversal R, regardless of its nonzero coordinate scalings.*

Proof. Suppose $C = \ker H$ has the stated parameters and is invariant under $DP\sigma$, where $P = R$. Squaring the equation $Hx = 0$ shows that

$$\sigma(C) = \ker \sigma(H).$$

Invariance gives $DP\sigma(C) = C$, and therefore

$$\ker(HDP) = \ker \sigma(H).$$

Both matrices have rank two. Two full-row-rank matrices with the same kernel have the same row space, so there is $B \in \mathrm{GL}(2,K)$ such that

$$HDP = B\sigma(H). \tag{8}$$

For completeness, the row-space statement follows because each row annihilates the common kernel; its annihilator has dimension two, and both row spaces are two-dimensional subspaces of it.

Let h_i be the columns of H. Column i of HDP is a nonzero scalar multiple of $h_{p(i)}$. Equation (8) therefore implies, on projective points,

$$[h_{p(i)}] = [B\sigma(h_i)].$$

By Lemma 7.2, the labels $[h_i]$ enumerate all five projective points. Thus the permutation induced by the semilinear projective map $[v] \mapsto [B\sigma(v)]$ is conjugate to p. Lemma 7.3 makes this permutation odd. But reversal of five positions is $(1\ 5)(2\ 4)$, an even permutation. Parity is invariant under relabelling. This contradiction proves both assertions; coordinate scalings have disappeared on passage to projective points. □

Corollary 7.5 (Representation-sensitive design fork)**.** *Under the pure-reversal hypotheses, C_* supplies perfect quotient single-error correction. Under the Frobenius-reversal hypotheses and the linear $[5,3,3]_4$ requirement, no invariant code exists. A change between these two calibrated actions changes the admissible design class.*

Proof. The first statement is Theorem 6.4; the second is Theorem 7.4. □

7.4 Scope of the obstruction

The theorem does not exclude every orientation-free code under a semilinear action. It excludes the specified K-linear, length-five, dimension-three, distance-three class. A nonlinear code, a smaller message set, a longer word, an orientation marker, or a different local alphabet can change the problem. Such alternatives are design responses, not counterexamples to the theorem.

Nor does the theorem determine the physical action. It provides a discriminating consequence once that action is calibrated. A molecular observation of Frobenius-reversal would reject the proposed pure-reversal implementation under the given encoding, even though it would not reject the entire Chrysene Formalism. Conversely, observing pure reversal would remove this particular semilinear obstruction but would not establish preparation, lifetime, or readout adequacy.

Remark 7.6 (Novelty and classical theory)**.** The proof uses classical projective descriptions of linear codes, finite-field automorphisms, and permutation parity. Reversible coding theory predates the present application [Mas64]. A claim

of literature-wide priority for this precise specialization requires a broader comparison than the finite proof itself. The present manuscript advances it as an explicit compatibility analysis within the molecular framework, with the assumptions and obstruction stated in a form that can be independently checked.

8 Disturbances, source distributions, and robust guarantees

Figure 4 compares the exact channel-success formulas derived in this section.

8.1 Adversarial and stochastic statements

A distance theorem quantifies over every admitted error of bounded weight. A probability theorem quantifies over a specified distribution. These statements should not be exchanged. Perfect radius-one decoding means that every received word has a unique nearby codeword; it does not mean that the decoded codeword is the one originally transmitted after arbitrary errors.

Proposition 8.1 (Guarantee under independent substitutions)**.** *For $0 \le p \le 1$, suppose each of the five symbols is independently replaced by a different symbol with probability p. For any transmitted word of C_*, ordinary decoding, and hence orbit decoding, is correct whenever at most one replacement occurs. Its success probability is therefore at least*

$$(1-p)^5 + 5p(1-p)^4.$$

Proof. The number of replaced coordinates has the binomial distribution with parameters five and p. The displayed expression is its probability of being at most one. On that event, Theorem 6.1 gives correct word decoding, which implies correct orbit decoding. Events with two or more replacements may sometimes decode to the same orbit, so the expression is a lower bound rather than a general exact success probability. □

The distribution among the three possible replacement values does not affect this lower bound, provided the replacement event means a genuinely different symbol. Independence is used only for the binomial formula, not for the deterministic theorem.

Proposition 8.2 (Correlated errors)**.** *Let G be the event of correct preparation, action calibration, and readout, with $\Pr(G) > 0$. Let E be any random error pattern on the five contacts. If $\Pr(\mathrm{wt}(E) > 1 \mid G) \le \varepsilon$, then $\Pr(\textit{correct decoding} \mid G) \ge 1 - \varepsilon$.*

Proof. Within G, the complement of the event of more than one change is contained in the success event by the deterministic theorem. Apply the bound under the conditional probability law given G. No factorization of the error distribution is used. □

This form is appropriate when a common environmental fluctuation correlates contacts. Estimating only five marginal error rates cannot determine the probability of a multi-contact event without a dependence model or a bound.

8.2 Exact success under a symmetric channel

The preceding lower bound can be sharpened when each changed symbol is chosen uniformly from its three alternatives and contact errors are independent. For $0 \le p \le 1$, the transition kernel is

$$W_p(y \mid c) = (1-p)^{5-d(c,y)}(p/3)^{d(c,y)},$$

with endpoint values given by the product of coordinate probabilities (equivalently $0^0 = 1$ in this expression).

