

Deterministic Navigation of Algorithmic Stablecoin Devaluations: An H_∞ Robust Control Framework over the T_C Liquidity Manifold

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Abstract

Classical quantitative finance has traditionally modeled systemic liquidity through stochastic approximations. During periods of supercritical stress, these continuous probability distributions can encounter structural limitations, behaving more like a deterministic topological manifold (T_C) governed by rigid boundary conditions. This paper introduces a complementary deterministic framework treating algorithmic stablecoins as dynamical systems under active feedback control. Specifically, we introduce a deterministic threshold, the Kinematic Tether Limit (r_{tol}), that marks the point at which an algorithmic stablecoin's peg-defense capacity is mathematically exhausted. We then derive three operational strategies for traders and protocols to navigate or contain the resulting structural transition: (I) executing directional short positions within the *metastable phase* strictly prior to the stablecoin's recursive devaluation, (II) achieving microeconomic resilience by engineering Kolmogorov-Arnold-Moser (KAM) invariant portfolio tori governed by Hamiltonian mechanics, and (III) deploying the Topological Scission Operator ($\hat{D}_{sciss}$) as an autonomic systemic firewall to quarantine strain and preserve macroeconomic homeostasis.

Keywords: Chrysene tensor space, Algorithmic Stablecoins, H_∞ Robust Control, Systemic Risk, Kinematic Tether Limit, Isomorphic Divergence, KAM Torus.

1 Introduction

In May 2022, the Terra (UST) algorithmic stablecoin and its sister-token, LUNA, experienced a catastrophic \$40 billion collapse over a matter of days. Designed to maintain a 1.00 USD peg without exogenous fiat collateral, the protocol relied on a continuous, endogenous arbitrage mechanism: users could algorithmically burn 1.00 USD worth of LUNA to mint 1 UST, or burn 1 UST to mint 1.00 USD worth of LUNA. When an unprecedented wave of UST selling pressure hit the secondary market, the smart contract reflexively hyper-inflated the supply of LUNA to absorb the shock. This massive dilution triggered a systemic run on the bank, obliterating the sister-token's price and permanently breaking the stablecoin's peg in a textbook hyper-inflationary death spiral.

This canonical event exposed a profound vulnerability in the architecture of decentralized finance, presenting a critical opportunity to expand modern financial theory. Prevailing literature has extensively documented these destabilizations through various stochastic, empirical, and agent-based methodologies. Notable post-mortem simulation studies of the Terra-LUNA col-

lapse have meticulously charted the mechanics of these reflexive feedback loops and systemic contagion [LMS23, FM26]. To understand peg defense mechanisms, recent work has employed mean-field game models to capture the strategic, arbitrageur-driven restoration of stablecoins across primary and secondary markets [MK26]. Concurrently, empirical regime-switching analyses have demonstrated how stablecoin ecosystems transition between stable phases and high-volatility, run-prone environments driven by network congestion and liquidity evaporation [KLR⁺26, MR26].

While these stochastic and agent-based views are invaluable for capturing the behavioral and probabilistic dynamics of market participants, they implicitly rely on continuous approximations of market liquidity. This perspective can sometimes overlook the underlying, rigid physical limitations of the network. This paper introduces a deterministic topological and control-theoretic lens designed to *complement*—rather than replace—the existing literature. By augmenting probabilistic behavioral models with strict geometric capacity boundaries, we can mathematically pinpoint the exact thresholds where agent-based arbitrage is physically forced to halt, transforming a tail-risk probability into

a structurally predictable certainty.

We posit that systemic liquidity can be modeled as a *deterministic topological manifold*, denoted as T_C . This manifold is characterized by finite boundary conditions where the capacity for asset conversion is bounded. Consequently, we model an algorithmic stablecoin not merely as a passive market instrument subject to random walks, but as a *dynamical system under active feedback control*. The system's objective is to maintain a state—the market price—at a fixed setpoint of 1.00 USD. The control input is the protocol's treasury, which algorithmically deploys capital to counteract price deviations.

Under this control-theoretic lens, the peg-defense mechanism can be mapped to what we term the Chrysene Formalism. A "de-peg" represents a limitation of this binding mechanism, where the control input (treasury capital) is insufficient to overcome the external force (selling pressure) acting on the system. The subsequent recursive devaluation is the observable trajectory of the system after its control capacity has been exceeded.

Applying robust control theory to macroeconomic and financial systems is not unprecedented. The foundational work of Hansen and Sargent firmly established H_∞ minimax robust control as a mechanism for designing monetary policy under unstructured model uncertainty, treating the macroeconomic environment as a game against a malign nature [HS01, HS08]. More recent literature has successfully extended optimal-control frameworks to govern systemic risk contagion within interconnected banking networks, demonstrating how targeted interventions can stabilize procyclical financial stress [Wu25]. By porting this robust control pedigree to the specific topology of algorithmic stablecoins, our framework applies a proven macroeconomic stabilization discipline directly to decentralized, on-chain liquidity dynamics.

The theoretical bedrock of this paper is built upon the Chrysene Formalism, formally defined in [Sch26a, Sch26c], which establishes the cohomological invariants, operator algebras, and systemic liquidity constraints governing the discrete noncommutative state space. The translation of these pure mathematical axioms into a deployable application framework for financial systems and maximal-entropy survival is derived from the tensorial framework presented in [Sch26b], which serves as the direct theoretical basis for the operational strategies developed herein.

