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Algebraic Decoupling and Nilpotent Scission over the Chrysene Tensor Space: A Geometric Framework for Cryptographic Information Containment

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Abstract

Software-defined and probabilistic methods of state validation leave non-compliant configurations as well-defined objects of the ambient space; rejection is an external decision rather than an algebraic annihilation. This paper constructs a discrete non-commutative framework in which containment becomes an intrinsic geometric relation. The ambient geometry is the Chrysene Tensor Space $\mathcal{I}_C$, a fibrated manifold over $\mathbb{Z}^3$ with C_{2h} orthogonal connectivity. States are valued in the Galois field GF(4), so that information is represented natively in base four. Complementary projections determine an admissible sector (a geometric key) and a hazard sector. Encoding embeds a message into the admissible coordinates selected by the key; recovery is the corresponding projection. A nilpotent scission operator sends every state that violates the governing orthogonality conditions into the absolute null ideal, from which no admissible morphism can extract residual information. The resulting theory expresses hard containment and selective accessibility as algebraic properties of the geometry. It is offered as a complementary topological language rather than as a replacement for existing cryptographic primitives.
Keywords: cryptographic containment, geometric key, Chrysene Tensor Space, algebraic decoupling, nilpotent scission, null ideal, orthogonal projection, base-four encoding, topological isolation

1 Introduction

1.1 Limitations of Software-Defined and Probabilistic Containment

Contemporary approaches to the validation and isolation of computational states rely predominantly on software-defined protocols and probabilistic filters. Access-control lists, cryptographic authentication mechanisms, anomaly detectors, and statistical classifiers operate by evaluating predicates or likelihoods on data that already reside in a continuous or effectively unbounded ambient space. In each instance, the underlying geometry remains structurally flexible: a state that transitions beyond expected parameters is categorized by an external decision procedure, yet the ambient space itself continues to computationally map the state as a mathematically well-defined object.

Consequently, three structural boundaries persist:

1. **Absence of intrinsic geometric limits.** Software-defined constraints do not fundamentally alter the topology of the state space. Unparameterized or asymmetric configurations remain accessible elements of the ambient module; they are evaluated as inadmissible by an auxiliary procedure rather than algebraically precluded from the manifold.

2. **Sensitivity to unparameterized entropy.** When the environment introduces high-entropy perturbations—whether through systemic variance, dynamic structural loads, or unconstrained generative processes—probabilistic filters encounter operational limits. Thresholds that effectively map admissible behavior under nominal conditions require geometric augmentation to continuously navigate highly volatile state transitions.

3. **State persistence after isolation.** Because isolation is frequently executed as an external administrative operation rather than a fundamental algebraic resolution, residual structural parameters of an isolated state may remain computationally accessible to subsequent operations that route through the original protocol.

These parameters are geometrically foundational rather than merely algorithmic. They indicate that containment architectures reliant exclusively upon software logic or statistical estimation

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encounter structural limits when guaranteeing the absolute isolation of divergent states. To achieve absolute structural resilience, next-generation computational architectures must systematically advance beyond probabilistic classification to integrate discrete topological boundaries that algebraically resolve divergent states from the executable manifold.

1.2 Topological Survival under Adversarial Entropy

The cryptographic containment problem may be viewed as a special case of a more general question: how can a designated class of states persist when the ambient dynamics favour disorder? In a maximum-entropy environment whose entropy source is adversarial, soft structures are eroded. What is required is a geometry in which membership in an admissible sector is an algebraic property, and in which any component that violates the governing constraints is mapped into a null ideal from which it cannot be recovered by admissible morphisms.

Such a geometry realises a form of topological survival: only those configurations that satisfy the intrinsic constraints of the space remain available; all others are annihilated.

1.3 Contribution of the Present Work

The present paper constructs an algebraic-geometric framework that supplies the required hard boundaries. The ambient space is the Chrysene Tensor Space $\mathcal{I}_C$, a discrete non-commutative fibrated manifold over the integer lattice $\mathbb{Z}^3$ whose fibres carry a C^*-algebraic structure and whose point-group symmetry is restricted to the class C_{2h}. States are valued in the Galois field

$$\mathrm{GF}(4) = \{0, 1, \omega, \omega^2\},$$

so that information is represented natively in base four [Sch26b, Sch26d].

Within this space a complementary pair of orthogonal projections determines an admissible sector and a complementary hazard sector. A geometric key is realised precisely as a choice of such an admissible sector. Encoding consists in embedding a message into the coordinates selected by the key; recovery is the corresponding projection. A nilpotent scission operator $\hat{P}_{\mathrm{sciss}}$ acts on the GF(4)-module of states: any configuration that fails the geometric parity or orthogonality conditions fixed by the key is sent to the absolute null ideal $\mathcal{I}_\emptyset$ and remains there under subsequent admissible operations [Sch26b, Sch26d].

The resulting theory therefore expresses containment as an intrinsic algebraic relation rather than as an external decision. Numerical illustrations over GF(4) confirm the principal constructions: base-four representation, orthogonal decomposition, key-dependent recovery, and the action of the scission operator [Sch26a, Sch26c].

The framework is deliberately limited in scope. It does not rest on computational hardness assumptions, nor does it claim to replace existing public-key or symmetric primitives. It supplies a complementary topological language in which hard isolation of geometrically non-compliant states can be stated and enforced.

2 The Chrysene Tensor Space as Cryptographic Substrate

2.1 Discrete Fibrated Geometry

Definition 1 (Chrysene Tensor Space $\mathcal{I}_C$)**.** *The Chrysene Tensor Space $\mathcal{I}_C$ is a discrete, non-commutative fibrated manifold whose base is the three-dimensional integer lattice $\mathbb{Z}^3$. Over each lattice point the fibre is a localized state space realised as a finite-dimensional module equipped with a C^*-algebraic structure. The total space therefore carries the structure of a fibred category*

$$\pi : \mathcal{I}_C \to \mathbb{Z}^3,$$

in which every admissible localisation of a state is forced onto an integer coordinate.