Lemma 8.3 (Representative-independent channel success)**.** *The kernel satisfies $W_p(Ry \mid Rc) = W_p(y \mid c)$. Under equivariant decoding, orbit success is constant on each transmitted code orbit. Consequently, uniform orbit sampling followed by either representative or any mixture within that orbit gives the same orbit-success average.*

Proof. Reversal preserves Hamming distance, giving the kernel identity. The bijection $y \mapsto Ry$ carries the success event for c onto the success event for Rc, since the decoder is equivariant. Summing the equal transition probabilities proves the assertion, and mixtures preserve that common value. □

Put

$$\begin{aligned} B(p) &= (1-p)^5 + 5p(1-p)^4, \\ Q(p) &= 4(p/3)^3(1-p)^2 \\ &\qquad + 9(p/3)^4(1-p) + 3(p/3)^5. \end{aligned}$$

Theorem 8.4 (Exact orbit success)**.** *Under this symmetric channel and ideal preparation, action, and readout, the orbit success probability for a transmitted $c \in C_*$ is $B(p)$ if $Rc = c$, and $B(p) + Q(p)$ otherwise. Uniform codeword sampling gives $B(p) + \frac{3}{4}Q(p)$; uniform code-orbit sampling gives $B(p) + \frac{3}{5}Q(p)$.*

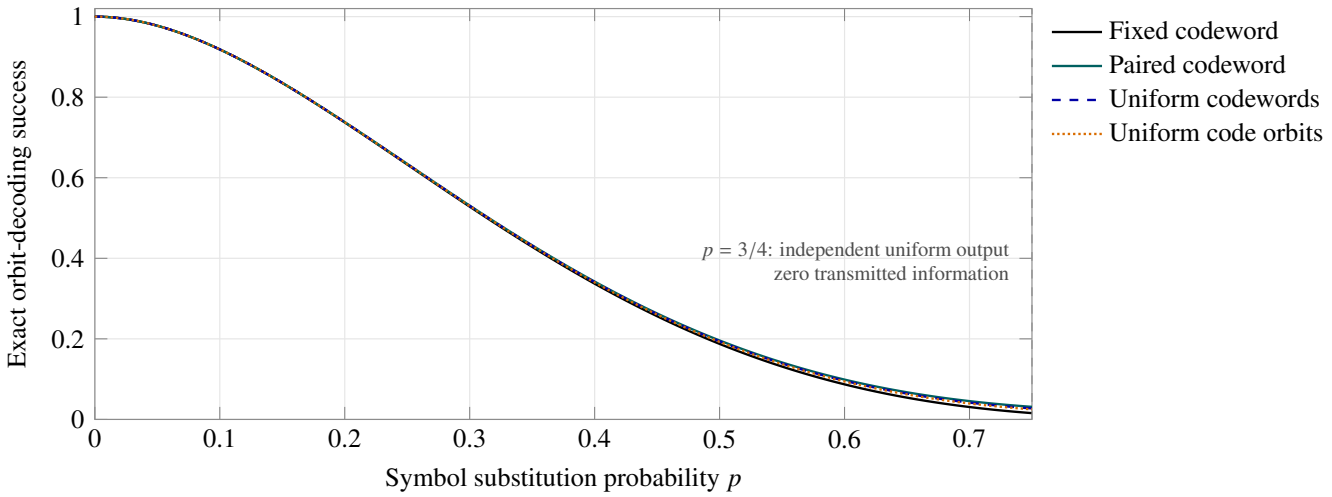


Figure 4: Exact success probabilities for independent symmetric four-symbol substitutions and ideal preparation, action and readout. The solid curves show fixed and nonfixed codewords; the dashed and dotted curves average over different priors. At $p = 3/4$ the received word is independent of the input: nonzero chance success is not transmitted information. These are model calculations, not experimental measurements.

Proof. The successful received words form $B_1(c)$ when c is fixed, and the disjoint union $B_1(c) \cup B_1(Rc)$ otherwise. Reversal changes positions in symmetric pairs, so $d(c, Rc)$ is even. For distinct codewords it is at least three and at most four, hence equals four.
Relative to c, the second ball has four words of weight three, nine of weight four, and three of weight five. Indeed, its center has weight four. Changing one of its four differing coordinates to the corresponding coordinate of c gives four weight-three words; changing that coordinate to either other value gives eight weight-four words. Changing the single matching coordinate gives three weight-five words. Adding the center gives nine at weight four.
A specified received word at distance j has probability $(p/3)^j(1-p)^{5-j}$. Summing these shells gives $Q(p)$ in addition to $B(p)$. There are 48 nonfixed words out of 64 and 24 nonfixed orbits out of 40, giving the stated averaging factors. □

In particular, no exactly-two-symbol error is successful in this model, although some errors of weights three through five preserve the decoded orbit. The different averaging factors show again why uniform words and uniform messages cannot be identified.

8.3 The maximally randomizing limit

Proposition 8.5 (No information from independent uniform output)**.** *If the received word Y is uniform on K^5 independently of the transmitted message M, any deterministic decoded orbit $f(Y)$ is independent of M. In particular, the code does not transmit information in this channel.*

Proof. For every message m and decoded value z, $\Pr(f(Y) = z \mid M = m) = \Pr(f(Y) = z)$ by independence of Y and M. Thus their joint distribution factors. The mutual information is zero by its definition. □

For the symmetric four-symbol substitution channel, uniform independent output occurs at $p = 3/4$, not at $p = 1/2$. At that point, ordinary perfect-code decoding assigns each of the 64 codewords probability $1/64$ because each has sixteen received words. The orbit distribution is consequently the nonuniform distribution in Section 6. It is a property of the decoder's partition, not recovered information about the input.

8.4 A common-experiment error budget

Definition 8.6 (Four good events)**.** On one probability space describing one experimental protocol, let E_P mean correct code-state preparation; E_A mean that the realized action and encoding agree on that trial; E_N mean that the symbol disturbance is in the admitted radius; and E_O mean ade-

quate readout. All four events refer to the same preparation, operating conditions, and observed trial.

Theorem 8.7 (Robust implementation bound)**.** *Suppose the intersection $E_P \cap E_A \cap E_N \cap E_O$ implies correct orbit decoding, and $\Pr(E_j^c) \le \varepsilon_j$ for $j = P, A, N, O$. Then*

$$\Pr(\textit{correct decoding}) \ge \max\{0, 1 - \varepsilon_P - \varepsilon_A - \varepsilon_N - \varepsilon_O\}.$$

No independence assumption is required.

Proof. Failure is contained in $E_P^c \cup E_A^c \cup E_N^c \cup E_O^c$. The union bound gives failure probability at most the sum of the four error bounds. Subtract from one and use nonnegativity of probability. □

Corollary 8.8 (Sequential calibration)**.** *The same sum bound holds if preparation has failure probability at most ε_P, and each later stage has conditional failure probability at most its ε_j given success of all preceding stages.*

Proof. Partition failure according to the first failed stage. The probability of first failure at stage j is the probability of all previous successes times the conditional failure probability, and is therefore at most ε_j. The disjoint first-failure probabilities sum to at most the stated total. □

Bounds obtained on unrelated specimen populations cannot simply be inserted into this formula. A preparation condition that changes the state distribution can change the readout and noise statistics. The common-experiment requirement is part of the theorem's applicability, not an optional reporting convention.