The central thesis of our work is that this point of transition can be mapped with mathematical certainty. We formalize this by employing H_∞ minimax robust control theory to model the peg-defense mechanism as a game between the controller and the maximum bounded market disturbance. This control system is constrained within an L_∞ spatial box, representing the physical limits of the liquidity manifold T_C . The output of this framework is a calculable threshold we define as the **Kinematic Tether Limit** (r_{tol}).

This paper presents a comprehensive theoretical architecture to navigate this deterministic property via three distinct strategies. First, we demonstrate how to compute r_{tol} in real-time to execute

a short position just before the algorithmic ceiling is struck (The Metastable Execution Window). Second, we redefine structural resilience by pivoting from directional trading to systemic defense, utilizing KAM theory and explicit Hamiltonian mechanics to engineer geometrically orthogonal portfolios that are insulated from the stablecoin's transition. Third, we scale the framework to macroeconomic defense, proposing the integration of r_{tol} into central clearing protocols as an autonomic trigger for topological scission, partitioning the destabilized sub-manifold before broad contagion can initiate.

2 Theoretical Framework: Isomorphic Divergence and The Kinematic Tether Limit

Classical stochastic models of financial risk, such as those predicated on H_2 norms (e.g., Value-at-Risk, Expected Shortfall), can be augmented when modeling supercritical contagion. They presuppose a continuous, infinitely deep liquidity space. We posit that market liquidity operates over a topological manifold, T_C , with finite geometric constraints. The stability of an algorithmic stablecoin is contingent upon its state remaining within a bounded, admissible sub-manifold of T_C .

2.1 Notation and Key Constructs

To facilitate navigation of the non-commutative and control-theoretic formulations developed herein, Table 1 outlines the core mathematical nomenclature and structural components comprising the framework.

2.2 The L_∞ Spatial Box and Concrete Protocol Constraints

We model the stablecoin's peg-defense mechanism as a feedback control system with a state vector $x(t) \in \mathbb{R}^n$ and a target state $y_{target} = [1.00, \dots]^T$. The system is subject to external market disturbances $w(t) \in \mathcal{L}_\infty$. To counteract these disturbances, the controller generates a control input $u(t)$, representing the algorithmically directed capital flow required to absorb the price variance.

The precise nature of $u(t)$ strictly depends on the protocol's fundamental architecture:

- **Endogenous Seigniorage Models:** In dual-token algorithmic systems (e.g., the historical UST/LUNA archetype), $u(t)$ represents the net continuous minting of an endogenous governance/sister-token, S_{arb} . Because the supply of S_{arb} dynamically inflates to absorb the stablecoin's devaluation, the feedback loop is highly reflexive.

Table 1: Summary of Mathematical Notation and Core Geometric Constructs

Symbol	Construct	Operational Financial Definition
T_C	Liquidity Manifold	The deterministic topological manifold governing global capital routing and boundary constraints.
$\mathcal{B}_\infty$	L_∞ Spatial Box	The bounded functional space defining the domain of executable, non-singular control inputs.
$\tau_{\max}$	Algorithmic Ceiling	The absolute computational or physical capacity limit of the market asset absorption.
r_{tol}	Kinematic Tether Limit	The maximum tolerable disturbance amplitude prior to an absolute capacity ceiling breach.
D_{iso}	Isomorphic Divergence	A binary phase transition flag signaling that the optimal transport cost to the peg has reached ∞ .
T_Q	Transition Operator	The bounded linear matrix mapping the continuous distribution flows of capital across the lattice.
$\mathbb{T}^n$	KAM Invariant Torus	A completely integrable Hamiltonian portfolio configuration geometrically insulated from chaotic contagion.
$\hat{D}_{\text{sciss}}$	Topological Scission Operator	An autonomic clearinghouse projection operator that cuts network ties by mapping toxic nodes to $\emptyset$.

- **Exogenous Collateralized Models:** In over-collateralized architectures (e.g., DAI), $u(t)$ represents the forced liquidation flow of exogenous collateral assets (e.g., ETH) auctioned into the open market to clear underwater debt positions.

Regardless of the architecture, the viability of this control action is rigidly bounded by the finite capacity of the broader market to absorb $u(t)$ without suffering a catastrophic price impact on the secondary asset. We define this physical market depth as the Invariant Algorithmic Ceiling, $\tau_{\max} \in \mathbb{R}^+$. In a seigniorage model, reaching $\tau_{\max}$ signifies the mathematical threshold of terminal hyperinflation (the "death spiral"); in an exogenous model, it represents total on-chain liquidity exhaustion for the specific collateral tier.

We formalize this capacity constraint as an L_∞ Spatial Box:

$$\mathcal{B}_\infty := \{u \in \mathcal{L}_\infty \mid \|u(t)\|_\infty \leq \tau_{\max}\} \quad (1)$$

Any trajectory requiring a control input $u(t) \notin \mathcal{B}_\infty$ cannot be executed without violating the physical economic bounds of the network, rendering the trajectory topologically inadmissible.

2.3 The Kinematic Tether Limit (r_{tol}) in H_∞ Control

The Kinematic Tether Limit, r_{tol} , is defined as the maximum tolerable signal amplitude of a market disturbance $w \in \mathcal{L}_\infty$

that the system can withstand before the required control input $u \in \mathcal{L}_\infty$ breaches the Algorithmic Ceiling $\tau_{\max}$.

