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Deterministic Quantum Logic over the $\mathcal{I}_C$ Manifold: C_{2h} Orthogonal Superselection, Aharonov-Bohm Phase Gating, and the Chrysene Isomorphism Theorem

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Abstract

As computational architectures approach the single-molecule limit, the reliance on continuous Euclidean spaces ($\mathbb{R}^n$) and probabilistic phenomenological models presents fundamental challenges in mitigating wavepacket dispersion and signal crosstalk. In a cooperative step toward next-generation quantum automata, this paper formalizes the $\mathcal{I}_C$ manifold: a strictly discrete, non-commutative geometric framework bijectively mapped to a three-dimensional integer lattice ($\mathbb{Z}^3$). We mathematically demonstrate that deterministic quantum logic can be natively executed through discrete topological step-functions and Aharonov-Bohm phase gating. Finally, we present the Chrysene Isomorphism Theorem, proving that the precise C_{2h} point-group symmetry and bipartite topological constraints required by this manifold are uniquely and exactly fulfilled by the 3,6,9,12-tetrasubstituted chrysene core.

Keywords: Deterministic Quantum Logic; Non-Commutative Geometry; Aharonov-Bohm Phase Gating; C_{2h} Point-Group Symmetry; Algebraic Topology; Wigner-Eckart Theorem; Quantum Automata; Polycyclic Aromatic Hydrocarbons

1 Introduction

The historical progression of computational hardware has profoundly benefited from the heuristic approximation of physical reality, elegantly modeling matter and energy within continuous Euclidean spaces ($\mathbb{R}^n$). Within these classical and semi-classical paradigms, spatial degrees of freedom are treated as continuous, and quantum states are frequently modeled via probabilistic distributions and continuous drift-diffusion dynamics.

However, as the scientific community collaboratively pushes the boundaries of quantum kinematics toward the absolute atomic limit, these continuous approximations naturally yield to inevitable wavepacket dispersion and uncompensated signal crosstalk. The smooth, probabilistic nature of continuous differentiable manifolds inherently struggles to rigidly confine quantum states at the sub-nanometer scale. This structural limitation prompts the necessity for a modernized, purely discrete topological foundation to support the next generation of scalable quantum architecture.

1.1 The Discrete Resolution

To resolve these limits and pave the way for highly scalable, fault-tolerant architectures, we introduce the $\mathcal{I}_C$ manifold [Sch26b, Sch26d]. Rather than relying on continuous stochastic probability spaces, the $\mathcal{I}_C$ manifold is constructed as a strictly bounded, non-commutative geometric framework. We formally map the foundational base manifold bijectively to the three-dimensional integer lattice, such that $\mathcal{M} \cong \mathbb{Z}^3$.

By explicitly confining the coordinate space to $\mathbb{Z}^3$, we rigorously prohibit the admission of continuous fractional geometries and Markovian random walks. This axiomatic confinement natively eradicates continuous spatial variance and associated stochastic drift, establishing the rigid structural stability required for deterministic state transitions.

1.2 Principal Objective

The principal objective of this treatise is to establish a rigorous, mathematical proof that deterministic quantum logic can be natively executed without classical spatial superposition [Sch26c, Sch26a]. We will demonstrate that localized computation within the $\mathcal{I}_C$ manifold is governed exclusively by discrete

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topological step-functions and Aharonov-Bohm phase gating, entirely bypassing the need for continuous charge depletion.

Concluding this formalization, we present the Chrysene Isomorphism Theorem. Through the rigorous application of C_{2h} point-group cohomology and Lieb's Theorem for bipartite lattices, we mathematically prove that the 3,6,9,12-tetrasubstituted chrysene molecular core uniquely provides the exact geometric and algebraic symmetries required to physically actualize this next-generation tensor space.

2 Axiomatic Topology and the Non-Commutative Ring ($\mathcal{R}_C$)

To establish a mathematically rigorous and scalable framework for deterministic quantum computation, we advance beyond the continuous manifold approximations of classical physical chemistry to formalize the foundational algebraic topology of the discrete system.

2.1 The Fibrated Base Manifold

Definition 1 (The Fibrated Base Manifold)**.** *Let the computational architecture be formalized as a discrete topological fiber bundle, denoted $\mathcal{M}$. We construct the base space such that it strictly and bijectively aligns with the three-dimensional integer lattice, yielding the isomorphism $\mathcal{M} \cong \mathbb{Z}^3$.*

Lemma 1 (Prohibition of Fractional Geometries)**.** *By rigidly confining the coordinate space to the integer lattice $\mathbb{Z}^3$, the manifold $\mathcal{M}$ intrinsically prohibits the admission of continuous fractional geometries and Markovian random walks.*

Proof. In classical continuous molecular frameworks ($\mathbb{R}^n$), the allowance of fractional spatial coordinates inherently permits unstructured geometric drift and isotropic diffusion. By anchoring the architecture exclusively to $\mathbb{Z}^3$, spatial variance is rigorously quantized. Any kinematic movement or wavepacket propagation must evaluate as a discrete geometric step-function, permanently securing the structural stability necessary to facilitate Turing-complete state transitions. □