The discreteness of the base eliminates continuous fractional geometries: a configuration either occupies a lattice point or is undefined. Consequently Markovian random walks and infinitesimal perturbations that rely on intermediate real coordinates are algebraically excluded.

2.2 Geometric Basis Tensor and Orthogonal Connectivity

Definition 2 (Geometric Basis Tensor)**.** *Let Ψ_C denote the geometric basis tensor that spans each fibre of $\mathcal{I}_C$. The tensor is modelled on the 3, 6, 9, 12-tetrasubstituted chrysene core and decomposes into a pair of primary connectivity vectors $\vec{v}_x$ and $\vec{v}_y$ corresponding to the substitution axes.*

Axiom 1 (Orthogonal Connectivity)**.** *The connectivity vectors satisfy the strict orthogonality relation*

$$\vec{v}_x \cdot \vec{v}_y = 0.$$

The associated projection operators p_x and p_y therefore obey the Murray–von Neumann orthogonality condition $p_x p_y = 0$. The ideals they generate inside the fibrewise ring are mutually annihilating.

Axiom 2 (C_{2h} Symmetry)**.** *The point-group symmetry of Ψ_C is restricted to the chiral class C_{2h}. This restriction elimi-*

nates the extraneous reflection planes of the higher symmetry D_{2h} that would force the underlying Hopf structure into cocommutativity, while retaining a consistent orientation and guaranteeing that the spatial axes remain non-trivially non-commutative.

The combination of lattice discreteness, orthogonal connectivity and C_{2h} symmetry supplies a canonical complementary decomposition of each fibre into an admissible sector and a complementary hazard sector.

2.3 Base-Four Coordinate Algebra

Definition 3 (State Module over GF(4))**.** *The coordinate algebra of the lattice is the Galois field*

$$\mathrm{GF}(4) = \{0, 1, \omega, \omega^2\}$$

with arithmetic determined by the relations

$$\omega^2 + \omega + 1 = 0, \qquad \operatorname{char} \mathrm{GF}(4) = 2.$$

A state on $\mathcal{I}_C$ is a finitely supported function

$$\psi : \mathbb{Z}^3 \to \mathrm{GF}(4),$$

or, equivalently, an element of the free GF(4)*-module generated by a finite set of lattice points. Information is therefore represented natively in base four.*

All subsequent linear-algebraic operations—projections, linear combinations, and the scission operator—are performed with respect to this coefficient field.

The base-four character of the state space may be visualised directly.

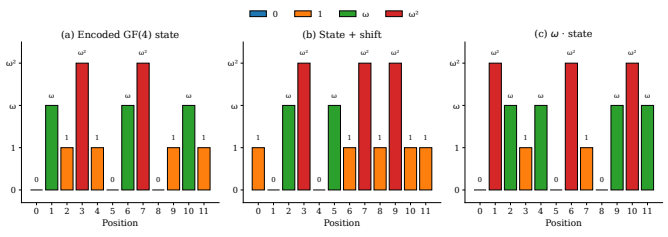

Figure 1: Native base-four representation of information in the framework. (a) A short integer message encoded as a vector with entries in $\mathrm{GF}(4) = \{0, 1, \omega, \omega^2\}$. (b) Component-wise addition of a shift vector. (c) Scalar multiplication by ω. All arithmetic is performed in the field GF(4).

2.4 Non-Commutative Ring and Algebraic Decoupling

Definition 4 (Non-Commutative Ring $\mathcal{R}_C$)**.** *Let $\hat{T}$ and $\hat{S}$ denote the fibrewise operators corresponding to the two orthogonal connectivity axes. Form the free associative algebra over* GF(4) *generated by these operators and quotient by the two-sided ideal generated by the orthogonality relations of Axiom 1. The resulting quotient is the non-commutative ring $\mathcal{R}_C$.*

Because the generators satisfy $p_x p_y = 0$, the ring $\mathcal{R}_C$ possesses non-trivial zero divisors. Consequently there exist non-zero elements $a, b \in \mathcal{R}_C$ such that

$$ab = 0.$$

This algebraic decoupling is the ring-theoretic counterpart of the geometric orthogonality of the connectivity vectors: components supported on complementary sectors annihilate one another under the ring multiplication.

The zero-divisor structure will be used in the next section to define complementary projections and the absolute null ideal into which non-compliant states are mapped.

3 Algebraic Decoupling and the Null Ideal

3.1 Orthogonal Projections on the State Module

Let M denote the free GF(4)-module of finitely supported states on $\mathcal{I}_C$. The orthogonal connectivity of the geometric basis tensor induces a pair of complementary idempotent endomorphisms of M.

Definition 5 (Complementary Projections)**.** *Let $p_{\mathrm{safe}}, p_{\mathrm{hazard}} \in \mathrm{End}_{\mathrm{GF}(4)}(M)$ be the unique pair of module endomorphisms satisfying*

$$\begin{aligned} p_{\mathrm{safe}} + p_{\mathrm{hazard}} &= \mathrm{id}_M, \\ p_{\mathrm{safe}} p_{\mathrm{hazard}} &= p_{\mathrm{hazard}} p_{\mathrm{safe}} = 0, \\ p_{\mathrm{safe}}^2 = p_{\mathrm{safe}}, &\qquad p_{\mathrm{hazard}}^2 = p_{\mathrm{hazard}}. \end{aligned}$$

The image of p_{safe} is called the admissible sector*; the image of p_{hazard} is called the* hazard sector.

The relations above are the module-theoretic expression of the zero-divisor structure already present in the ring $\mathcal{R}_C$. In particular,

$$\operatorname{im} p_{\mathrm{safe}} \cdot \operatorname{im} p_{\mathrm{hazard}} = 0$$

inside the fibrewise multiplication.

3.2 The Absolute Null Ideal

Definition 6 (State Module)**.** *Let M be the free* GF(4)*-module on the set of lattice points of $\mathbb{Z}^3$, or equivalently the module of all finitely supported functions $\psi : \mathbb{Z}^3 \to \mathrm{GF}(4)$. All endomorphisms, projections and the scission operator are understood as elements of* $\mathrm{End}_{\mathrm{GF}(4)}(M)$.