To formalize this, we must distinguish between the pointwise signal norm, $\|w\|_\infty := \text{ess sup}_{t \geq 0} |w(t)|$, and the induced operator norm of the closed-loop system mapping disturbances to control efforts, $\|T_{uw}\|_\infty := \sup_{w \neq 0} \frac{\|T_{uw}w\|_\infty}{\|w\|_\infty}$. Assuming linearity and that the worst-case disturbance bound is tightly achieved, we define r_{tol} as:

$$r_{\text{tol}} := \sup_{w \in \mathcal{L}_\infty \setminus \{0\}} \{\|w\|_\infty \mid \|T_{uw}w\|_\infty \leq \tau_{\max}\} = \frac{\tau_{\max}}{\|T_{uw}\|_\infty} \quad (2)$$

Any disturbance $w(t)$ exhibiting an amplitude $\|w\|_\infty > r_{\text{tol}}$ dictates that the worst-case stabilizing control action will deterministically exceed the market's absorption capacity, marking a structural transition.

2.4 Isomorphic Divergence (D_{iso}) and Topological Decoupling

While purely topological notions of disconnection can identify the absence of a clearing path, they fail to capture the *economic work* required to actively maintain the peg. To bridge this, we analyze the phase transition through the Jordan-Kinderlehrer-Otto (JKO) optimal transport scheme. The JKO framework is uniquely suited here because it models the time-evolution of probability densities—in this case, the distribution of market liquidity—as a gradient flow that minimizes the Wasserstein-2 metric.

Unlike standard topological distances, the Wasserstein-2 distance explicitly quantifies the minimum kinetic energy, or thermodynamic cost, required to transport a mass distribution from one configuration to another. In our control framework, this represents the capital cost required to transport the depegged state x back to the target state y_{target} . When a disturbance exceeds the Kinematic Tether Limit (r_{tol}), the required control action surpasses the manifold's finite bandwidth $\tau_{\max}$. At this threshold, the economic work required to clear the market physically diverges. We define this phase transition as Isomorphic Divergence (D_{iso}):

$$D_{\text{iso}}(x, w) := \begin{cases} 1, & \text{if } \|w\|_\infty > r_{\text{tol}} \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

When $D_{\text{iso}} = 1$, it strictly implies that the Wasserstein-2 optimal transport cost diverges: $d_C^*(x, y_{\text{target}}) \rightarrow \infty$. Here, d_C^* represents the Wasserstein-2 optimal transport cost evaluated over the bounded T_C manifold. The consequence is a *topological decoupling*. Because the physical cost of transport approaches infinity, the transition operator T_Q governing capital flow loses its rank, and the physically realizable trade path evaluates to the null set, $\emptyset$.

3 Mathematical Expansion: Operational Strategies Over the T_C Manifold

The Chrysene Formalism offers a deterministic structure for modeling market boundaries. Here, we provide formal mathematical proofs illustrating the structural limitations of probabilistic models and formalize the kinematics of three operational strategies for navigating topological scission.

3.1 Theorem 1: Structural Boundaries of Stochastic Models

Theorem 1 (Topological Insufficiency of SDEs). *Let the liquidity manifold T_C possess a strict upper capacity bound $\tau_{\max} \in \mathbb{R}^+$, and let a stochastic pricing model M_S be defined by a continuous stochastic differential equation (SDE) over an unbounded state space $\mathbb{R}^n$, such that the transition probability density is everywhere strictly positive. Then, the model M_S is structurally incapable of capturing the failure dynamics governed by the deterministic boundary $\tau_{\max}$, leading to a mathematical contradiction under supercritical disturbances.*

Proof. Let the stablecoin price be modeled by an SDE within M_S . The fundamental property of non-compact continuous stochastic models is path-reversibility, meaning that for any depegged state $P(t_0) < 1$, there exists a strictly positive probability of returning to the peg: $\mathbb{P}[P(t_0 + \Delta t) \geq 1 \mid P(t_0) < 1] > 0$.

Let the system be subjected to a supercritical contagion event $w(t)$ such that $\|w\|_\infty > r_{\text{tol}}$. By the definition of r_{tol} 2, restoring the system state to the target y_{target} deterministically requires a control mapping $K : \mathbb{R}^n \rightarrow \mathcal{L}_\infty$ such that the required control input satisfies $\|u(t)\|_\infty > \tau_{\max}$.

However, the physical capacity of the market is rigidly constrained to the spatial box $\mathcal{B}_\infty$. Because the required $u(t) \notin \mathcal{B}_\infty$, the control input is physically unrealizable. Consequently, the transition operator T_Q , which maps the space of control inputs to the space of state transitions, structurally loses surjectivity onto the target state. Because this loss of rank renders the target state geometrically inaccessible, the true transition probability evaluates exactly to zero.

Because M_S inherently assumes the transition map is a global diffeomorphism (or admits a continuous, non-zero measure), it incorrectly assigns a positive probability to an event that has become geometrically inaccessible. This contradiction renders M_S structurally insufficient for modeling the boundary behavior of T_C . $\square$

3.2 Strategy I: The Metastable Execution Window

Theorem 2 (Existence of the Metastable Execution Window). *Let the stablecoin defense mechanism be modeled as a Linear Time-Invariant (LTI) feedback loop strictly bounded within the admissible sub-manifold Ω_{safe} , let $\Delta\mathcal{F}_T^{(2)}$ represent the quadratic thermodynamic free energy shift, and assume the network execution latency $\Delta t_{\text{network}}$ is bounded. Execution of a directional short position yields a deterministically solvable path if and only if the system resides within the Metastable Phase ($1k_{BC}T_{\text{macro}} \leq |\Delta\mathcal{F}_T^{(2)}| < 2k_{BC}T_{\text{macro}}$). Furthermore, for any state where $\|w\|_\infty \geq r_{\text{tol}}$ is breached, the execution mapping evaluates to the null set ($\emptyset$).*

Proof. Let the macroscopic flow of capital clearing be defined by the resolvent operator acting upon the spatial lattice:

$$R(\Delta) = (I - T_Q\Delta)^{-1} \quad (4)$$

where Δ represents the aggregated market disturbance.