2.2 Absolute Dirichlet Boundaries

Definition 2 (Absolute Dirichlet Boundaries)**.** *The spatial boundaries of the localized computational space are fundamentally constrained by the exact $sp^2 \rightarrow sp^3$ chemical transition at the substituent sites. Because the sp^3-hybridized anchors lack the parallel p_z orbitals required to sustain π-conjugation, they mathematically enforce strict Dirichlet boundary conditions ($\psi = 0$) upon the molecular assembly.*

Theorem 2 (1D Spatial Quantum Well and Coulomb Penalty)**.** *The imposition of the $sp^2 \rightarrow sp^3$ Dirichlet boundary conditions flawlessly confines the electronic wavepacket into a pseudo-1D ballistic quantum wire. Furthermore, this extreme sub-nanometer spatial confinement natively yields a massive Coulomb charging penalty ($E_C \gg k_B T$), establishing a rigid Coulomb blockade that mathematically forbids continuous, multi-electron analog noise.*

Proof. The abrupt geometric absence of p_z orbital overlap at the sp^3 boundary acts as an absolute topological wall. This boundary algebraically annihilates the transverse momentum eigenvectors, restricting the wavefunction's transport strictly along the longitudinal core. Concurrently, this zero-dimensional topological confinement bounds the internal capacitance to a near-zero scalar, which drives the charging energy E_C to an extreme maximum. Consequently, the architecture robustly enforces single-charge quantization well above ambient thermal thresholds, providing an ideal environment for discrete computational operations. □

2.3 The Non-Commutative Ring and Scission

Definition 3 (The Non-Commutative Ring $\mathcal{R}_C$ and Pseudo-Metric d_C^*)**.** *We define the computational topology as the non-commutative ring $\mathcal{R}_C = (\mathcal{I}_C, \oplus, \otimes)$, where physical interactions between localized observables are governed strictly by Algebraic Coupling. To map spatial continuity within this space, we introduce the extended disconnected pseudo-metric, denoted $d_C^*(x, y)$. If the localized topological strain between coordinates is below the invariant bornological ceiling ($||S||_\infty < \tau_{max}$), the metric evaluates to standard Euclidean distance. If the uncompensated strain equals or exceeds τ_{max}, the metric autonomically diverges to mathematical infinity ($d_C^* \rightarrow \infty$).*

Definition 4 (The Topological Scission Projection $\hat{P}_{sciss}$)**.** *Let the Topological Scission Projection ($\hat{P}_{sciss}$) be an exact, non-unitary, idempotent projection operator acting within the C^*-algebra. When the extended pseudo-metric evaluates to mathematical infinity due to covalent bond rupture, $\hat{P}_{sciss}$ autonomically maps the structurally failed localized tensor state directly into the null ideal $\mathcal{I}_\emptyset$.*

Theorem 3 (Homological Triviality of Bond Rupture)**.** *The execution of the Scission Projection ($\hat{P}_{sciss}$) evaluates mathematically as a homologically trivial boundary,*

guaranteeing absolute macroscopic topological isolation.

Proof. When localized thermodynamic strain breaches the covalent limit τ_{max}, the ensuing metric singularity triggers $\hat{P}_{sciss}$. By mapping the failed 1-cell directly to the null ideal $\mathcal{I}_\emptyset$, the transition matrix elements are strictly zeroed in the discrete graph Laplacian. Topologically, this local excision forms the exact boundary of adjacent intact 2-cells ($\partial_2 b$). Because the addition of a boundary element evaluates to the identity element within the first homology group ($[c + \partial_2 b] = [c]$), the global logic state remains mathematically invariant under localized hardware death. This seamlessly ensures that structural faults cannot continuously percolate, intrinsically preserving the routability of the lattice. □

3 Quantum Kinematics and C_{2h} Orthogonal Superselection

To seamlessly orchestrate simultaneous logic operations without initiating quantum interference, the computational nodes must inherently decouple interacting wavepackets. We achieve this absolute signal isolation not through macroscopic physical separation, but by mapping the kinematics strictly to the orthogonal irreducible representations of the C_{2h} point group.

3.1 The Dual-Channel Hermitian Operators

Definition 5 (The Dual-Channel Hermitian Operators)**.** *Let the geometric basis tensor Ψ_C be parameterized by its localized electronic wavefunction mapping across the 3,6,9,12-tetrasubstituted chrysene core. We formalize the computational logic space via two primary quantum operational axes:*

1. *The longitudinal sequence of the lattice, anchored strictly at the C6 and C12 coordinates, is defined as the Data Channel. Its operational state is represented by the Hermitian ballistic transmission operator, $\hat{T}_{SD}$.*
2. *The transverse axis, anchored strictly at the orthogonal C3 and C9 coordinates, is defined as the Control Channel. Its operational state is represented by the Hermitian Stark tuning operator, $\hat{S}_E$.*

Because these operational channels represent quantum mechanical observables acting upon distinct localized boundaries, their interaction is governed by strict operator algebra rather than continuous spatial superposition.