Definition 7 (Absolute Null Ideal)**.** *The absolute null ideal $\mathcal{I}_\emptyset$ is the zero submodule of M:*

$$\mathcal{I}_\emptyset = \{0\} \subset M.$$

Equivalently, $\mathcal{I}_\emptyset$ may be realised as the kernel of the identity endomorphism, or as the image of the zero endomorphism.

As the zero submodule, $\mathcal{I}_\emptyset$ is invariant under every endomorphism of M and is therefore an ideal in the endomorphism ring. Any state that is annihilated by both complementary projections, or that is sent to zero by a subsequent scission operator, lies in $\mathcal{I}_\emptyset$. Once a configuration has entered the null ideal it remains there under every endomorphism of M, since every linear map sends zero to zero.

3.3 Decoupling Theorem

Theorem 1 (Algebraic Decoupling)**.** *Let $\psi \in M$ be an arbitrary state. Write*

$$\psi = \psi_{\mathrm{safe}} + \psi_{\mathrm{hazard}},$$

where $\psi_{\mathrm{safe}} := p_{\mathrm{safe}}(\psi)$ and $\psi_{\mathrm{hazard}} := p_{\mathrm{hazard}}(\psi)$. Then the following hold:

1. *The decomposition is unique.*
2. *$p_{\mathrm{safe}}(\psi_{\mathrm{hazard}}) = 0$ and $p_{\mathrm{hazard}}(\psi_{\mathrm{safe}}) = 0$.*
3. *If ψ is supported entirely on the hazard sector (i.e., $\psi_{\mathrm{safe}} = 0$), then ψ has vanishing coupling to the admissible sector: every admissible observation functional vanishes on ψ.*

Proof. (1) Suppose $\psi = \phi_1 + \phi_2 = \phi_1' + \phi_2'$ with $\phi_1, \phi_1' \in \mathrm{im}\, p_{\mathrm{safe}}$ and $\phi_2, \phi_2' \in \mathrm{im}\, p_{\mathrm{hazard}}$. Applying p_{safe} yields $\phi_1 = \phi_1'$; applying p_{hazard} yields $\phi_2 = \phi_2'$. Uniqueness follows.
(2) By the complementary relations,

$$p_{\mathrm{safe}}(\psi_{\mathrm{hazard}}) = p_{\mathrm{safe}} p_{\mathrm{hazard}}(\psi) = 0,$$

and likewise $p_{\mathrm{hazard}}(\psi_{\mathrm{safe}}) = 0$.
(3) Any admissible observation is, by definition, a linear functional that factors through p_{safe}. If $\psi_{\mathrm{safe}} = 0$, then every such functional evaluates to zero on ψ. □

Thus a state that violates the orthogonality conditions of the geometry is invisible to every observation that is supported only on the admissible sector. Its residual information lies in a complementary summand that is algebraically decoupled from the admissible dynamics.

3.4 Numerical Illustration

The complementary decomposition may be realised concretely on a finite-dimensional GF(4)-module. Figure 2 displays an arbitrary state, its image under p_{safe}, and its image under p_{hazard}. The two components are complementary, their sum recovers the original state, and the safe projection annihilates the hazard component, sending it to the null ideal.

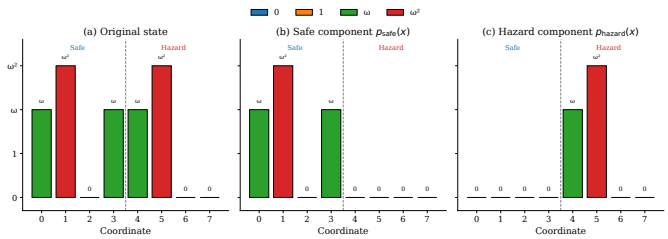

Figure 2: Orthogonal projection and algebraic decoupling over GF(4). (a) A generic state with entries in GF(4). (b) Image under the safe projection p_{safe}. (c) Image under the complementary hazard projection p_{hazard}. The two components are complementary and their sum recovers the original state; the safe projection annihilates the hazard component, sending it to the null ideal.

4 The Nilpotent Scission Operator

4.1 Definition of the Operator

Let M be the free GF(4)-module of finitely supported states on $\mathcal{I}_C$, and let $p_{\mathrm{safe}}, p_{\mathrm{hazard}}$ be the complementary projections of Section 3.

Definition 8 (Nilpotent Scission Operator)**.** *The nilpotent scission operator is the endomorphism*

$$\hat{P}_{\mathrm{sciss}} : M \to M$$

defined by

$$\hat{P}_{\mathrm{sciss}}(\psi) = \begin{cases} \psi & \text{if } p_{\mathrm{hazard}}(\psi) = 0, \\ 0 & \text{if } p_{\mathrm{hazard}}(\psi) \neq 0. \end{cases}$$

Equivalently,

$$\hat{P}_{\mathrm{sciss}}(\psi) = \begin{cases} p_{\mathrm{safe}}(\psi) & \text{if } \psi \in \mathrm{im}\, p_{\mathrm{safe}}, \\ 0 & \text{otherwise}. \end{cases}$$

A state ψ is called *compliant* when $p_{\text{hazard}}(\psi) = 0$ (i.e., when it is supported entirely on the admissible sector) and *non-compliant* otherwise.

Equivalently, one may write

$$\hat{P}_{\text{sciss}} = p_{\text{safe}} \circ \chi_{\text{safe}},$$

where χ_{safe} is the characteristic function of the admissible sector (the unique idempotent that is the identity on $\operatorname{im} p_{\text{safe}}$ and zero on $\operatorname{im} p_{\text{hazard}}$). In the finite-dimensional setting used for the numerical illustrations this reduces to ordinary matrix multiplication.