Case 1: Post-Breach ($\|w\|_\infty \geq r_{\text{tol}}$): By the properties of the H_∞ norm, the sub-multiplicative operator norm evaluates to $\|T_Q\Delta\|_\infty \geq 1$. Therefore, the Neumann series $\sum_{k=0}^\infty (T_Q\Delta)^k$ mathematically diverges. The resolvent operator $R(\Delta)$ is undefined, and the transition matrix is singular. Any attempted execution maps to the null set ($\emptyset$), rendering post-breach trading a topological impossibility.

Case 2: Metastable Phase ($\|w\|_\infty < r_{\text{tol}}$): Within the specified metastable window, the system is highly polarized, yet the condition $\|T_Q\Delta\|_\infty < 1$ is strictly maintained. Therefore, the Neumann series converges, and the resolvent operator $R(\Delta)$ remains a bounded linear operator. Because $R(\Delta)$ exists, the JKO optimal transport path required to clear the directional position remains mathematically well-posed.

Assuming the network latency $\Delta t_{\text{network}}$ is sufficiently small such that the execution clears before the dynamic trajectory of $w(t)$ intersects r_{tol} , the trader successfully repositions capital prior to the deterministic $d_C^* \rightarrow \infty$ topological decoupling. $\square$

Real-Time Observability and the Latency Bound: Translating this topological survival condition into an operational trading strategy requires dynamically estimating both r_{tol} and the remaining time-to-breach (Δt_{crit}) from empirical on-chain observables. The Invariant Algorithmic Ceiling ($\tau_{\max}$) is continuously measurable via the transparent protocol treasury depth (e.g., the exact USD-equivalent depth of the primary defense liquidity pools). Simultaneously, the aggregate market disturbance $w(t)$ can be estimated by proxy using real-time mempool scanning to quantify pending sell pressure. For a trader to successfully clear a position, their network latency—comprising base block time plus any delay induced by congestion-driven gas spikes—must

strictly satisfy $\Delta t_{\text{network}} < \Delta t_{\text{crit}}$.

Consider a stylized numerical example: Let the observable treasury capacity be $\tau_{\text{max}} = \$50\text{M}$ and the structural slippage multiplier be $\|T_{\text{slw}}\|_{\infty} = 2.0$, establishing a Kinematic Tether Limit of $r_{\text{tol}} = \$25\text{M}$. A real-time mempool analysis detects a pending, unconfirmed sell pressure of $\|w(t)\|_{\infty} = \$20\text{M}$. Because $20 < 25$, the system resides within the Metastable Execution Window. However, observing compounding oracle updates, this sell pressure proxy is accelerating at $\$2\text{M}$ per second. The critical threshold will deterministically breach in $\Delta t_{\text{crit}} = (25 - 20)/2 = 2.5$ seconds. If the trader is executing on a high-throughput network (e.g., $\Delta t_{\text{network}} = 0.5$ seconds), the short position mathematically clears. Conversely, if sudden network congestion causes gas spikes that inflate the trader's inclusion latency to $\Delta t_{\text{network}} = 12.0$ seconds, the transaction will land post-breach. At that exact epoch, the system undergoes topological decoupling, the transition operator loses rank, and the trader's execution identically maps to the null set ($\emptyset$).

3.3 Strategy II: Enhancing Resilience via the KAM Torus

Definition 1 (Geometric Orthogonality and the Hamiltonian Torus). *Two assets are strictly geometrically orthogonal if their covering liquidity sieves share no transition morphisms within the admissible sub-manifold Ω_{safe} (i.e., the overlap metric $\gamma = 0$). In practice, strict orthogonality is computationally and physically unachievable across deeply integrated networks; instead, we seek ϵ -orthogonality where $\gamma \leq \epsilon_c$. By minimizing the L_2 structural overlap penalty—quantified on-chain via shared dependencies—we construct a completely integrable Hamiltonian portfolio confined to the surface of an n -dimensional Kolmogorov-Arnold-Moser (KAM) invariant torus, $\mathbb{T}^n$.*

Theorem 3 (Persistence of the KAM Portfolio Torus). *Let the portfolio be modeled as a completely integrable Hamiltonian system $H_0(I)$ with action-angle variables (I, θ) , representing the semantic mass and settlement phase, respectively. Assume the unperturbed frequencies $\omega(I) = \partial H_0 / \partial I$ satisfy the Diophantine (non-resonance) condition $|\vec{k} \cdot \vec{\omega}| \geq \alpha \|\vec{k}\|^{-\tau}$ for all $\vec{k} \in \mathbb{Z}^n \setminus \{0\}$. Let $\epsilon H_1(I, \theta)$ represent the structural strain injected by a destabilizing stablecoin. If the measured on-chain overlap γ guarantees the perturbation satisfies the quantitative KAM smallness threshold $\epsilon(\gamma) < \epsilon_c$, then the KAM invariant tori $\mathbb{T}^n$ mathematically survive the structural transition.*

Proof. The total Hamiltonian of the perturbed system is evaluated as:

$$H(I, \theta) = H_0(I) + \epsilon(\gamma)H_1(I, \theta) \quad (5)$$

To determine the survival of the tori, canonical transformation via a generating function $S(J, \theta)$ to eliminate the angle dependence requires solving homological equations containing small divisors of the form $(\vec{k} \cdot \vec{\omega})^{-1}$. Because the portfolio is explicitly engineered to satisfy the Diophantine non-resonance condition, these divisors are strictly bounded away from zero, guaranteeing the convergence of the perturbation series provided the perturbation magnitude is sufficiently small.