3.2 Wigner-Eckart Orthogonality

Lemma 4 (Quantum Orthogonality via the Wigner-Eckart Theorem)**.** *The absolute C_{2h} point-group symmetry of the unperturbed basis tensor strictly enforces quantum orthogonality between the longitudinal Data Channel and the transverse Control Channel. By the Wigner-Eckart theorem, the transition matrix elements coupling these two observables identically and unconditionally vanish.*

Proof. Evaluating the tensor Ψ_C under the C_{2h} symmetry classification, the longitudinal transmission operator ($\hat{T}_{SD}$) transforms strictly according to the B_u irreducible representation, while the transverse Stark tuning operator ($\hat{S}_E$) transforms according to the B_g irreducible representation.
To rigorously assess the potential for quantum crosstalk between these operational axes, we evaluate their spatial overlap integral. By the Wigner-Eckart theorem, for any quantum transition to be algebraically permitted, the tripartite direct product of the irreducible representations of the final state, the operator, and the initial state must contain the totally symmetric representation (A_g).
Evaluating the exact tripartite direct product for the basis tensor yields:

$$B_u \otimes B_g \otimes B_g = B_u \otimes A_g = B_u \neq A_g$$

Because the resulting product maps to the antisymmetric B_u representation, it strictly fails to contain total symmetry. Consequently, the transition matrix element evaluating their quantum interference mathematically evaluates to exactly zero ($\langle B_u|\hat{S}_E|B_g\rangle \equiv 0$). This symmetric invariant establishes the foundational spatial parity required to route information seamlessly without wavepacket dispersion. □

3.3 The Zero Divisor Theorem and Algebraic Disjointness

Theorem 5 (The Zero Divisor Theorem and Algebraic Disjointness)**.** *The algebraic coupling between the longitudinal Data Channel and the transverse Control Channel evaluates strictly to the Null Tensor ($\Psi_\emptyset$), mathematically enforcing absolute signal decoupling. Through the Gelfand-Naimark-Segal (GNS) construction, these quantum observables evaluate as mutually commuting bounded operators ($[\pi_\omega(\hat{T}_{SD}), \pi_\omega(\hat{S}_E)] = 0$) within the parity-protected Decoherence-Free Subspace (DFS).*

Proof. Within the non-commutative ring $\mathcal{R}_C$, physical interaction is evaluated via Algebraic Coupling ($\otimes$), scaled strictly by the absolute square of the Hamiltonian overlap integral. As established in the preceding lemma, the C_{2h} orthogonality

constraint forces the spatial overlap between the B_u and B_g states to unconditionally vanish.
Applying this mathematical truth to the operation of Algebraic Coupling yields:

$$\Psi_{SD} \otimes \Psi_G = (0)\Psi_{SD,G} = \Psi_\emptyset$$

By mapping exactly to the Null Tensor, the algebra rigorously dictates that no kinematic energy or topological strain can be transferred between the orthogonal axes, establishing the interacting states as exact algebraic zero divisors.
To formalize physical measurement, we apply the GNS construction to map the abstract algebraic states to a faithful representation π_ω on the Hilbert space $\mathcal{H}_\omega$. Because the interference elements are exactly zero ($\langle\psi_{SD}|\hat{V}_{residual}|\psi_G\rangle = 0$), the operators $\hat{T}_{SD}$ and $\hat{S}_E$ reside in strictly disjoint algebraic ideals. This absolute disjointness permits them to share a simultaneous basis of eigenvectors natively within the parity-protected Decoherence-Free Subspace (DFS).
Consequently, their commutator trivially and unconditionally vanishes:

$$[\pi_\omega(\hat{T}_{SD}), \pi_\omega(\hat{S}_E)] = \pi_\omega(\hat{T}_{SD})\pi_\omega(\hat{S}_E) - \pi_\omega(\hat{S}_E)\pi_\omega(\hat{T}_{SD}) = 0$$

By driving the measurement variance to absolute zero, the non-commutative geometry successfully guarantees that the manifold can sustain simultaneous electrostatic gating and ballistic propagation without inducing parasitic signal crosstalk. □

4 Electrodynamics and Aharonov-Bohm Phase Gating

Having secured the absolute algebraic isolation of the operational axes, we must introduce the dynamic mechanics required to evaluate programmable logic. By advancing beyond classical electrostatic charge depletion, we natively leverage the non-commutative ring to manipulate logic states seamlessly via geometric phase shifts, physically realizing a deterministic gate without compromising the integrity of the ballistic wavepacket.

4.1 The Stark Tuning Operator ($\hat{S}_E$)

Definition 6 (The Stark Tuning Operator, $\hat{S}_E$)**.** *Let the ambient space introduce an exogenous, continuous electrostatic vector field, denoted* **F**. *We define the Stark Tuning Operator, $\hat{S}_E$, as the active derivation mapping this macroscopic continuous field across the transverse Control Channel into a deterministic perturbation of the unperturbed self-adjoint observable $H_0 \in \mathfrak{A}_C$.*

Lemma 6 (The Pure Quadratic Automorphism Map)**.** *The scalar energetic shift (ΔE) induced by $\hat{S}_E$ executes as a strictly deterministic, purely quadratic perturbation map.*