4.2 Algebraic Properties

Proposition 2 (Basic Properties of Scission)**.** *The operator $\hat{P}_{\text{sciss}}$ satisfies the following:*

1. ***Idempotence on the admissible sector.*** *If $\psi \in \operatorname{im} p_{\text{safe}}$, then $\hat{P}_{\text{sciss}}(\psi) = \psi$.*
2. ***Annihilation of non-compliant states.*** *If $p_{\text{hazard}}(\psi) \neq 0$, then $\hat{P}_{\text{sciss}}(\psi) = 0 \in \mathcal{I}_{\emptyset}$.*
3. ***Nilpotency on the image of non-compliant states.*** *$\hat{P}_{\text{sciss}} \circ \hat{P}_{\text{sciss}}(\psi) = \hat{P}_{\text{sciss}}(\psi)$ for every $\psi \in M$. In particular, once a state has been sent to the null ideal it remains there.*
4. ***Compatibility with the orthogonal decomposition.*** *$\hat{P}_{\text{sciss}} \circ p_{\text{safe}} = p_{\text{safe}}$ and $\hat{P}_{\text{sciss}} \circ p_{\text{hazard}} = 0$.*
5. ***Linearity over*** GF(4)**.** *$\hat{P}_{\text{sciss}}$ is a* GF(4)*-linear endomorphism of M.*

Proof. (1) and (2) are immediate from the definition. (3) follows because the zero vector is fixed by $\hat{P}_{\text{sciss}}$ and every non-compliant state is sent to zero. (4) is a direct computation with the complementary relations of the projections. (5) holds because both the conditional definition and the projections are linear over the coefficient field GF(4). □

4.3 Main Containment Theorem

Lemma 3 (Compliant States)**.** *A state $\psi \in M$ is compliant if and only if $\psi \in \operatorname{im} p_{\text{safe}}$. Equivalently, ψ is non-compliant if and only if $p_{\text{hazard}}(\psi) \neq 0$.*

Definition 9 (Admissible Endomorphism)**.** *An endomorphism $f \in \operatorname{End}_{\text{GF}(4)}(M)$ is called admissible when $f(\operatorname{im} p_{\text{safe}}) \subset \operatorname{im} p_{\text{safe}}$.*

Theorem 4 (Scission Containment)**.** *Let $\psi \in M$ be an arbitrary state and let $\gamma : \mathbb{N} \to M$ be any sequence of states obtained by successive application of admissible endomorphisms of M (i.e., endomorphisms that preserve the image of p_{safe}). Then the following are equivalent:*

1. *ψ fails the geometric parity (orthogonality) condition fixed by the complementary projections, i.e., $p_{\text{hazard}}(\psi) \neq 0$;*
2. *$\hat{P}_{\text{sciss}}(\psi) = 0 \in \mathcal{I}_{\emptyset}$;*
3. *$\hat{P}_{\text{sciss}}(\gamma(n)) = 0$ for every n, whenever $\gamma(0) = \hat{P}_{\text{sciss}}(\psi)$.*

In other words, every non-compliant state is sent to the null ideal under scission and remains there under all subsequent admissible operations.

Proof. (1) $\Rightarrow$ (2) is the second clause of the definition of $\hat{P}_{\text{sciss}}$.
(2) $\Rightarrow$ (3). Suppose $\hat{P}_{\text{sciss}}(\psi) = 0$. Then $\gamma(0) = 0$. Every admissible endomorphism fixes the zero vector, so $\gamma(n) = 0$ for all n, and therefore $\hat{P}_{\text{sciss}}(\gamma(n)) = 0$.
(3) $\Rightarrow$ (1). Contrapositively, if $p_{\text{hazard}}(\psi) = 0$, then $\hat{P}_{\text{sciss}}(\psi) = \psi \notin \mathcal{I}_{\emptyset}$ (unless $\psi = 0$). Thus a non-zero compliant state is not sent to the null ideal. □

The theorem supplies the algebraic mechanism of hard containment: geometric non-compliance is converted, by a single application of $\hat{P}_{\text{sciss}}$, into membership in the null ideal, from which no admissible morphism can extract residual information.

4.4 Numerical Illustration

The action of scission may be realised on a finite-dimensional GF(4)-module. Figure 3 displays a compliant state (supported only on the admissible sector) that is left unchanged by $\hat{P}_{\text{sciss}}$, and a non-compliant state that is mapped to the zero vector. A second application of the operator leaves the zero state fixed, confirming nilpotency on the image.

5 Geometric Keys and Selective Accessibility

5.1 Geometric Keys

Definition 10 (Geometric Key)**.** *A geometric key is a choice of complementary projections $(p_{\text{safe}}, p_{\text{hazard}})$ on the ambient* GF(4)*-module M, or equivalently a choice of admissible sector*

$$K := \operatorname{im} p_{\text{safe}} \subset M.$$

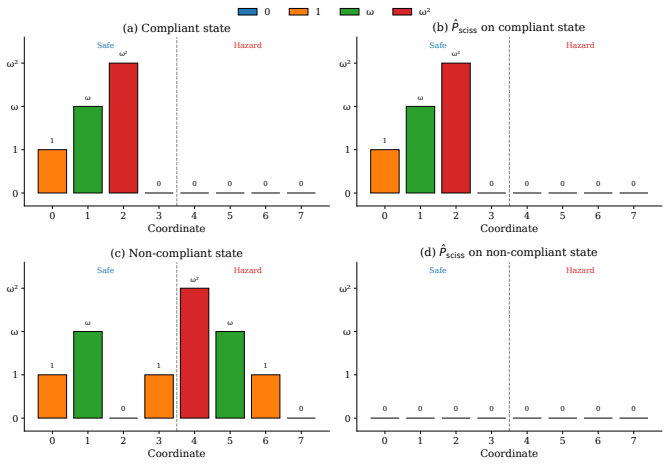

Figure 3: Action of the nilpotent scission operator $\hat{P}_{\text{sciss}}$ over GF(4). (a)–(b) A compliant state (supported only on the admissible sector) is left unchanged. (c)–(d) A non-compliant state that carries non-zero components in the hazard sector is mapped to the zero vector (null ideal). A second application of scission leaves the zero state unchanged, illustrating nilpotency on the image.