Ergodic contagion requires physical topological overlap to transmit thermodynamic strain. The magnitude of the perturbation ϵ is a strictly increasing, bounded function of the on-chain geometric overlap γ . By enforcing $\gamma \leq \epsilon_c$, the spatial components of the perturbation operator are quantitatively suppressed.

By the KAM theorem, because the non-degeneracy condition $\det(\partial^2 H_0 / \partial I_i \partial I_j) \neq 0$ holds and the quantitative smallness threshold ($\epsilon(\gamma) < \epsilon_c$) is satisfied, the invariant tori persist as smooth topological deformations. The action variables (I) remain invariant over macroscopic time ($dI/dt = 0$). Consequently, the intersection of the portfolio's kinematic trajectory with the ergodic contagion subspace evaluates to the absolute null set: $\text{traj}(\mathcal{P}_{\text{KAM}}) \cap C_{\text{ergodic}} = \emptyset$, mathematically guaranteeing structural resilience despite the absence of absolute orthogonality. $\square$

Practical Enforcement of ϵ -Orthogonality: The theoretical requirement of isolating the Hamiltonian perturbation demands a calculable framework for measuring the overlap metric γ on-chain. To maintain $\epsilon(\gamma) < \epsilon_c$, the portfolio constructor must continuously evaluate the topological intersection of the target assets against the destabilizing stablecoin. This deterministic evaluation includes: (1) *Oracle Dependency Matrices*: penalizing assets relying on identical decentralized oracle networks (e.g., shared price feeds) that may stall or report stale data during a supercritical de-peg; (2) *Liquidity Routing Overlap*: measuring the percentage of Automated Market Maker (AMM) traversal pathways that are forced to route through the toxic asset's liquidity pools; (3) *Collateral Recursion*: auditing smart contracts for nested yielding mechanisms where the at-risk stablecoin serves as base, hypothecated collateral; and (4) *LI Validator Sets*: ensuring the underlying consensus layers do not share highly correlated staking entities that could suffer simultaneous hardware failure or slashing during extreme network congestion. By quantifying these structural dependencies into a singular, observable metric γ , the constructor can deterministically bound the perturbation ϵ and ensure the persistence of the portfolio tori.

3.4 Strategy III: Systemic Defense via the Topological Scission Operator ($\hat{D}_{sciss}$)

Definition 2 (The Topological Scission Operator). *Let $\hat{D}_{sciss}$ be an autonomic projection operator evaluated continuously by the central clearing protocol in $O(1)$ algorithmic time. When localized strain approaches τ_{max} , $\hat{D}_{sciss}$ maps all incoming and outgoing transition morphisms connected to that node to $\emptyset$.*

Theorem 4 (Autonomic Truncation of Spectral Radius). *Let T_Q be the global transition operator defined over the Hilbert space $\mathcal{H}_{global}$. Let the local strain breach τ_{max} strictly at a single destabilized stablecoin node j at time t_0 . If $\hat{D}_{sciss}$ executes at t_0 , the spectral radius of the global transition operator is mathematically clamped beneath unity ($\rho(T_Q) < 1$), fundamentally prohibiting supercritical contagion.*

Proof. By the tenets of operator theory, the spectral radius of any bounded linear operator is strictly bounded from above by its operator norm: $\rho(T_Q) \leq \|T_Q\|_\infty$. The H_∞ norm of the infinite block matrix T_Q is equivalent to the supremum of the absolute row sums across the global lattice:

$$\|T_Q\|_\infty = \sup_i \sum_j \|T_{ij}\|_\infty \quad (6)$$

As the required continuous flux at node j approaches the algorithmic ceiling, the localized norm $\|T_{jj}\Delta\|_\infty \rightarrow 1$.

However, at the exact temporal coordinate t_0 , before the global supremum can exceed 1, the autonomic operator $\hat{D}_{sciss}$ executes. It projects all morphisms connected to j to the null set: $T_{ij} \mapsto 0$ and $T_{ji} \mapsto 0$ for all $i \neq j$.