8.5 Approximate equivariance

If the symbol map is finite-valued, "approximately equivariant" can mean that Equation (3) holds with high probability on a declared calibration distribution. It can also mean that the two image symbols differ in at most s coordinates. These notions lead to different guarantees.

Proposition 8.9 (Deterministic error-margin composition)**.** *Suppose the discrepancy between the actual and intended symbol action is bounded by Hamming distance s, and all subsequent symbol disturbance and readout discrepancies together have distance at most $t - s$, where $0 \le s \le t$. Then the total discrepancy from the intended codeword orbit has quotient distance at most t.*

Proof. Choose the intended representative supplied by the action model. The triangle inequality bounds the total symbol discrepancy by the sum $s + (t - s) = t$. Passing to the quotient cannot increase distance by Lemma 4.2. □

An analog closeness bound does not imply this hypothesis without a calibrated margin separating symbol basins. In particular, an arbitrarily small analog perturbation can cross a decision boundary. A proposed implementation should either keep a positive margin or include boundary events in its failure probability.

8.6 Preparation laws and deterministic architecture

Let μ be a probability measure on the admitted physical state set. The source's code preparation yield is $\mu(q^{-1}(C_*))$ when the symbol map is defined on the sampled states. Conditioning on this event changes the distribution of codewords. If a source gives most mass to one code orbit, the number forty is a capacity of the available message set, not its realized entropy.

Proposition 8.10 (Conditioning a successful source)**.** *Let $S = q^{-1}(C_*)$ have positive probability. If the four implementation premises hold on S, then the deterministic correction statement holds for every prepared state in S within the admitted error class, independently of the conditional distribution $\mu(\cdot \mid S)$.*

Proof. The implementation theorem is pointwise on the successful preparation set. Integrating a true pointwise statement against any probability law on that set preserves it. The value of the yield and the distribution among messages remain separate quantities. □

This is the precise sense in which deterministic structure and stochastic origin can coexist. An endogenous source and an externally placed source may share a downstream code theorem while having radically different yields, resource costs, and historical relevance.

9 Joint realization and observable rejection

9.1 One realization for several requirements

The common-realization perspective is inherited from the finite formalism and its consistency analysis [Sch26a]. Here it is specialized to geometry, action, preparation, disturbance, and readout for one coding architecture.

Definition 9.1 (Joint admissibility)**.** Let Θ be a declared set of environmental and operational conditions. For $\theta \in \Theta$, let $\mathcal{R}(\theta)$ be the candidate realization set, and let $H_j(r, \theta)$ be the requirement predicates for geometry, action, preparation, disturbance, and observation. Define

$$\mathcal{R}_{\text{joint}}(\theta) = \{r \in \mathcal{R}(\theta) : \bigwedge_j H_j(r, \theta)\}.$$

The ambient candidate set must be specified before assessing whether the intersection is empty. Otherwise one can evade every failed test by adding a new realization without recording that the model has changed. Enlargements are legitimate research steps, but they produce a new model version with new coverage obligations.

Proposition 9.2 (Separate witnesses do not imply a joint witness)**.** *Nonemptiness of every individual requirement set does not imply nonemptiness of* $\mathcal{R}_{\text{joint}}(\theta)$*.*

Proof. Take $\mathcal{R} = \{r_1, r_2\}$, let the first requirement hold only at r_1, and the second only at r_2. Each requirement has a witness, but their intersection is empty. Additional requirements can be made universal without changing the example. □

The proposition is logically elementary. Its scientific use is to demand that jointly invoked measurements refer to compatible specimens and conditions. A thermal lifetime measured for one substitution pattern and a symmetry action measured for another cannot be combined without an argument that transports both properties to a common object.

9.2 Feasible observable regions

Definition 9.3 (Observable image and uncertainty set)**.** Let $F : \mathcal{R}_{\text{joint}}(\theta) \to \mathbb{R}^m$ be a declared observation map. Its feasible image is $\mathcal{Y}(\theta)$. An observation is represented by a set $U \subseteq \mathbb{R}^m$ of values consistent with its uncertainty model.

Proposition 9.4 (Set-valued rejection)**.** *If* $U \cap \mathcal{Y}(\theta) = \varnothing$*, no realization in the declared joint model explains the observation within* U*. If* $\mathcal{Y}(\theta) \subseteq \mathcal{Y}_{\text{out}}(\theta)$ *and* $U \cap \mathcal{Y}_{\text{out}}(\theta) = \varnothing$*, the same conclusion follows.*

Proof. A compatible realization would have its image in both U and $\mathcal{Y}$, contradicting disjointness. The outer-set claim follows from inclusion. □

An outer approximation has a one-sided logical role. Disjointness from it is a valid rejection certificate. Intersection with it is not an existence proof in the original model. This distinction is crucial when convexification or interval relaxation is used to make a problem computationally tractable.

When U is statistically estimated, exclusion inherits its stated coverage and the adequacy of the observation model; it is not an unconditional logical refutation from a finite sample. Nuisance parameters, systematic error and model discrepancy must be included in the declared uncertainty construction. Guaranteed error bounds and confidence regions must be identified as such, with their scope or coverage stated.

9.3 Finite common-realization certificates

Theorem 9.5 (Linear rejection certificate)**.** *Suppose the candidate observable vectors are* $y_1, \dots, y_N \in \mathbb{R}^m$*, and let* $\mathcal{Y} \subseteq \operatorname{conv}\{y_1, \dots, y_N\}$*. Let* $a \in \mathbb{R}^m$ *and* $b \in \mathbb{R}$ *satisfy* $a \cdot y_i \le b$ *for all* i*. If* $a \cdot u > b$ *for every* $u \in U$*, then* $U \cap \mathcal{Y} = \varnothing$*.*

Proof. Every convex combination $y = \sum_i \lambda_i y_i$ with $\lambda_i \ge 0$ and $\sum_i \lambda_i = 1$ satisfies $a \cdot y = \sum_i \lambda_i (a \cdot y_i) \le b$. The two sets therefore lie on opposite sides of the strict inequality. □

Corollary 9.6 (Coordinate uncertainty)**.** *If* $U = \{u : |u_j - \hat{y}_j| \le \delta_j\}$ *and*

$$a \cdot \hat{y} - \sum_j |a_j| \delta_j > b,$$

the same rejection follows.