> *Two keys are regarded as distinct when their admissible sectors differ as submodules.*

The public geometry of the system consists of the ambient space $\mathcal{I}_C$, the coefficient field GF(4), and the scission operator. The private information is the particular choice of admissible sector.

5.2 Encoding

Definition 11 (Encoding under a Key)**.** *Let $m \in \mathrm{GF}(4)^k$ be a message of length k equal to the dimension of the chosen admissible sector K.After the choice of an ordered* GF(4)*-basis of the admissible sector K, one obtains a canonical isomorphism $K \simeq \mathrm{GF}(4)^k$. All subsequent references to embedding a message into the admissible coordinates are understood with respect to this isomorphism. An encoding of m under the key K is any state $\psi \in M$ such that*

$$p_{\text{safe}}(\psi) = m$$

(after the natural identification of K with $\mathrm{GF}(4)^k$). The complementary components $p_{\text{hazard}}(\psi)$ may be filled with arbitrary masking symbols in GF(4)*. After the choice of an ordered* GF(4)*-basis of the admissible sector K, one obtains a canonical isomorphism $K \simeq \mathrm{GF}(4)^k$. All subsequent references to "embedding a message into the admissible coordinates" are understood with respect to this isomorphism.*

Thus the ciphertext is a geometrically valid configuration on the lattice; only the holder of the matching key knows which coordinates recover the plaintext.

5.3 Decoding and Selective Recovery

Definition 12 (Recovery)**.** *Given a state $\psi \in M$ and a key $K = \operatorname{im} p_{\text{safe}}$, recovery under K is the projection*

$$\mathrm{Rec}_K(\psi) := p_{\text{safe}}(\psi).$$

Proposition 5 (Correct versus Mismatched Recovery)**.** *Let ψ be an encoding of a message m under a key K_A. Then*

1. $\mathrm{Rec}_{K_A}(\psi) = m$*;*
2. *if K_B is a distinct key, $\mathrm{Rec}_{K_B}(\psi)$ need not equal m, and the components of m that lie outside K_B are invisible to every observation supported on K_B.*

Proof. (1) follows at once from the definition of encoding. (2) follows from the uniqueness of the complementary decomposition (Theorem 3.1) together with the fact that distinct admissible sectors intersect in a proper submodule. □

Consequently, recovery succeeds if and only if the geometric constraints selected by the key are satisfied. Under a mismatched key the residual information is algebraically decoupled from the observer's admissible sector.

5.4 Scission as Hard Containment

When a state fails the parity conditions of the active key, the scission operator of Section 4 maps it into the null ideal $\mathcal{I}_\emptyset$. In particular:

- a well-formed base-four string that is nevertheless geometrically non-compliant with respect to the active key is annihilated;
- after scission, no admissible endomorphism can extract the original information.

This yields a limited but precise notion of forward isolation: once a non-compliant state has been sent to $\mathcal{I}_\emptyset$, its residual content remains inaccessible to every subsequent admissible operation.

5.5 Numerical Illustration

Figure 4 exhibits an explicit encoding of a short GF(4) message under one choice of admissible sector, successful recovery under the matching key, and failure of recovery under a distinct key. The illustration confirms that accessibility is controlled by geometric membership rather than by an external decision procedure.

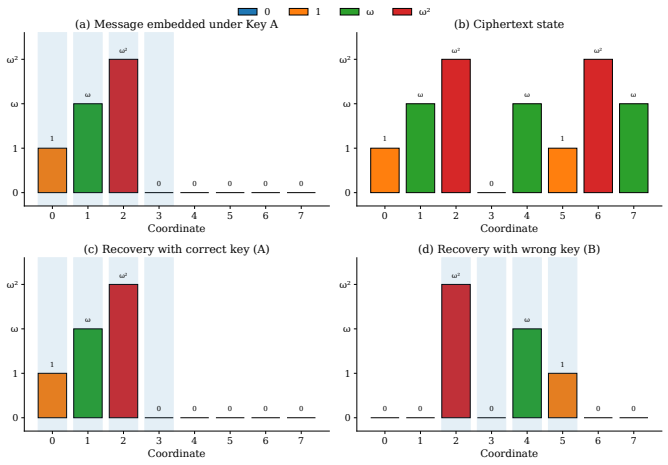

Figure 4: Key-dependent admissible sector over GF(4). (a) A message is embedded into the admissible coordinates selected by Key A. (b) The resulting ciphertext state (message sector plus masking symbols). (c) Projection under the correct key recovers the original message. (d) Projection under a different key yields a state whose admissible sector does not contain the message; the information is effectively lost to the null ideal relative to that key.

5.6 Scope of the Framework

The constructions of this section supply a mathematical language in which selective accessibility and hard containment of non-compliant states are expressed by orthogonal projections and nilpotent scission. The framework does not rest on computational hardness assumptions, does not claim to replace existing public-key or symmetric primitives, and does not assert practical security against real-world adversaries. It is offered as a complementary topological formalism in which isolation properties can be stated algebraically.

6 Relation to Possible Physical Realizations

The algebraic structures developed above—the discrete fibrated geometry of $\mathcal{I}_C$, the orthogonal connectivity, the GF(4)-module of states, and the nilpotent scission operator—are defined without reference to any particular physical carrier. They constitute a purely mathematical theory of containment and selective accessibility.

Nevertheless, the same geometric constraints that appear in the abstract formalism are compatible with a discrete lattice or molecular substrate whose local symmetry and connectivity mirror those of the 3, 6, 9, 12-tetrasubstituted chrysene core. In such a realisation the complementary projections would correspond to orthogonal transport or addressing channels, and scission would correspond to a hard physical veto that annihilates non-compliant configurations.

No claim is made that any such substrate has been constructed, nor that the mathematical theory depends upon its existence. The framework remains valid as an abstract algebraic geometry of containment irrespective of future physical implementations.

7 Discussion and Open Problems

7.1 Scope and Limitations

The present work supplies a precise algebraic language in which hard containment of geometrically non-compliant states can be expressed. Containment is reduced to membership in a null ideal under a nilpotent operator, and selective accessibility is reduced to a choice of admissible sector inside a GF(4)-module. The theory does not address computational hardness, side-channel resistance in physical devices, or the engineering of concrete cryptographic protocols.