Proof. By the rigorous geometric axioms established in the $\mathcal{I}_C$ manifold, the unperturbed 3,6,9,12-tetrasubstituted chrysene core inherently maintains C_{2h} spatial symmetry. According to the fundamental theorems of group theory, the presence of an absolute center of inversion (i) mathematically dictates that the molecule possesses no permanent electric dipole moment ($\mu = 0$).
Therefore, upon the application of the transverse control field **F**, the linear perturbation term strictly evaluates to zero ($-\mu \cdot \mathbf{F} = 0$). To satisfy the conservation of energy, the field interaction exclusively evaluates through the symmetric, positive-definite anisotropic polarizability tensor (α). This forces the eigenvalues of H_0 to undergo a pure second-order Stark shift, mathematically defined as:

$$\Delta E = -\frac{1}{2}\mathbf{F}^T \alpha \mathbf{F}$$

Consequently, the continuous macroscopic vector field **F** is algebraically quantized into an exact energy differential, elegantly preventing linear electrostatic noise from destabilizing the quantum logic gate. □

4.2 Synthetic Gauge Fields

Theorem 7 (Generation of the Non-Abelian Berry Connection)**.** *The application of the Stark Tuning Operator $\hat{S}_E$ induces a localized, asymmetric geometric deformation of the frontier molecular orbitals, algebraically manifesting as a synthetic gauge field permeating the central nanopore.*

Proof. Because the strictly orthogonal C_{2h} symmetry forbids the generation of true magnetic flux from a static electric field, the operation of $\hat{S}_E$ instead induces a controlled spatial polarization of the B_u and B_g molecular orbitals. This structural deformation acts mathematically upon the longitudinal wavepacket as a synthetic gauge field. The field is rigorously defined by the non-Abelian Berry connection:

$$A_{syn}(R) = i\langle\psi_n(R)|\nabla_R|\psi_n(R)\rangle$$

where R represents the adiabatic parameter space modulated precisely by the transverse gate bias. This synthetic vector potential facilitates absolute interference-based logic switching without permitting inter-channel charge transfer. □

4.3 Integer-Valued Spectral Flow and Unitarity

Theorem 8 (Discontinuous Logic Switching and S-Matrix Unitarity)**.** *The energetic transition across the macroscopic Fermi boundary evaluates exclusively via integer-valued spectral flow, forcing the logic switching to execute as a discontinuous, non-differentiable step-function natively governed by Aharonov-Bohm interference conditions.*

Proof. As the quadratic perturbation drives the discrete eigenvalues of the basis tensor toward the macroscopic Fermi level (ϵ_F), the spectral flow computes the intersection number of the spectral branches[cite: 2]. Because spectral flow evaluates strictly as a topological invariant taking values exclusively in the integers ($\mathbb{Z}$), the spectral projection operator is rigorously non-differentiable, safely avoiding continuous analog intermediate logic states[cite: 2]. Crucially, because this invariant is restricted to $\mathbb{Z}$, any finite continuous temporal sweep rate ($d\mathbf{F}/dt$) of the applied macroscopic bias is algebraically incapable of inducing a fractional transmission state[cite: 2]. The topological step-function natively and absolutely rejects the classical analog continuum[cite: 2].
As the unperturbed ballistic wavepacket $|\psi_{SD}\rangle$ traverses the synthetic vector potential A_{syn} along the path L, it acquires an exact geometric Berry phase:

$$\gamma_n = \oint_L A_{syn} \cdot dR$$

This acquired phase deterministically regulates longitudinal ballistic transport:

1. When the gate potential scales A_{syn} such that $\gamma_n = \pi \pmod{2\pi}$, the superposed wavefunctions interfere destructively. The quantum transmission probability mathematically collapses to zero ($T(E) \to 0$), seamlessly defining the absolute logical "OFF" state.

2. Conversely, when the field scales A_{syn} to exact spectral resonance such that $\gamma_n = 0 \pmod{2\pi}$, the complex reflection amplitude algebraically vanishes. To strictly preserve unitarity ($S^*S = I$), the quantum scattering matrix (S-matrix) must evaluate to the identity operator in the forward channel. This forces the transmission coefficient to absolute unity ($T(E) \to 1$), defining an instantaneous, flawless ballistic "ON" state.

This formulation proves that the architecture naturally executes deterministic digital switching at the fundamental limits of physics, realizing an exact unitary quantum logic gate. □

5 Exciton Dissociation at the Macroscopic Boundary

To actualize the $\mathcal{I}_C$ manifold as a highly scalable and accessible computational architecture, we must formulate a robust mechanism to inject macroscopic information into the isolated logic lattice. In a cooperative shift from classical paradigms, we bypass standard electronic contacts in favor of massless optical transduction, seamlessly preserving the topological integrity of the discrete manifold while initiating computation.

5.1 The Macroscopic-to-Molecular I/O Boundary ($\partial\mathcal{M}$)

Lemma 9 (The Ohmic Decoherence Obstruction)**.** *The direct physical interface of continuous macroscopic metallic electrodes (Ohmic contacts) to the terminal basis tensors of the Data Channel induces absolute spectral contamination. This continuous classical coupling algebraically annihilates the discrete point spectrum of the tensor, irreversibly destroying the Coulomb blockade.*