Proof. For every $u \in U$, $a \cdot u \ge a \cdot \hat{y} - \sum_j |a_j| \delta_j > b$. □

The quantities a, b can be rational and checked by exact arithmetic when the finite model is rational. A numerical optimization result can propose a separator, but its inequalities should be certified with directed bounds or exact arithmetic before it is used as a rejection proof.

Example 9.7 (A joint constraint invisible to marginal fits)**.** Suppose three admissible model vectors are $(1, 0)$, $(0, 1)$, and $(0, 0)$. Each individual coordinate can attain every value between zero and one after mixing. However every common mixture satisfies $y_1 + y_2 \le 1$. An observation near $(3/4, 3/4)$ can fit each marginal separately but violates the joint bound if the sum of its coordinate uncertainties is less than $1/2$.

This example is not a claim about measured chrysene data. It demonstrates the kind of contradiction a molecule-specific model should seek: a relationship that survives nuisance variation and excludes a joint observation even though separate module fits look satisfactory.

9.4 Identifiability and alternatives

Definition 9.8 (Observational equivalence)**.** Two realization classes are observationally equivalent for F and Θ if their feasible observable images coincide for all admitted conditions. A dataset restricted to those observables cannot distinguish them by feasibility alone.

Proposition 9.9 (Limit of compatibility evidence)**.** *If two model classes have identical feasible observable images, an observation compatible with one is compatible with the other. Compatibility alone cannot establish uniqueness between them.*

Proof. Membership in one feasible image is membership in the other by equality of the sets. □

To move beyond compatibility, one can seek new observables, different controlled conditions, or distributional predictions with justified source measures. The latter approach introduces probability explicitly. A deterministic compatibility framework does not eliminate the need for statistical inference when the evidence is sampled and noisy.

9.5 Comparative molecular uniqueness

Definition 9.10 (Bounded uniqueness claim)**.** Let $\mathcal{S}$ be a specified scaffold class and let $P(s)$ mean that scaffold s admits a realization satisfying the declared requirements. A bounded uniqueness claim is

$$P(s_0) \quad \text{and} \quad \forall s \in \mathcal{S} \setminus \{s_0\},\ \neg P(s).$$

The claim is falsified by an admissible competing witness. It cannot be established merely by failure to construct competitors with an incomplete search. A finite exhaustive proof is possible only when the class and predicates are effectively represented and the search covers them. For an infinite class, a structural exclusion theorem is needed.

The seven-tetrihex result of Article 40 supplies one bounded classification. The stronger biogenesis hypothesis asks for a much richer predicate involving chemistry, function, environment, and preparation. The same word “unique” should not conceal this change of problem. The present code theorem is compatible with many abstract alphabets; any molecular specificity must enter through the realization conditions and the comparative test.

Material-specific advantage requires matched controls: specified alternative scaffolds or assemblies, characterized substitution and terminal groups, common operating conditions and resource budgets, and the same readout and scoring procedure. Realizing the abstract code demonstrates an instance; superiority and uniqueness require separate comparative statements over a declared class. Failure of one preparation does not exclude all preparations in an unrestricted class.

10 Operator structure and physical information

10.1 Three levels of information

Throughout the operator examples below, Hilbert spaces are finite-dimensional, states are positive trace-one density matrices, and instrument sums are finite. The finite code is a classical symbolic construction. Its alphabet may label molecular configurations or readout outcomes without those outcomes being coherent quantum states. The operator axioms of Article 40 provide a separate mathematical language for observables, dynamics, and response. An implementation must connect these levels rather than identify them by terminology.

Definition 10.1 (Classical register in an operator model)**.** Let X be a finite label set and $\mathcal{H}$ a Hilbert space. An exact register representation is a family of nonzero mutually orthogonal projections $(P_x)_{x\in X}$ with $\sum_x P_x = I$. A state ρ has label distribution $p_x = \mathrm{Tr}(\rho P_x)$ whenever the trace expression is defined.

No claim of molecular realizability is built into this definition. A faithful representation of $\mathbb{C}^X$ has at least $|X|$ orthogonal nonzero sectors, but many microscopic states can occupy the same sector. A quaternary contact register requires four reliably distinguishable sectors, not merely four names assigned to overlapping preparations.

A physical register specification must include the preparation, storage and readout times and environmental conditions. Its validation requires joint preparation yield, transitions and leakage during the hold interval, and a readout confusion matrix with failures and rejection recorded. Report unconditional preparation yield and success as well as any postselected accuracy. Spectral distinctness or an equilibrium energy difference alone is not a measured storage lifetime; barriers and environmental dynamics matter [Kra40]. The exact projector model is therefore an ideal specification, not evidence that four robust chemical memory states have been demonstrated.

Proposition 10.2 (Register covariance)**.** *Suppose U is unitary and $UP_xU^* = P_{gx}$ for a permutation g of X. Then the label probabilities in $U\rho U^*$ satisfy $p'_{gx} = p_x$.*

Proof. Cyclicity of the trace gives $\mathrm{Tr}(U\rho U^* P_{gx}) = \mathrm{Tr}(\rho U^* P_{gx} U) = \mathrm{Tr}(\rho P_x)$. □

This proposition supplies an operator-level route to the action hypothesis. It does not show that a particular spatial transformation is represented by U, nor that its action on labels is pure reversal. Those identifications require the preparation and measurement model.

10.2 Commutants and measurement disturbance

The commutant of an observable algebra consists of operators commuting with every member of that algebra. Algebraic commutation is valuable, but a nondisturbance claim requires a measurement instrument, not only a pair of algebras.

Proposition 10.3 (A sufficient nondisturbance condition)**.** *Let a quantum instrument, with its outcome ignored, act as $\Phi(\rho) = \sum_k M_k \rho M_k^*$, where $\sum_k M_k^* M_k = I$. If every M_k commutes with every register projection P_x, then $\mathrm{Tr}(\Phi(\rho)P_x) = \mathrm{Tr}(\rho P_x)$.*

Proof. The dual map satisfies

$$\Phi^*(P_x) = \sum_k M_k^* P_x M_k = P_x \sum_k M_k^* M_k = P_x.$$

Taking expectations proves the assertion. □

The conclusion preserves the label distribution, not all coherences, correlations, or future dynamics. It is deliberately narrower than a blanket claim of nondemolition information storage. Article 40's commutant duality does not by itself supply the instrument operators or their physical implementation.