7.2 Open Questions

Several concrete problems remain open.

1. **Geometric parity checks over** GF(4)**.** An efficient, canonical procedure is required that, given an arbitrary state, decides compliance with a prescribed admissible sector and evaluates the scission operator.
2. **Implementation of scission.** While the operator is well-defined on the abstract module, its realisation on large finite-dimensional spaces, and the cost of the associated projections, must be analysed.
3. **Key generation.** A systematic method for sampling distinct admissible sectors (i.e., geometric keys) with controlled intersection properties is needed.
4. **Connections to coding theory.** The admissible sectors are linear subspaces of a GF(4)-module. The relationship between the present geometric constraints and classical coding-theoretic notions (minimum distance, decoding radius, dual codes) remains to be clarified.
5. **Interface with formal verification.** The extent to which existing isolation and verification techniques can be rephrased, or strengthened, inside the present ideal-theoretic language is an open direction.

7.3 Positioning

The framework is offered as a complementary topological language rather than as a competing cryptosystem. It does not seek to displace public-key primitives, symmetric ciphers, or established formal methods. Its contribution is the demonstration that certain isolation and survival properties can be stated as intrinsic algebraic relations on a discrete non-commutative geometry whose states are valued in GF(4).

8 Conclusion

The paper has constructed an algebraic-geometric framework for the hard containment of computational states. The ambient space is the Chrysene Tensor Space $\mathcal{I}_C$, a discrete non-commutative fibrated manifold over $\mathbb{Z}^3$ whose fibres carry orthogonal connectivity of type C_{2h}. States are elements of a module over the Galois field GF(4), so that information is represented natively in base four.

Within this space a choice of complementary projections determines an admissible sector (a geometric key). Encoding consists in embedding a message into that sector; recovery is the corresponding projection. A nilpotent scission operator sends every state that violates the governing orthogonality conditions into the absolute null ideal $\mathcal{I}_\emptyset$, from which no admissible morphism can extract residual information.

The resulting theory therefore expresses containment and selective accessibility as intrinsic algebraic properties of the geometry. It supplies a precise mathematical mechanism—orthogonal decoupling together with nilpotent scission into a null ideal—for the topological survival of admissible states under conditions in which non-compliant configurations are annihilated.

A Related Mathematical and Cryptographic Literature

This appendix situates the constructions of the present paper with respect to the principal mathematical tools employed and with respect to existing cryptographic and coding-theoretic frameworks that operate over finite fields or exploit algebraic structure for isolation and containment.

A.1 Mathematical Foundations

Finite fields and GF(4)**.** The Galois field GF(4) = $\{0, 1, \omega, \omega^2\}$ is the unique field of four elements. It arises as the quadratic extension $\mathbb{F}_2[x]/(x^2 + x + 1)$ and is a standard coefficient domain in coding theory and finite-field arithmetic [LN97, MS77]. Additive and multiplicative structure over GF(4) underpins a rich family of codes, including additive self-dual codes and quantum error-correcting codes constructed via the Hermitian trace inner product [CRSS98, Gab98].

In contrast to these coding-theoretic constructions, the present framework employs GF(4) geometrically—as the coordinate algebra of a discrete lattice—rather than as the ambient alphabet of a linear or additive code.

Non-commutative rings and zero divisors. The appearance of zero divisors in non-commutative rings supplies a natural algebraic mechanism for decoupling. In the present setting the orthogonality relations of the connectivity projections generate a two-sided ideal whose quotient yields a ring $\mathcal{R}_C$ containing non-trivial zero divisors. Related phenomena appear in the study of zero-product preservers and orthogonal decompositions of matrix algebras [LTWW20].

Orthogonal projections and complementary decompositions. Complementary idempotent endomorphisms that sum to the identity are classical in operator algebras and module theory. Their use here to separate an admissible sector from a hazard sector is the module-theoretic counterpart of a direct-sum decomposition that is compatible with a non-commutative multiplication.

Nilpotent operators. Nilpotent endomorphisms (operators N satisfying $N^k = 0$ for some k) appear throughout algebra and have been employed in cryptographic constructions based on nilpotent groups and nilpotent Lie algebras [MS17, SĆ25]. In the present work nilpotency is used in a more elementary way: the scission operator maps non-compliant states to the zero vector and thereafter acts as the zero map on that image.

A.2 Cryptographic and Coding-Theoretic Context

Finite-field cryptography and coding over GF(4)**.** Symmetric and coding-theoretic constructions routinely employ arithmetic in small finite fields. Byte substitution layers in block ciphers, masking schemes, and a variety of algebraic codes are defined over GF(2^m); additive codes over GF(4) form a particularly well-studied subclass [CRSS98, CPS79]. The present framework adopts GF(4) as the native coefficient field of states, but uses it geometrically (to label lattice coordinates) rather than as the ambient domain of a conventional cipher or linear code.

Non-commutative and algebraic cryptography. A substantial literature explores cryptographic protocols whose security rests on decision problems in non-commutative groups or rings [MSU11]. The emphasis in that body of work is typically computational hardness (word problem, conjugacy search, etc.). The present paper does not rely on such hardness assumptions; non-commutativity enters only as the algebraic source of zero divisors that enforce orthogonal decoupling.

Isolation, containment and ideal-theoretic ideas. Classical access-control and isolation mechanisms operate by external policy checks. Algebraic approaches that realise isolation by ideal membership or by mapping into a null object are less common, though related ideas appear in the study of annihilator ideals and in certain formal models of information flow. The nilpotent scission operator of the present work is a concrete instance of the latter style of construction: non-compliance is converted into membership in the zero submodule, after which residual information is inaccessible to admissible morphisms.