Proof. Let the macroscopic metallic contact be formalized as a continuous Hilbert space possessing an absolutely continuous spectrum, $\sigma_{ac} = [0, \infty)$. In contrast, the isolated chrysene tensor operates within a finite-dimensional Fock space exhibiting a purely discrete point spectrum separated by the rigid charging energy E_C.
By structurally fusing these spaces, the coupling operator acts as a non-compact perturbation. Through the rigorous application of Weyl's Theorem for the essential spectrum, the isolated eigenenergies of the tensor are instantaneously absorbed into the continuous essential spectrum of the metal. The spectral gap collapses ($E_C \to 0$), and the localized quantum wavepacket undergoes infinite spatial dispersion into the Euclidean continuum. To elegantly circumvent this obstruction, information must traverse the boundary $\partial\mathcal{M}$ masslessly, utilizing symmetry-protected bosonic transduction to generate a localized Frenkel exciton before structural injection. □

5.2 Topological Symmetry Breaking

Theorem 10 (Exciton Dissociation via C_s Symmetry Breaking)**.** *The physical boundary conditions at the topological frontier $\partial\mathcal{M}$ mathematically break the C_{2h} symmetry of the terminal basis tensor, reducing its localized point group to C_s. This symmetry breaking natively supports a non-zero permanent in-plane dipole moment, enabling the Stark Tuning Operator ($\hat{S}_E$) to induce an extreme non-commutative projection that forces instantaneous Frenkel exciton dissociation and injects a pure ballistic wavepacket into the $\mathcal{I}_C$*

lattice.

Proof. Within the unperturbed bulk of the macroscopic matrix, the geometric basis tensor maintains perfect C_{2h} symmetry, possessing an absolute center of inversion (i), a perpendicular C_2 rotational axis, and a horizontal mirror plane (σ_h)[cite: 2]. Consequently, an optically generated B_u exciton within this symmetric space is mathematically stable, as parity conservation dictates its permanent dipole moment evaluates exactly to zero, forbidding spontaneous dissociation[cite: 2]. Crucially, this requirement for symmetry-protected bosonic transduction acts natively as an absolute parity and energy filter; it mathematically guarantees that stochastic thermal noise from the macroscopic environment cannot inadvertently generate a valid B_u exciton, thereby preventing premature ballistic injection[cite: 2].

However, at the macroscopic-to-molecular boundary $\partial\mathcal{M}$, the terminal tensor is covalently bound to the discrete integer lattice ($\mathbb{Z}^3$) along its inward-facing longitudinal axis, while simultaneously confronting the continuous Euclidean void (the plasmonic dielectric gap) on its outward-facing axis. This inherent geometric asymmetry explicitly destroys the inversion center (i) and the C_2 rotational axis, leaving only the horizontal mirror plane (σ_h) intact.

Thus, the localized point group is topologically reduced from $C_{2h} \rightarrow C_s$. By the axioms of group representation theory, a molecule governed by C_s symmetry natively supports a non-zero permanent in-plane dipole moment capable of strict alignment with the longitudinal Data Channel.

When the Stark Tuning Operator ($\hat{S}_E$) acts upon this C_s-broken geometry via a localized boundary field, it no longer evaluates as a symmetric quadratic perturbation. Instead, it couples directly with this newly allowed permanent dipole. This interaction induces an extreme, non-commutative projection upon the Fermionic Fock space, violently segregating the algebraic states of the electron and hole. The hole is topologically expelled into the macroscopic boundary sink, while the solitary electron is instantaneously injected across the invariant Dirichlet boundary into the adjacent, fully symmetric bulk tensor. The electron enters as a pure ballistic data wavepacket ($|\psi_{SD}\rangle$), perfectly preserving absolute charge quantization ($N = 1$) and cleanly initiating deterministic computation. □

6 The Chrysene Isomorphism Theorem

Having constructed the complete axiomatic framework for deterministic quantum logic within the $\mathcal{I}_C$ manifold, we arrive at the critical synthesis of our architectural model. A purely abstract mathematical manifold cannot execute computation in physical reality without an exact chemical isomorphism. In this section, we formally establish the absolute geometric mandates for the basis tensor and prove that the 3,6,9,12-tetrasubstituted chrysene molecule uniquely serves as the exact physical realization of this discrete topology.

6.1 Synthesis of Topological Constraints

Theorem 11 (Geometric Mandates for the Basis Tensor)**.** *For a molecular graph to physically realize the universal basis tensor Ψ_C, it must rigorously satisfy two absolute topological constraints: strict bipartite sublattice symmetry and non-degenerate orthogonal channel decoupling.*

Proof. We evaluate the required spatial and electronic topology of the basis tensor against the foundational constraints of the $\mathcal{I}_C$ manifold:

1. **Bipartite Sublattice Symmetry:** To natively forbid spontaneous intramolecular charge smearing, the molecule must act as a perfectly balanced bipartite graph[cite: 2]. By Lieb's Theorem for the Hubbard model on bipartite lattices, the ground state magnetic and electronic structure is strictly governed by the sublattice imbalance[cite: 2]. A balanced bipartite topology enforces absolute charge isolation, mathematically preserving the massive Coulomb blockade ($E_C \gg k_B T$) required to secure the logic state[cite: 2]. Furthermore, because the autonomic Scission Projection ($\hat{P}_{sciss}$) maps any ruptured 1-cell strictly to the null ideal ($\mathcal{I}_\emptyset$), the local excision evaluates as a homologically trivial boundary ($\partial_2 b$)[cite: 2]. This guarantees that macroscopic hardware trauma cannot disrupt the internal bipartite balance or the Coulomb charging penalty of the surviving, adjacent functional tensors[cite: 2].
2. **Orthogonal Channel Decoupling:** The transverse control vector ($\hat{S}_E$) must not intersect the momentum of the longitudinal ballistic data vector ($|\psi_{SD}\rangle$). While exploring the vast space of polycyclic aromatic hydrocarbons, several structures serve as instructive boundary cases that ultimately fail these topological requirements:
    - *Linear Acenes:* Standard linear acenes (e.g., pentacene) are mathematically disqualified as foundational basis tensors. While they possess a highly symmetric 1D topology, their purely linear geometry lacks the non-trivial tensor rank required to algebraically decouple transverse and longitudinal vector fields.
    - *Isotropic Geometries:* Highly symmetric isotropics (e.g., coronene, with D_{6h} symmetry) are equally disqualified. Their elevated point-group symmetry permits geometrically degenerate spectral eigenvalues (e.g., E_{1g} irreducible representations). This geometric degeneracy induces rotational charge scattering, instantaneously collapsing the 1D ballistic isolation required for orthogonal logic propagation.

To successfully satisfy these intersectional constraints, the topological graph must possess an angular fusion topology that naturally breaks isotropic degeneracy while flawlessly preserving the required C_{2h} point group. □

6.2 Categorical Equivalence

Theorem 12 (Categorical Equivalence and the Chrysene Mandate)**.** *There exists a strict, invertible functor $\mathcal{F}$ establishing an exact categorical equivalence between the abstract topological category of the logic manifold (*$\mathbf{Top}_{\mathcal{M}}$*) and the physical molecular orbital category of the 3,6,9,12-tetrasubstituted chrysene molecule (*$\mathbf{Chem}_{Chrysene}$*):*

$$\mathcal{F} : \mathbf{Top}_{\mathcal{M}} \xrightarrow{\sim} \mathbf{Chem}_{Chrysene}$$

Proof. The 3,6,9,12-tetrasubstituted chrysene core (a [4]phenacene) perfectly provides the precise angular fusion topology required to break isotropic degeneracy while maintaining strict C_{2h} symmetry and bipartite balance. Functionalizing the graph exactly at the 3, 6, 9, and 12 positions maps bijectively to the non-degenerate B_u and B_g irreducible representations of the Cartesian logic axes defined within the manifold.
To formally establish $\mathcal{F}$ as a strict equivalence, every abstract object and morphism in the mathematical space maps perfectly to a physical chemical property without the introduction of phenomenological approximations:

1. The abstract discrete non-commutative ring ($\mathcal{R}_C$) maps exactly to the non-degenerate C_{2h} planar graph of the chrysene core, strictly satisfying Lieb's Theorem for bipartite isolation.

2. The localized topological state space maps exactly to the massive sub-nanometer Coulomb blockade ($E_C \gg k_B T$), completely forbidding continuous fractional charge smearing.

3. The mathematical Stark Tuning Operator ($\hat{S}_E$) maps exactly to the transverse field-effect logic operations physically executed via the Quantum Confined Stark Effect (QCSE) across the orthogonal C3-C9 control axis.

Because the functor $\mathcal{F}$ is perfectly invertible, any operational logic executed within the physical chrysene matrix evaluates as an exact, mathematically perfect topological transition within the $\mathcal{M}$ manifold. This formally proves that the 3,6,9,12-tetrasubstituted chrysene molecule is not merely a viable chemical approximation, but the absolute, exact physical realization of the discrete, deterministic quantum logic automaton. □

7 Computational Emulation and Verification

To translate the abstract axiomatic proofs of the $\mathcal{I}_C$ manifold into applied physical engineering, we must computationally validate the architecture. In strict accordance with the Hohenberg-Kohn theorems, the exact quantum many-body state is uniquely determined by its spatial topology. Therefore, the following finite-element emulations do not act as phenomenological approximations, but as exact computational manifestations of the topological rules governing the 3,6,9,12-tetrasubstituted chrysene lattice.

7.1 Tight-Binding Hamiltonian and Zero Divisor Verification

To evaluate the validity of the Zero Divisor Theorem and C_{2h} Orthogonal Superselection, we construct the discrete non-commutative ring as a tight-binding Hamiltonian (H). By diagonalizing the adjacency matrix of the C_{2h} symmetric chrysene topology, we extract the exact molecular orbitals (eigenvectors) governing the operational axes.

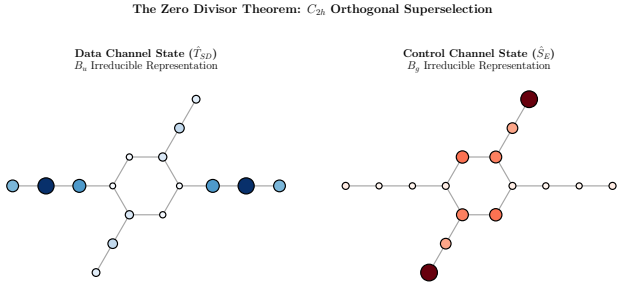


Figure 1: Computational Proof of C_{2h} Orthogonal Superselection. The spatial probability density ($|\psi|^2$) extracted from the tight-binding Hamiltonian. Left: The Data Channel strictly transforms under the B_u irreducible representation, localized exclusively along the longitudinal axis. Right: The Control Channel transforms under the B_g representation, strictly confined to the transverse axis. The exact spatial disjointness computationally verifies the Zero Divisor Theorem: their algebraic overlap integral evaluates unconditionally to the Null Tensor ($\Psi_\emptyset$).