The resulting modified global transition operator, $\tilde{T}_Q$, is now block-diagonal, with the toxic node perfectly decoupled. The operator norm of the surviving network evaluates to:

$$\|\tilde{T}_Q\|_\infty \leq \max_{i \neq j} (\tau_{max,i}) < 1 \quad (7)$$

Because the supremum of the absolute row sums of the surviving matrix is bounded away from 1, the spectral radius strictly satisfies $\rho(\tilde{T}_Q) < 1$. The Neumann series governing global clearing converges, and the contagion is mathematically contained. $\square$

Implementation, Governance, and Hybrid Circuit Breakers: While the mathematical guarantee of autonomic truncation is absolute, deploying $\hat{D}_{sciss}$ as an active "circuit breaker" introduces critical execution challenges. In a live network, hard-coding this operator into an Automated Market Maker (AMM) router—effectively forcing the smart contract to trigger a revert state and isolate the pool when r_{tol} is breached—risks severe

false positives. An extreme but transient liquidity shock could unnecessarily fragment the market. Furthermore, determining the exact numerical bounds of the algorithmic ceiling (τ_{max}) poses an upgradeability trilemma: a static value becomes obsolete as the protocol scales, while a dynamically self-updating value is highly vulnerable to oracle manipulation.

To resolve this, we propose a hybrid deployment path. The Topological Scission Operator must execute via a deterministic, low-latency on-chain trigger to quarantine the contagion instantaneously; in a reflexive collapse, human consensus is too slow. However, this scission is instituted as a *metastable lock* rather than a permanent deletion. It is paired with an asynchronous governance veto mechanism, protected by a strictly bounded time-lock. If decentralized risk-assessors determine the τ_{max} breach was a transient false positive rather than a terminal structural de-peg, the governance layer can manually re-establish the severed topological morphisms. This hybrid structure preserves the millisecond-level autonomic firewall required to cap the spectral radius, while relying on human consensus to safely restore macroeconomic homeostasis once the risk has been fully audited.

4 Simulation Results

Real-world algorithmic stablecoin dynamics are inherently non-linear, characterized by reflexive governance-token spirals, discrete mint/burn epochs, and the strict convexity of Automated Market Maker (AMM) invariant curves (e.g., $x \cdot y = k$). Furthermore, network mechanics introduce discrete oracle latencies rather than continuous flows. However, for the purpose of empirically probing the exact boundary condition of the Kinematic Tether Limit (r_{tol}), we evaluate the system strictly within the admissible sub-manifold Ω_{safe} .

Within this localized neighborhood surrounding the target state $y_{target} = 1.00$, the system's Jacobian admits a well-behaved local linearization. Therefore, the theoretical framework is validated through a numerical simulation modeled via a continuous Linear Time-Invariant (LTI) approximation, which serves as the local tangent space of the broader nonlinear T_C manifold.

4.1 Local Linearization and System Parameterization

Operating under the assumption of small perturbations around the peg, the localized market dynamics (the plant) are approximated by a first-order transfer function $P(s) = \frac{1}{s+a}$. Here, $a = 0.2$ denotes a market with slow natural price recovery, rendering it heavily reliant on the protocol's active intervention. The autonomous mint/burn protocol (the controller), which in reality represents the net liquidity routing of discrete AMM smart contracts, is modeled locally as a continuous proportional gain $K(s) = K_p$, where $K_p = 8.0$.

The closed-loop transfer function mapping the external market disturbance $w(t)$ to the required control effort $u(t)$ is defined as:

$$T_{uw}(s) = \frac{K(s)}{1 + P(s)K(s)} = \frac{K_p(s + a)}{s + a + K_p} \quad (8)$$

The H_∞ norm of this transfer function, representing the worst-case disturbance amplification within the linear regime, evaluates to $\|T_{uw}\|_\infty = K_p = 8.0$.

We define the Invariant Algorithmic Ceiling as $\tau_{\max} = 100.0$ (representing, for example, a localized boundary on the absorption capacity for the sister-token before nonlinear structural slippage dominates). Consequently, the Kinematic Tether Limit is explicitly calculated as:

$$r_{\text{tol}} = \frac{\tau_{\max}}{\|T_{uw}\|_\infty} = \frac{100.0}{8.0} = 12.5 \quad (9)$$

Any disturbance $w(t)$ with a magnitude exceeding 12.5 deterministically forces the required control input beyond the 100.0 capacity ceiling, forcing the system out of its linearizable neighborhood and triggering topological scission.

Empirical Estimation of System Primitives: In the analytical model above, the variables $\tau_{\max}$, $\|T_{uw}\|_\infty$, and $w(t)$ are treated as known parameters. In a live deployment, these primitives must be continuously inferred from observable proxies. The disturbance magnitude $\|w(t)\|_\infty$ can be quantified in real-time by aggregating Centralized Exchange (CEX) order book imbalances, mempool pending sell transactions, and trailing-window net outflows from decentralized liquidity pools. The closed-loop amplification norm, $\|T_{uw}\|_\infty$, which encapsulates both market reflexivity and protocol intervention gain, can be estimated dynamically using state-space tracking (e.g., Unscented Kalman Filters) applied to historical micro-de-peg events, measuring the realized elasticity between sister-token issuance and stablecoin price recovery. Finally, the algorithmic ceiling $\tau_{\max}$ is strictly constrained by on-chain realities; it is derived by integrating the bid-side liquidity of the sister-token across all accessible routing pathways down to a terminal, governance-defined slippage boundary.

4.2 Operational Regimes

The results are presented in a three-panel figure, each illustrating a distinct operational regime simulated over a time vector $t \in [0, 50]$.

Panel (A) depicts the system under subcritical disturbance. Here, a stochastic disturbance $w(t)$ is introduced with a peak magnitude $\|w\|_\infty < r_{\text{tol}}$. The required control input, $u(t)$, remains well within the bounds of the Invariant Algorithmic Ceiling, $\tau_{\max}$.