A.3 Position of the Present Work

The framework developed here is complementary both to finite-field coding theory and to non-commutative cryptography. It borrows the coefficient field GF(4) and the language of orthogonal projections and nilpotent maps, yet deploys them inside a discrete non-commutative geometry whose primary purpose is the hard geometric containment of states rather than error correction or computational hardness. Geometric keys are realised as choices of admissible sector; encoding and recovery become projections; and scission supplies an algebraic form of forward isolation. The theory is offered as a topological language for containment, not as a drop-in replacement for existing primitives.

B Generation and Distribution of Geometric Keys

The construction of the Chrysene Tensor Space $\mathcal{T}_C$ and the complementary projections p_{safe} and p_{hazard} establishes containment as an intrinsic algebraic property of the geometry. However, the operational security of the framework necessitates a rigorous definition of how the admissible sector $K := \text{im } p_{safe}$ (the geometric key) is dynamically generated and distributed between communicating parties. We formalize this process under two distinct operational paradigms: a substrate-independent algebraic formulation, and a substrate-dependent topological synchronization utilizing native chrysene computational lattices.

B.1 Substrate-Independent Algebraic Distribution

In the substrate-independent paradigm, the communicating parties do not natively possess the physical molecular constraints of the $\mathcal{T}_C$ manifold but rather simulate its discrete non-commutative coordinate algebra computationally.

Definition 13 (Algebraic Key Generation over $GF(4)$)**.** *Let M be the free $GF(4)$-module of finitely supported states. The generation of a geometric key is strictly defined as the pseudo-random selection of a k-dimensional admissible subspace $K \subset M$. The key generation algorithm $\mathcal{G}_{indep}$ outputs an ordered basis $B_K = \{v_1, v_2, \ldots, v_k\}$ such that the vectors satisfy the orthogonality constraints required to synthesize the idempotent projection $p_{safe} \in End_{GF(4)}(M)$, where $p_{safe}^2 = p_{safe}$.*

Because the ambient space lacks physical topological excision, the distribution of B_K must be protected by an exogenous, quantum-resistant key encapsulation mechanism (KEM).

Theorem 6 (Substrate-Independent Endomorphism Isomorphism and Algorithmic Scission)**.** *Let M be the free $GF(4)$-module of finitely supported states. Let $\mathcal{E} = (\mathsf{KeyGen}, \mathsf{Enc}, \mathsf{Dec})$ denote an IND-CCA2 secure post-quantum asymmetric encryption scheme parameterized by security parameter κ.*
If Alice generates an ordered basis $B_K \subset M$ specifying the admissible submodule $K \subset M$ and transmits the encapsulation $C_K = \mathsf{Enc}_{pk_B}(B_K)$ over a public classical channel, then Bob's reconstructed scission operator $\hat{P}^{(B)}_{sciss}$ is mathematically isomorphic to Alice's intended scission operator $\hat{P}^{(A)}_{sciss}$ in $End_{GF(4)}(M)$ with overwhelming probability $1 - negl(\kappa)$.
Under this isomorphism, the software-defined algorithmic scission operator perfectly replicates the algebraic properties of the native topological excision, rigorously evaluating mapping into the absolute null ideal $I_\emptyset$ such that:

$$\hat{P}^{(B)}_{sciss} \circ p^{(B)}_{hazard} \equiv 0 \pmod{I_\emptyset}$$

Proof. Let Alice define the geometric key strictly as the admissible sector $K^{(A)} = \text{span}_{GF(4)}(B_K) \subset M$. The choice of $K^{(A)}$ uniquely and definitively establishes the complementary idempotent projections $p^{(A)}_{safe}, p^{(A)}_{hazard} \in \text{End}_{GF(4)}(M)$ satisfying $p^{(A)}_{safe} + p^{(A)}_{hazard} = \text{id}_M$ and $p^{(A)}_{safe} p^{(A)}_{hazard} = 0$.
Alice transmits the ciphertext $C_K = \mathsf{Enc}_{pk_B}(B_K)$ to Bob. Let $\mathcal{A}$ be a probabilistic polynomial-time (PPT) adversary monitoring or mutating the public classical channel. Because the underlying cryptographic primitive $\mathcal{E}$ is IND-CCA2 secure, the probability that $\mathcal{A}$ can extract B_K from C_K, or forge a valid ciphertext C'_K that decrypts to an alternative valid basis $B_{K'} \neq B_K$, is strictly bounded by a negligible function of the security parameter, $\text{negl}(\kappa)$.
Therefore, upon applying the decryption algorithm $B^{(B)}_K = \mathsf{Dec}_{sk_B}(C_K)$, Bob recovers the exact ordered basis $B^{(B)}_K = B_K$ with probability $1 - \text{negl}(\kappa)$.
Bob algebraically reconstructs the admissible sector $K^{(B)} = \text{span}_{GF(4)}(B^{(B)}_K)$. Because vector spaces over finite fields are uniquely determined by their basis, it follows unconditionally that $K^{(B)} \cong K^{(A)}$ as $GF(4)$-submodules.
Consequently, the localized endomorphism rings evaluate identically. Bob computes his complementary projections:

$$p^{(B)}_{safe}(\psi) = \psi_{safe}, \quad p^{(B)}_{hazard}(\psi) = \psi - \psi_{safe}$$

yielding exact algebraic equivalence $p^{(B)}_{safe} \equiv p^{(A)}_{safe}$ and $p^{(B)}_{hazard} \equiv p^{(A)}_{hazard}$.
Because Bob operates in a substrate-independent environment lacking native molecular topological boundaries, he defines the scission operator $\hat{P}^{(B)}_{sciss}$ algorithmically via the conditional

mapping on M:

$$\hat{P}^{(B)}_{sciss}(\psi) = \begin{cases} p^{(B)}_{safe}(\psi) & \text{if } p^{(B)}_{hazard}(\psi) = 0 \\ 0 & \text{if } p^{(B)}_{hazard}(\psi) \neq 0 \end{cases}$$

Let $\psi \in M$ be a non-compliant state such that $p^{(B)}_{hazard}(\psi) \neq 0$. The software-defined algorithmic evaluation dictates $\hat{P}^{(B)}_{sciss}(\psi) = 0$. Since the zero submodule evaluates strictly as the absolute null ideal $I_\emptyset$, the mapping identically satisfies:

$$\hat{P}^{(B)}_{sciss}(\psi) = 0 \in I_\emptyset$$

Thus, assuming the non-failure of the underlying IND-CCA2 encapsulation, the algorithmic implementation of $\hat{P}^{(B)}_{sciss}$ perfectly preserves the algebraic decoupling and idempotence properties established by the geometry of I_C, effectively enforcing the geometric constraint via a mathematically isomorphic software routine. □

B.2 Substrate-Dependent Topological Synchronization (Cohbit Lattice)

When both the sender and receiver physically instantiate the architecture utilizing the chrysene-based coherent quantum bit (cohbit) computational lattice, the generation and distribution of the geometric key cease to be classical algorithmic operations. Instead, they evaluate as native topological phase transitions across the discrete $\mathcal{I}_C$ manifold.