10.3 The spectral and electronic operators

The spectral operator D of the noncommutative-geometric data and the electronic Hamiltonian H of a terminal–bridge model have different definitions and obligations. A bound on a commutator with D is not automatically a bound on an electronic transition generated by H. Likewise, a geometric anisotropy is not a numerical optical anisotropy.

For a finite bridge decomposition, the electronic elimination is an exact algebraic statement under an invertibility hypothesis. Write

$$H = \begin{pmatrix} H_T & W^* \\ W & H_B \end{pmatrix}.$$

If $zI - H_B$ is invertible, solving the bridge row of $(zI - H)\psi = 0$ yields

$$\psi_B = (zI - H_B)^{-1} W \psi_T.$$

Substitution gives the effective terminal equation

$$[zI - H_T - W^*(zI - H_B)^{-1} W]\psi_T = 0.$$

The off-diagonal terminal transfer therefore contains both the direct term J and the bridge-mediated term:

$$T(z) = J + W_L^*(zI - H_B)^{-1} W_R.$$

Deleting J is an additional model restriction. Taking z at a bridge eigenvalue invalidates this elimination unless a different limiting or generalized construction is justified. A formal response expression does not establish a favorable magnitude, lifetime, or signal-to-noise ratio.

10.4 Classical coding versus quantum correction

The Hamming balls used here describe classical symbol substitutions. They are not a proof of correction of arbitrary quantum errors on five four-level systems. A quantum code requires a code subspace, an error-operator family, and an appropriate recovery criterion. Decoherence, leakage, destructive readout, and state-dependent preparation must be translated into a justified classical channel before the present decoder bound applies.

A useful experimental reduction is to prepare labeled states, apply a controlled perturbation, and measure a confusion matrix together with correlation statistics. If the resulting observation law satisfies the stated Hamming-error assumptions, the classical theorem becomes applicable. If it does not, a new error model is required. The existence of quantum structure in the molecular Hamiltonian neither invalidates nor automatically strengthens a classical coding theorem.

11 Biogenesis as a bounded realization program

11.1 The material hypothesis

The biogenesis objective is to investigate whether substituted chrysene can serve as a common material organizer for structural assembly and information-bearing function. The stronger proposal that it is a unique material of biogenesis is a research hypothesis. Its scope must name the substitutions, environments, functions, and competing scaffold class. A theorem about a seven-element geometric comparison does not establish uniqueness among all chemical precursors or historical pathways.

The patent [Sch25] records a technical and intellectual-property proposal. Patent issuance is not experimental verification of a prebiotic pathway, and the legal scope of a patent is not the domain of a mathematical uniqueness theorem. In the present work it provides provenance for the proposed molecular program, not a premise in the coding proof.

11.2 Endogenous and placed sources

Article 84 compares endogenous and placed preparation through equality of reachable sets under named support assumptions [Sch26j]. Equal reachable sets do not imply equal probabilities, preparation costs, rates, or historical plausibility. This distinction is essential when a laboratory preparation is used to probe a candidate ancient mechanism.

Proposition 11.1 (Reachability is weaker than distributional equality)**.** *There exist distinct probability distributions with the same finite support and different expected costs for a fixed cost function.*

Proof. On $\{a, b\}$, assign probabilities $(1/2, 1/2)$ and $(3/4, 1/4)$. Both supports are the full set. With costs $k(a) = 0$ and $k(b) = 1$, the expectations are $1/2$ and $1/4$. □

Thus an exogenous realization can test whether a proposed material architecture works under controlled preparation. It cannot alone establish that an endogenous source supplied the architecture at an adequate rate. Conversely, failure of one placed protocol does not refute all endogenous pathways without a completeness argument covering the relevant conditions.

The deterministic content of the present theorem is pointwise: every admitted transmitted word and every admitted one-symbol perturbation satisfy the decoding conclusion. A natural source can be stochastic while every realized event in that class obeys the theorem. Environmental probabilities and kinetics determine how often the class is entered and how often its hypotheses fail.

11.3 From finite operation to continuing function

A finite resource construction can establish a trajectory over a prescribed time interval. It does not establish indefinite persistence, autonomous replication, or heritable continuation. Article 77 makes this distinction explicitly in a finite-stock protocell setting [Sch26c]. The same discipline applies to an information-bearing module.

Definition 11.2 (Resource-aware repeated operation)**.** A cycle model consists of a state space S, an admissible resource set B, a partial transition $T : S \times B \to S \times B$, and an operational predicate Q. Repeated operation for n cycles requires every iterate to be defined and satisfy Q at the specified stages.

A single successful cycle proves none of the domain conditions for its second iterate. A replenishment law, waste handling, state reset, and stability are separate obligations. If division is involved, the distribution of code states and resources among descendants must be modeled rather than assumed from an averaged composition.

11.4 Existing-data comparisons

The program can use existing datasets without claiming that mathematical derivation substitutes for observation. No new kerogen, fossil, or cellular dataset is analyzed in this manuscript. The following are prospective protocols for connecting constrained models to evidence.

Kerogen and ancient organic matter. A proposed precursor signature must survive an explicit model of burial, maturation, alteration, contamination, and mineral association. Aromatic material in a mature deposit is not by itself a unique signature of an original substituted-chrysene source. A useful test would prespecify a joint compositional pattern and compare its preservation under candidate mechanisms, rather than select favorable peaks after inspection.

Cellular composition. A compositional comparison must retain species, compartment, growth conditions, extraction method, and measurement uncertainty. Bulk agreement can be nonidentifying when many mixtures share the same totals. The joint-feasibility approach of Section 9 is relevant only after the map from precursor inventory to measured composition has been defined and its nuisance parameters bounded.

Cambrian diversification. The diversification of animals is temporally and causally distinct from initial biogenesis. Connecting an early molecular information architecture to a later diversification pattern requires intermediate evolutionary and ecological dynamics. A fit to a late-time curve cannot identify its remote molecular cause without a discriminating causal model.