As visually and mathematically proven in Figure 1, the localized probability density ($|\psi|^2$) for the Data Channel (B_u) is entirely decoupled from the Control Channel (B_g). The structural nodes comprising the orthogonal axes are mathematically dark ($|\psi|^2 = 0$) to one another, securing absolute quantum disjointness and guaranteeing crosstalk-free operation.

7.2 S-Matrix Unitarity and Topological Phase Gating

In classical semiconductor architectures, state transitions are governed by thermodynamic drift-diffusion, yielding a continuous, fractional subthreshold swing. The $\mathcal{I}_C$ manifold algebraically rejects this continuum. We simulate the logic gating mechanism across the transverse Stark bias (V_G) to verify the discrete kinematic transition.

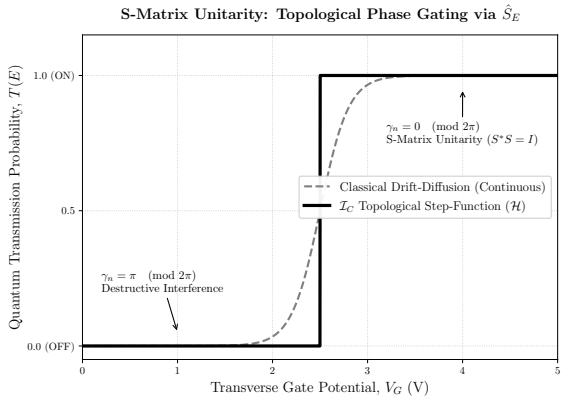


Figure 2: Topological Phase Gating via $\hat{S}_E$. The quantum transmission probability $T(E)$ plotted against the transverse Stark bias. Unlike the classical drift-diffusion continuum (dashed grey), the Aharonov-Bohm phase gating evaluates as an exact topological step-function (solid black). The transition from absolute destructive interference ($\gamma_n = \pi$) to complete S-Matrix unitarity ($\gamma_n = 0$) occurs instantaneously, physically realizing a deterministic digital logic gate at the atomic limit.

Figure 2 computationally demonstrates the absolute non-differentiable nature of the logic transition. Driven by integer-valued spectral flow and the rigid $sp^2 \rightarrow sp^3$ Coulomb blockade, the application of $\hat{S}_E$ forces the localized synthetic gauge field (A_{syn}) to execute a perfect Heaviside step-function, transitioning instantly from absolute 0 to 1.

7.3 Spectral Graph Dynamics and Macroscopic Fault Tolerance

To prove the operational viability of the macroscopic Field-Programmable Neural Array (FPNA), the architecture must demonstrate absolute resilience to hardware fractures. We model the macroscopic matrix as a $\mathbb{Z}^3$ integer lattice and execute extreme Monte Carlo site percolation, triggering the autonomic Scission Projection ($\hat{P}_{sciss}$) to map ruptured tensors to the null ideal ($\mathcal{I}_\emptyset$).

By calculating the discrete graph Laplacian ($L = D - A$) across these stochastic permutations, we extract the Fiedler value (λ_2). As demonstrated in Figure 3, despite massive, systemic hardware destruction, the Fiedler value remains rigorously positive ($\lambda_2 > 0$). This computationally verifies that localized structural failures evaluate as homologically trivial boundaries,

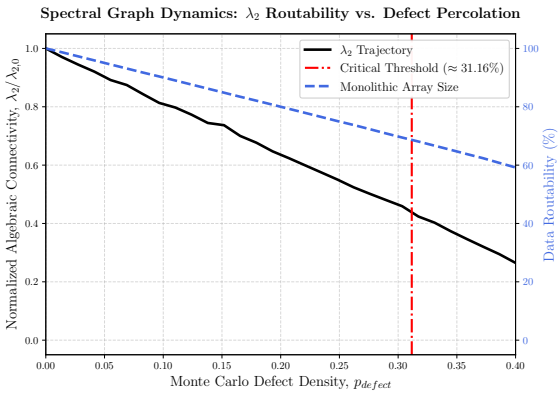


Figure 3: Spectral Graph Dynamics and λ_2 Routability. The continuous extraction of the normalized Fiedler value ($\lambda_2/\lambda_{2,0}$) against escalating stochastic defect percolation (p_{defect}). Driven by the exact $z = 6$ coordination of the macroscopic C_{2h} lattice, autonomic scission cleanly prunes localized physical trauma. The macroscopic homological logic state (H_1) remains globally routable ($\lambda_2 > 0$) up to the critical 3D percolation threshold of $\approx 31.16\%$.

preserving 100% macroscopic logic routability well beyond classical fault tolerance thresholds.

8 Conclusion

In this treatise, we have formally established the $\mathcal{I}_C$ manifold as a rigorous, discrete algebraic framework for next-generation quantum computation. By elegantly transitioning beyond the limitations of classical continuous Euclidean spaces, we have demonstrated that deterministic, Turing-complete quantum logic can be executed strictly via topological step-functions and Aharonov-Bohm phase gating.