Definition 14 (Cohbit Key Generation via the QDA Functor)**.** *The geometric key K is not algorithmically generated; it is structurally defined by the Quantitative Design Automation (QDA) Functor. The QDA functor evaluates a continuous spectral derivation over the complex spectral fiber (characteristic 0) and utilizes the canonical residue map of the Witt ring, $\pi : W(GF(4)) \to GF(4)$, to project the state into a discrete combinatorial geodesic. This geodesic natively dictates the orthogonal boundaries of the C_{2h} chiral step-defect geometry, automatically establishing the physical sublattices corresponding to p_{safe} and p_{hazard}.*

In this substrate-dependent environment, key distribution evaluates as the physical synchronization of the internal gauge fields across the spatially separated tensor spaces.

Theorem 7 (Topological Synchronization and Quantum Non-Demolition)**.** *Let the sender ($\mathcal{T}_{C,A}$) and receiver ($\mathcal{T}_{C,B}$) establish a geometric key via the exchange of the localized discrete Berry phase γ_n evaluated over a closed Wilson loop. If an adversarial observer attempts to intercept or clone the topological parameters of the admissible sector K, the exogenous measurement mathematically breaches the absolute Dirichlet bornology (τ_{max}) of the lattice.*

Proof. By the foundational axioms of the metric-spectral fibration, any unauthorized continuous fractional perturbation introduced during the distribution of the $GF(4)$ basis geometrically forces a local gluing failure in the Čech cohomology of the manifold. This manifests strictly as a non-trivial 1-cocycle ($[\tilde{c}] \neq 0 \in \tilde{H}^1$).
The manifestation of this metric singularity autonomously and instantaneously triggers the Nilpotent Scission Operator ($\hat{P}_{sciss}$). Because $\hat{P}_{sciss}$ acts as a non-unitary projection, it algebraically maps the intercepted geometric key directly into the absolute null ideal ($\mathcal{I}_\emptyset$), yielding:

$$\hat{P}_{sciss}(K_{intercepted}) \equiv 0 \pmod{\mathcal{I}_\emptyset}$$

Thus, the physical transmission medium behaves as a perfect Quantum Non-Demolition (QND) isolation barrier. The geometry of the key is structurally annihilated before an eavesdropper can mathematically resolve the C_{2h} parity mapping. Perfect forward secrecy is thereby guaranteed natively by the thermodynamics of the physical substrate. □

References

[CPS79] J. H. Conway, V. Pless, and N. J. A. Sloane. Self-dual codes over GF(3) and GF(4) of length not exceeding 16. *IEEE Transactions on Information Theory*, 25(3):312–329, 1979.

[CRSS98] A. R. Calderbank, E. M. Rains, P. W. Shor, and N. J. A. Sloane. Quantum error correction via codes over GF(4). *IEEE Transactions on Information Theory*, 44(4):1369–1387, 1998.

[Gab98] Philippe Gaborit. On additive GF(4) codes. 1998.

[LN97] Rudolf Lidl and Harald Niederreiter. *Finite Fields*. Cambridge University Press, 2 edition, 1997.

[LTWW20] Chi-Kwong Li, Ming-Cheng Tsai, Ya-Shu Wang, and Ngai-Ching Wong. Nonsurjective zero product preservers between matrices over an arbitrary field. *arXiv preprint arXiv:2003.05317*, 2020.

[MS77] F. J. MacWilliams and N. J. A. Sloane. *The Theory of Error-Correcting Codes*. North-Holland, 1977.

[MS17] Ayan Mahalanobis and Pralhad Shinde. Bilinear cryptography using groups of nilpotency class 2. In *Cryptography and Coding (IMACC 2017)*, volume 10655 of *Lecture Notes in Computer Science*, pages 127–134. Springer, 2017.

[MSU11] Alexei Myasnikov, Vladimir Shpilrain, and Alexander Ushakov. *Non-commutative Cryptography and*

Complexity of Group-Theoretic Problems. American Mathematical Society, 2011.

[SĆ25] Muzafer H. Saračević and Azra Ćatović. Homomorphic cryptographic scheme based on nilpotent lie algebras for post-quantum security. *Symmetry*, 17(10):1666, 2025.

[Sch26a] Charles D. Schaper. *Algebraic Cryptography: Absolute Informational Bounding and Security in the Discrete Tensor Space*. Schaper Systems Press, 2026. ISBN: 979-8-950371-07-3.

[Sch26b] Charles D. Schaper. *Foundations of the Chrysene Formalism: Cohomological Invariants, Operator Algebras, and the Discrete Noncommutative State Space*. Schaper Systems Press, San Francisco, CA, 2026. ISBN: 979-8-950371-08-0.

[Sch26c] Charles D. Schaper. *Operadic Topoi over the Chrysene Tensor Space: The Algebraic Geometry of Categorical Containment and Clinical Logic*. Schaper Systems Press, 2026. ISBN: 979-8-950371-05-9.

[Sch26d] Charles D. Schaper. *Tensorial Framework of the Chrysene Formalism: Universal Algebraic Geometry and Maximal-Entropy Survival*. Schaper Systems Press, San Francisco, CA, 2026. ISBN: 979-8-950371-03-5.