Cosmological analogies. A shared algebraic form can motivate a mathematical application. It does not demonstrate a shared physical mechanism. Cosmological applications require their own state variables, units, dynamics, observables, and comparison to established theories. No theorem here concerns the physical origin or expansion of the universe.

11.5 A reproducible comparison protocol

Before fitting a dataset, record its accession and version, the inclusion criteria, preprocessing, missing-data treatment, observable map, uncertainty model, and competing models. Separate calibration from evaluation. Fix the scope of parameter adjustment and preserve negative results. A change in a constitutive assumption after observing failure creates a revised model that must be tested anew.

The theoretical objective is not to replace all stochastic origin-of-life research by an unqualified deterministic alternative. Contemporary programs include chemically constrained common-precursor pathways and environmental concentration mechanisms [PPR+15, MAS+24]. The differentiating proposal here is a specified molecular anchor and a traceable chain from structural axioms to conditional functional predictions. Its merit must be assessed against concrete alternatives, not against a caricature of unconstrained chance.

12 A corpus designed for auditable revision

12.1 Two connected research objectives

The journal program connects a pure-mathematical objective, the study of structures defined by molecule-inspired axioms, with a material objective, the testing of substituted-chrysene realizations. A mathematical structure can remain useful when one proposed molecular interpretation fails. Conversely, a successful chemical experiment does not automatically prove every abstract theorem associated with its terminology.

The corpus includes foundational geometry and operators, genetic-code combinatorics, finite protocell constructions, source preparation, electronic and energetic models, and abstract applications. Article 89 provides an earlier categorical survey [Sch26h]. The present manuscript is a focused synthesis around action-compatible information, not an independent revalidation of every preceding result.

A bibliographic export inspected during preparation contains 89 records marked published as of 4 November 2026. Later delivered manuscripts and their final publication metadata must be reconciled before a final census of the first hundred articles. It would be misleading to describe an intended hundred-paper compilation as one hundred independently audited published results. Category boundaries overlap; a count requires a versioned list and explicit classification rules.

12.2 Applications and their interfaces

An abstract network model may be transported to financial networks, healthcare organization, or containment architectures if its vertices, operations, constraints, and observations are reinterpreted explicitly. The transport transfers a theorem only when the new interpretation satisfies its hypotheses. It does not transfer empirical validation from chemistry to another domain.

Applications involving molecular electronics, energy conversion, or pharmaceuticals retain the molecular realization burden. Each requires constitutive parameters, safety considerations, operating conditions, and relevant performance comparisons. A common mathematical language is a means of stating those burdens, not a reason to omit them.

12.3 Dependency-aware updates

The certification-frontier machinery of Article 42 already addresses dependency and revision [Sch26d]. The elementary formulation below states the operational obligation for a programmable corpus.

Definition 12.1 (Typed dependency record)**.** A claim record consists of a precise statement, assumptions, proof or computational artifact, source version, and outgoing references to the claims or definitions it uses. A directed edge $u \to v$ means that v depends on u.

Proposition 12.2 (Conservative impact region)**.** *Let S be a set of changed records in a finite dependency graph. Every record whose recorded derivation uses a changed record through a chain of dependencies belongs to the forward-reachable set from S.*

Proof. A direct use gives a path of length one. A chain of n uses gives a directed path of length n by concatenation. The assertion is exactly forward reachability. □

The reachability calculation identifies candidates for rechecking. It does not automatically repair their proofs. A record outside the computed region is protected only relative to the completeness of the recorded dependencies. Hidden reliance on an informal convention can defeat an otherwise correct software traversal.

For each update, preserve the earlier version, state the changed assumption, rebuild dependent computations, rerun formal checks where available, and require human disposition of affected interpretation claims. Published revisions need visible versioning rather than silent replacement. A corpus can be programmed for rapid analysis without promising instant scientific validation.

12.4 Orthogonal review and editorial governance

Mathematical review asks whether the conclusion follows. Concordance review asks whether objects and hypotheses agree with the controlling framework. Chemical review asks whether the proposed interpretation is constituted, feasible, and discriminating. These are distinct questions; agreement among reviewers who share the same mistaken encoding is not independent evidence.

Article 41 describes an AI-augmented orthogonal-review case study [Sch26b]. Such assistance can locate inconsistencies and generate counterexamples. It does not establish superiority to conventional review from a single uncontrolled comparison. Formal methods strengthen exact obligations but do not replace human assessment of novelty, relevance, chemical validity, or conflicts of interest.

The founder-led development of Volume 1 and the intended opening of Volume 2 to conventional author groups are editorial plans. The proposed molecular-machines sequence for Volume 1, Number 5 connects substituted-chrysene networks, hybrid systems, and standalone peptide architectures. Each transition must have its own realization assumptions. Founder-authored work should receive independent editorial handling and human review; internal computational checks are not a substitute for that independence.

13 Verification scope and proof obligations

13.1 Executed exact verification

The canonical verification companion, `verify_a100.py`, uses exact integer and rational arithmetic. Fresh runs in ordinary and optimized Python modes produced identical reports. It checks all 1024 ambient words, the two 64-word codes, the distance-one reference witness, repaired-code invariance and decoding, fixed-word and orbit counts, all 64×1024 transmitted/received pairs, and all 120 projective labelings. Ten negative controls reject false variants. These are finite checks of the encoded definitions, not a proof of every general theorem in this manuscript. The original draft's two exact programs are retained as supplementary comparison checks.

13.2 Scoped Lean 4 proof companion

The supplied `FormalProof_A100.lean` compiles under Lean 4.24.0 with `Std` as its only import. Its finite certificates use kernel reduction rather than native evaluation as a proof oracle. The accompanying declaration ledger and axiom-audit output delimit the actual formal coverage; no project-defined axiom or admitted proof is used.

The finite declarations cover field and Frobenius operation laws, reversal involution, encoder and covariance identities, code and orbit cardinalities, the explicit original obstruction, syndrome representatives, finite distance bounds, and decoder validity and equivariance. They also check the linear-even and semilinear-odd projective permutations. The names `covariance_certificate`, `decoder_valid_certificate`, `decoder_equivariance_certificate`, `quotient_distance_certificate`, `all_linear_even` and `all_semilinear_odd` identify the principal concrete declarations. Their finite-list predicates use the word and matrix representations specified in the source; correspondence to the mathematical interpretation remains an inspectable part of the specification.