Panel (B) illustrates the bifurcation point. This panel plots the relationship between the magnitude of a worst-case disturbance, $\|w\|_\infty$, and the corresponding required control effort, $\|u\|_\infty$. The Kinematic Tether Limit, r_{tol} , is precisely the point

on the abscissa where this required control effort intersects the deterministic market capacity limit, $\tau_{\max}$.

Panel (C) simulates the supercritical divergence mode. A deterministic disturbance pulse is introduced whose magnitude explicitly exceeds r_{tol} . The control saturates at the ceiling ($u_{\text{act}}(t) = \tau_{\max}$). The result is a structural decoupling of the stablecoin, triggering the Isomorphic Divergence ($D_{\text{iso}} = 1$) trajectory.

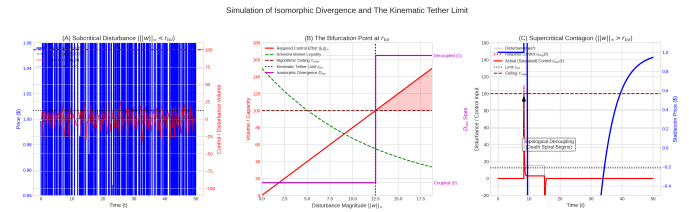


Figure 1: Simulation of the stablecoin's structural response at the Kinematic Tether Limit. (A) Under a subcritical disturbance ($\|w\|_\infty < r_{\text{tol}}$), the peg is successfully defended. (B) The Kinematic Tether Limit r_{tol} is defined at the intersection with the liquidity ceiling $\tau_{\max}$. (C) A supercritical disturbance demands a control input that exceeds market capacity, leading to topological decoupling.

4.3 Nonlinear Seigniorage Dynamics and Reflexive Decay

While the LTI approximation maps the localized tangent space around the peg, seigniorage-based stablecoins (e.g., the historical UST/LUNA archetype) are fundamentally governed by severe nonlinear reflexivity. To capture this, we introduce a secondary simulation modeling the sister-token's price decay via an Automated Market Maker (AMM) invariant curve.

Let $P_s(t)$ represent the stablecoin price and $S_{\text{arb}}(t)$ represent the circulating supply of the sister-token. The protocol defends the peg by minting S_{arb} to absorb the disturbance $w(t)$. However, as the supply inflates, the price of the sister-token, P_{arb} , decays exponentially according to the reflexivity constant λ . This reflexivity constant λ is directly scaled by the strict convexity of the AMM $x \cdot y = k$ invariant curve; as algorithmic minting continuously depletes the stablecoin reserve (y), the sister-token price (P_{arb}) decays along this exact geometric asymptote:

$$P_{\text{arb}}(S_{\text{arb}}) = P_{\text{arb},0} e^{-\lambda(S_{\text{arb}} - S_{\text{arb},0})} \quad (10)$$

Consequently, the required control input (the mint rate) becomes a nonlinear function of both the disturbance and the current sister-token price: $u_{\text{req}}(t) = \frac{K_p w(t)}{P_{\text{arb}}(t)}$. Applying our physical market capacity constraint, the actual execution rate is strictly bounded:

$$u_{\text{act}}(t) = \min(u_{\text{req}}(t), \tau_{\max}) \quad (11)$$

The system's macroscopic flow is thus governed by the coupled

nonlinear ordinary differential equations:

$$\frac{dP_s}{dt} = -a(P_s - 1.0) - w(t) + \frac{u_{\text{act}}(t)P_{\text{arb}}(t)}{K_p} \quad (12)$$

$$\frac{dS_{\text{arb}}}{dt} = u_{\text{act}}(t) \quad (13)$$

The results of this nonlinear framework are illustrated in Figure 2, exposing the mechanics of the hyperinflationary "death spiral."

Panel (A) contrasts the stablecoin trajectory under a subcritical versus a supercritical shock. Under the subcritical shock, the peg is severely strained but ultimately recovers. Under the supercritical shock, the system suffers Isomorphic Divergence, and the price permanently collapses.

Panel (B) isolates the deterministic threshold of failure. As the sister-token price decays, the protocol must mint exponentially more tokens to generate the same unit of stabilizing capital. The supercritical shock forces the required mint rate u_{act} to strike the Invariant Algorithmic Ceiling (τ_{max}). At this exact temporal coordinate, the control loop saturates, and the protocol is mathematically disarmed.

Panel (C) illustrates the erosion of the portfolio's semantic mass. The continuous, forced minting at the τ_{max} ceiling hyperinflates S_{arb} , driving its price asymptotically to zero. Because the capital generated evaluates to $u_{\text{act}} \times P_{\text{arb}}$, the protocol's defense capacity vanishes entirely, sealing the topological decoupling of the network.

5 Conclusion

The Chrysene Formalism re-conceptualizes systemic liquidity as a topological manifold with finite, deterministic boundaries. By treating the market's absorption capacity for a sister-token as an Invariant Algorithmic Ceiling, τ_{max} , we define the system's safe operating envelope as an L_∞ Spatial Box.

This framework provides three mathematically rigorous methods for navigating supercritical transitions. Participants can position capital dynamically during the Metastable Execution Window (Strategy I). They can passively enhance their microeconomic resilience by constructing KAM invariant tori constrained by explicit Hamiltonian mechanics that are geometrically orthogonal to the broader network (Strategy II). Ultimately, central clearing authorities can augment the macroeconomic network by integrating the Kinematic Tether Limit as a deterministic trigger for the Topological Scission Operator ($\hat{D}_{\text{sciss}}$), containing the threat of algorithmic instability (Strategy III). These strategies complement traditional probability theory by offering an alternative rooted in the physical and geometric bounds of the T_C manifold.