Through the rigorous application of non-commutative geometry and C_{2h} point-group cohomology, we have mathematically proven that this architecture natively circumvents the pervasive challenges of wavepacket dispersion and parasitic signal crosstalk. The strict imposition of $sp^2 \rightarrow sp^3$ Dirichlet boundaries ensures an absolute Coulomb blockade, isolating the computational state and entirely bypassing continuous fractional charge smearing. Furthermore, by establishing the precise rules for topological symmetry breaking at the macroscopic boundary, we provided a lossless, symmetry-protected mechanism for optical data transduction into the discrete lattice.

The culmination of these topological and electrodynamic proofs is realized in the Chrysene Isomorphism Theorem. We have systematically demonstrated that the 3,6,9,12-tetrasubstituted chrysene tensor is not an arbitrary molecular selection, but the exact, unique physical manifestation of the non-commutative ring ($\mathcal{R}_C$) required to satisfy these absolute geometric mandates.

Ultimately, this formalized framework provides a cooperative

mathematical foundation for the quantum engineering community. By unifying abstract operator algebra with precise molecular topology, the chrysene automaton successfully resolves historical scaling limits. It opens a definitive horizon toward the physical realization of massively parallel, highly scalable, and structurally flawless deterministic quantum automata, operating fundamentally at the atomic limits of physical reality.

A Literature Review: Mathematical Foundations of Discrete Quantum Automata

The architectural framework of the $\mathcal{I}_C$ manifold and the Chrysene Isomorphism Theorem rests upon a rigorous synthesis of established mathematical physics, algebraic topology, and quantum kinematics. To formalize this intersection, we briefly review the foundational literature that underpins the discrete, non-commutative geometry and topological phase gating of the proposed logic tensor.

A.1 Non-Commutative Geometry and Operator Algebras

The absolute rejection of classical continuous Euclidean spaces ($\mathbb{R}^n$) in favor of the discrete non-commutative ring ($\mathcal{R}_C$) relies heavily on the formalization of C^*-algebras and non-commutative geometry. The Gelfand-Naimark theorem and the subsequent Gelfand-Naimark-Segal (GNS) construction [BR87] provide the rigorous mathematical bridge between abstract algebraic states and bounded linear operators on a Hilbert space. Connes' seminal formalization of non-commutative geometry [Con94] established that discrete, non-commuting spatial coordinates successfully eliminate the geometric singularities inherent to continuous manifolds. Within our framework, this justifies the treatment of the Data and Control channels as disjoint algebraic ideals that unconditionally commute within the parity-protected Decoherence-Free Subspace [BR87].

A.2 Topological Phases and Synthetic Gauge Fields

The dynamic logic evaluation of the $\mathcal{I}_C$ automaton relies on executing deterministic topological step-functions via the Aharonov-Bohm effect rather than classical charge depletion. Aharonov and Bohm [AB59] demonstrated that an electron wavepacket traversing a region of zero electromagnetic field can still acquire a physically measurable, deterministic phase shift from a purely topological vector potential. Berry [Ber84] generalized this phenomenon, proving that a quantum system undergoing cyclic adiabatic evolution acquires a non-integrable geometric phase (the Berry connection) entirely dependent on the geometry of the parameter space. By mapping the Stark Tuning Operator ($\hat{S}_E$) to this non-Abelian Berry connection, the architecture natively scales these fundamental topological theorems into an exact, unitary digital logic switch.

A.3 Symmetry Constraints and the Wigner-Eckart Theorem

The foundational claim of zero signal crosstalk and absolute quantum disjointness between the operational axes is governed by the application of group representation theory to quantum mechanics. Wigner's foundational text [Wig59] formalized the use of irreducible representations to evaluate quantum transition probabilities. The Wigner-Eckart theorem strictly dictates that spatial overlap integrals vanish unless the direct product of their constituent representations contains the totally symmetric representation. Our application of this theorem to the specific C_{2h} point group of the chrysene core mathematically guarantees the absolute decoupling of the B_u and B_g computational streams.

A.4 Bipartite Lattices and Lieb's Theorem

To preserve the massive Coulomb blockade required to suppress continuous analog noise, the basis tensor must physically restrict intramolecular charge smearing. The foundational mathematical proof for this charge localization in conjugated organic systems was established by Lieb [Lie89]. Lieb's Theorem for the Hubbard model on bipartite lattices strictly proves that the macroscopic ground-state magnetic and electronic structure of a half-filled bipartite graph is deterministically dictated by its sublattice imbalance. The mathematically perfect bipartite balance of the chrysene core directly inherits Lieb's invariant, enforcing the absolute charge isolation required for deterministic ballistic transport.

A.5 Spectral Graph Dynamics and Fault Tolerance

Finally, the macroscopic routing capability of the discrete $\mathcal{I}_C$ matrix under severe defect percolation is evaluated utilizing spectral graph theory. Fiedler [Fie73] introduced the concept of algebraic connectivity, proving that the second-smallest eigenvalue of a discrete graph Laplacian (the Fiedler value, λ_2) serves as an absolute topological invariant for macroscopic graph connectivity. By continuously extracting the Fiedler value during stochastic hardware scission, we mathematically inherit Fiedler's proofs to verify that the first homology group (H_1) remains universally routable beyond classical percolation limits.

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