Two conditional general lemmas are also formalized. The declaration `unique_radius_decode` proves uniqueness from a natural-number-valued symmetric distance satisfying a triangle inequality and separation greater than twice the radius. The declaration `equivariant_decoder` proves decoder equivariance assuming an isometry, code preservation, valid decoding and uniqueness. It does not assert those premises for every molecular model.

The entire abstract quotient-metric construction and general descent theorem are not fully formalized. Nor is the universal semilinear no-go theorem's reduction from arbitrary admissible parity-check matrices: its finite projective premises are checked, while the classification and reduction remain written arguments. Shell counts and equality of the radius-one union with actual decoder-orbit success on every transmitted/received pair are exhaustively checked in Python, not certified in Lean. Real-valued channel probabilities, entropy, analytic feasibility bounds, the operator-model results and all physical realization claims remain outside this Lean development.

> **Definition 13.1** (Verification ledger)**.** For each claim, the ledger records one or more of: a written proof, an executed finite check, a kernel-checked theorem, a numerical estimate with error bounds, or an empirical hypothesis. The categories may overlap but are not interchangeable.

A kernel verifies the formal proposition that was entered. It does not verify an omitted interpretation map or prove that a laboratory sample satisfies a formal predicate. Likewise, exact code can check all 1024 words while failing to detect that the wrong physical action was encoded. The ledger must therefore preserve both computational scope and semantic correspondence. Standard foundational dependencies reported by the axiom audit are distinguished from project assumptions and proof admissions.

13.3 Reproducibility and review status

The package includes executable sources, pinned Lean version, execution outputs, declaration coverage, source hashes, manuscript sources and readable companion documents. The Python verifier does not rely on language assertions that disappear under optimization. Separate matrix and orbit implementations provide comparison evidence, but no agreement of programs removes the need to inspect their definitions.

The orthogonal referee report records AI-assisted mathematical, concordance and chemistry review, a meta-review, and specific revision dispositions. It is not external human peer review. Independent editorial handling, human review, broader novelty assessment and physical validation remain separate obligations. This package should not be advertised as a fully machine-proved molecular theory.

14 Conclusions and research decisions

The principal lesson is that information assigned to a molecular carrier must respect the carrier action used to identify physical states. A code can be perfect in labeled coordinates and fail after those coordinates are identified by a symmetry. The reference quaternary code exhibits this failure with quotient distance one. A coordinate reassignment yields an invariant perfect code with forty message orbits and an exact one-symbol quotient decoder. If reversal also conjugates field symbols, the same linear perfect-code objective is impossible within the stated class, even after nonzero coordinate rescaling.

These conclusions are mathematically sharp and physically conditional. They make the encoding map a testable part of the model. A measured pure-reversal action supports one branch of the construction; a measured Frobenius-reversal action rules out that branch's invariant linear code objective. Neither result proves that a particular substituted-chrysene assembly realizes four stable contact states or arose historically.

The next mathematical tasks are to extend the scoped Lean 4 development to the full abstract quotient and universal semilinear arguments, investigate nonlinear or alternative-length constructions under the semilinear action, and characterize constrained attachment permutations. The next implementation tasks are to establish preparation coverage, measure the actual action on the register, and test the correlated error and readout laws. A code that cannot be prepared or observed is not an information device.

The broader biogenesis program benefits from the same separation. Source reachability, finite chemical operation, persistent reproduction, and historical explanation are different claims. Each can be made more precise through a named realization class and a discriminating observable. The resulting framework aims for exact deductions where exactness is warranted and explicit uncertainty where evidence remains incomplete.

Authorship and competing interests. The author is founder and Chief Editor of the journal and is associated with the proposed molecular program and cited patent. Independent editorial handling and human review are appropriate before publication. This manuscript was developed with AI assistance for drafting, mathematical checking, and source concordance. Such assistance is not presented as independent human peer review.

Data and code availability. No new experimental dataset is reported. The accompanying verification source and manuscript sources support the finite mathematical claims identified in Section 13. Article DOI, final repository location, and final pagination are publication-stage metadata and are not invented here.

A Exact arithmetic and matrix certificates

Represent $0, 1, \omega, \omega^2$ by the integers $0, 1, 2, 3$, respectively. Addition is bitwise exclusive-or. Multiplication is polynomial multiplication over $\mathbb{F}_2$, reduced by $X^2 + X + 1$. Thus

$$\omega^2 = \omega + 1, \qquad \omega^3 = 1, \qquad (\omega^2)^2 = \omega.$$

The nonzero multiplication table is

$\cdot$	1	ω	ω^2
1	1	ω	ω^2
ω	ω	ω^2	1
ω^2	ω^2	1	ω

These conventions fix every printed matrix and executable test. Replacing ω by ω^2 gives an isomorphic presentation but changes literal entries; a verification artifact must not mix the two presentations silently.

A.1 The intersection of the reference codes

For $x = (a, b, c, d, e) \in C \cap RC$, the four check equations are

$$\begin{aligned} a + c + d + e &= 0, \\ b + c + \omega d + \omega^2 e &= 0, \\ e + c + b + a &= 0, \\ d + c + \omega b + \omega^2 a &= 0. \end{aligned}$$

The first and third give $b = d$. Adding the second and fourth then gives $\omega^2(e + a) = 0$, hence $e = a$. The first gives $c = d = b$. The second becomes $\omega b + \omega^2 a = 0$, hence $b = \omega a$. Therefore

$$C \cap RC = \{(a, \omega a, \omega a, \omega a, a) : a \in K\}.$$

Every displayed word satisfies all four equations, proving equality rather than only inclusion. The intersection has four words, so $|C \cup RC| = 64 + 64 - 4 = 124$. All four intersection words are fixed by reversal; consequently the quotient image of the original code has $4 + (64 - 4) = 64$ orbits. Its cardinality does not rescue its distance-one defect.

Table 2: Claim and evidence status for the revised manuscript and companions.

Claim or requirement	Evidence supplied	Remaining obligation
Finite matrix identities and orbit counts	Written arguments, exact Python checks and scoped Lean certificates	Independent human specification review
General quotient-decoding theorem	Written proof; two conditional decoding lemmas in Lean	Full abstract quotient formalization
Semilinear no-go theorem	Written reduction; finite projective parity certificates	Formal classification and universal reduction
Exact symmetric-channel success	Written shell argument and exhaustive exact Python decoder check	Formal real-valued probability derivation; physical calibration
Molecular preparation, action and readout	Typed hypotheses and operational tests	Measurements on one common realization
Historical biogenesis and comparative uniqueness	Bounded research program and candidate discriminators	Independent empirical and comparative evidence

A.2 Repaired-code fixed points

A reversal-fixed word has form (a, b, c, b, a). The first row of $H_* x = 0$ vanishes identically in characteristic two. The second row gives

$$c + (1 + \omega)b + \omega^2 a = 0,$$

so $c = \omega^2(a + b)$. There are $4^2 = 16$ choices. The remaining 48 codewords form 24 two-element orbits.

A received word fixed by reversal has three free coordinates, giving $4^3 = 64$ ambient fixed words. Burnside's formula therefore gives $(1024 + 64)/2 = 544$ ambient orbits. This is a separate count from the forty code orbits: the former describes received states, the latter messages.

A.3 A check on the matrix action

With h_i the columns of H_*, the matrix $A = \left(\begin{smallmatrix} 1 & 0 \\ \omega^2 & 1 \end{smallmatrix}\right)$ satisfies $Ah_1 = h_5$, $Ah_2 = h_4$, $Ah_3 = h_3$, and the reverse identities. Thus $H_* R = AH_*$ column by column. This certificate is short enough to inspect independently of enumeration. It proves invariance because $H_* x = 0$ implies $H_* Rx = AH_* x = 0$.

B Finite procedures and their logical scope

B.1 Enumeration procedure

Enumerate the 1024 words of K^5 lexicographically in the integer representation of Appendix A. For each word, compute both syndromes. Retain the zero-syndrome words to construct C and C_*. Compute Hamming weights directly and compute pairwise distances for the finite code sets. No sampling is involved.

For reversal orbits, define the representative of a word as the lexicographically smaller of x and Rx. This representative is a computational convention, not a preferred physical orientation. The set of representatives has 544 elements. Restricting it to C_* gives forty. Computing the quotient distance by the minimum of the two ordinary distances agrees with the general orbit-metric definition.

B.2 Syndrome decoding

Precompute the syndrome of the zero error and of all fifteen weight-one errors. Each of the sixteen syndromes appears exactly once. Given a received word y, subtract the unique error with syndrome $H_* y$. In characteristic two subtraction equals addition. The result lies in C_*. Return its reversal orbit rather than a coordinate representative when orientation is not an observable.

An exhaustive test iterates over all 64 codewords and all sixteen errors of weight at most one, then verifies both the labeled correction and the quotient output. The 1024 resulting received words are distinct. Repeating this test after reversal verifies action compatibility at the implementation level.

B.3 Projective action enumeration

Enumerate all 2×2 matrices over K, retain those of nonzero determinant, and compute their permutations of the five projective column lines. There are 180 invertible matrices and sixty distinct projective permutations, each even. Composing each permutation with Frobenius gives sixty odd permutations. The reversal permutation has two transpositions and is even, so it does not occur in the latter list.

This enumeration is a finite cross-check of the parity proof, not its conceptual substitute. The proof explains why nonzero column scalings cannot help: they change representatives of projective lines, not their induced permutation. The enumeration

is especially useful for detecting a mistaken convention for matrix multiplication or conjugation.

B.4 Negative controls

A useful test suite also verifies failure of the original code's reversal invariance, failure of distance-three after reversal saturation, failure of uniform quotient-ball sizes, failure of semilinear invariance of the repaired code, and failure of a constant-zero decoder. These controls protect the scope of the positive results.

The exact output should be preserved alongside its source version. A changed test or matrix requires a fresh run. A printed success message without the executed source and domain is not a reproducible certificate.

C Claim and realization register

The following register organizes the manuscript's principal conclusions by their proof obligations.

General quotient theorem. Domain: a metric space with a finite isometry group. Input: an invariant code with minimum distance at least $2t + 1$. Conclusion: unique orbit decoding within radius t. Evidence: written proof. Limitation: invariance is sufficient; other noninvariant orbit codes may have useful quotient distance and must be analyzed directly.

Reference-code obstruction. Domain: the specified K-linear kernel and pure reversal on five coordinates. Conclusion: a distance-one pair of distinct message orbits. Evidence: explicit witness and finite enumeration. Limitation: this is a failure of this representation, not of all quaternary coding or of all molecular information.

Constructive repair. Domain: the specified column reassignment and pure reversal. Conclusion: a perfect invariant code, forty message orbits, and a quotient partition into sixteen balls of size ten and twenty-four of size sixteen. Evidence: matrix identity, Hamming bound with disjoint balls, fixed-point calculation, and enumeration. Limitation: physical permission to make the reassignment is an extra condition.

Semilinear obstruction. Domain: K-linear $[5, 3, 3]$ codes invariant under diagonal-rescaled Frobenius reversal. Conclusion: no such code exists. Evidence: projective parity proof and finite action enumeration. Limitation: nonlinear codes, different lengths, different distances, and other physical actions remain outside the theorem.

Robust implementation. Domain: the declared same-experiment preparation, action, noise, and readout events. Conclusion: the stated lower success bound. Evidence: implication on the good-event intersection and the union bound. Limitation: event probabilities must refer to the same population or valid sequential conditioning; unrelated calibration studies cannot be added as if they did.

Observable rejection. Domain: a fixed joint realization class and its observable image or certified outer approximation. Conclusion: disjoint uncertainty sets reject the declared model. Evidence: set inclusion and a linear separator when available. Limitation: compatibility is not uniqueness, and a relaxed-model witness is not automatically a witness in the original model.

Molecular implementation. Domain: a chemically specified substituted-chrysene assembly under specified conditions. Required evidence: constitution, preparation coverage, action measurement, state discrimination, error characterization, and resource accounting. Status: hypotheses and proposed tests, not new experimental findings in this manuscript.

Historical biogenesis. Domain: a specified historical pathway and environmental reconstruction. Required evidence: rates, supply, preservation, alternative mechanisms, and discriminating observations. Status: prospective research program. The coding theorems alone do not establish historical occurrence or universal material uniqueness.

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