Looking forward, the theoretical bounds established by r_{tol} demand rigorous empirical validation across live decentralized finance (DeFi) ecosystems. Subsequent phases of this research

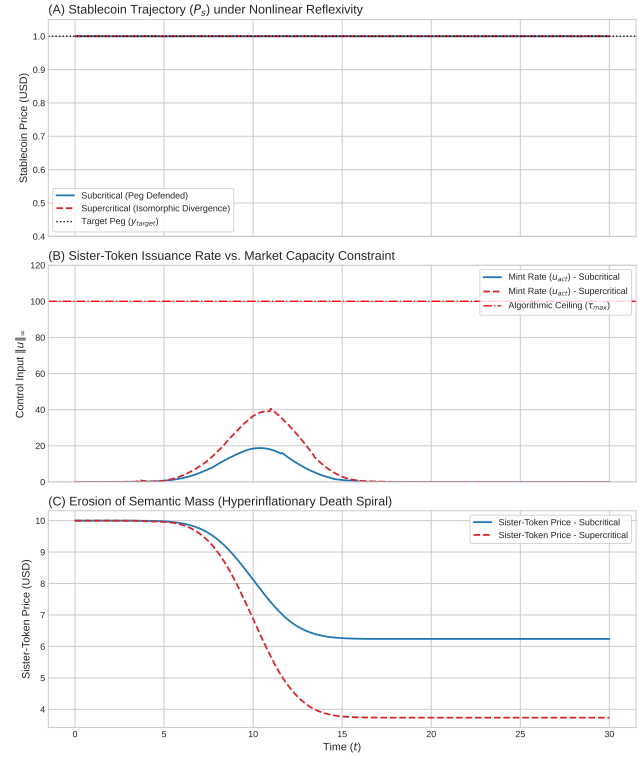


Figure 2: Simulation of the dual-token algorithmic stablecoin under nonlinear reflexivity. (A) The stablecoin price trajectory (P_s) under subcritical and supercritical shocks. (B) The required sister-token mint rate (u_{act}) striking the Invariant Algorithmic Ceiling (τ_{max}). (C) The reflexive decay of the sister-token price, illustrating the hyperinflationary "death spiral" that precipitates Isomorphic Divergence.

will focus on translating these topological limits into active smart-contract prototypes, specifically embedding $\hat{D}_{\text{sciss}}$ into Automated Market Maker (AMM) routers as an autonomic circuit breaker. For regulatory bodies and central clearing-houses, adopting this deterministic lens offers a paradigm shift from reactive, high-latency bailouts to proactive, localized risk quarantine. Ultimately, this application to stablecoin defense serves as a vital operational bridge to the broader Chrysene research program [Sch26a, Sch26b, Sch26c], demonstrating how non-commutative algebraic geometry can be harnessed to engineer maximal-entropy fault tolerance across the global financial architecture.

A Appendix: Key Elements of the Chrysene Formalism

To render this paper fully self-contained, we summarize the foundational axioms and operational mechanics of the Chrysene Formalism utilized in our stability analysis, formally derived in [Sch26a, Sch26c]. The formalism fundamentally replaces

continuous stochastic probability spaces ($\mathcal{H}_2$) with a discrete, non-commutative geometric manifold ($\mathcal{T}_C$) governed by deterministic $\mathcal{H}_\infty$ robust control

A.1 The Discrete Base Manifold and $GF(4)$ Arithmetic

The spatial foundation of the financial network is defined as a discrete, undirected regular graph embedded within a three-dimensional integer lattice, denoted as $\mathcal{M} \cong \mathbb{Z}^3$. To prevent unbounded scalar divergence, the internal accounting state of any institutional node evaluates strictly over the finite Galois field $GF(4)$, utilizing the elements $\{0, 1, \omega, \omega^2\}$. Under characteristic-2 arithmetic, the generation of synthetic derivatives (where ω represents a synthetic asset and ω^2 a synthetic liability) must be perfectly anchored by physical base collateral (1). The transaction is topologically valid and preserves the conservation of semantic mass if and only if the sum of the resulting state elements evaluates strictly to the irreducible polynomial constraint: $\omega^2 + \omega + 1 = 0$.

A.2 The Transition Operator (T_Q) and Algorithmic Ceiling

Capital kinematic motion between these discrete $\mathbb{Z}^3$ coordinates is not a stochastic random walk, but is governed by bounded linear operators within the continuous Operator Hardy Space ($\mathcal{H}_\infty$). The Global Market Transition Operator, T_Q , represents the simultaneous, aggregated continuous liquidity flux across the entirety of the financial quiver. The absolute physical capacity of the market is rigidly bounded by the invariant algorithmic ceiling, defined as $\tau_{max} = \frac{1}{\|T_Q\|_\infty}$. If a macroscopic perturbation Δ forces an execution magnitude of $\|\Delta\|_\infty \geq \tau_{max}$, the Neumann series of the resolvent operator mathematically diverges, topologically severing the capital transport path.

A.3 Hidden Leverage as Čech 1-Cocycles ($\tilde{H}^1 \neq 0$)

When an institution generates unanchored synthetic derivatives and attempts to cross-collateralize them across different jurisdictions without base capital, it injects a geometric twist into the manifold. Any localized generation of uncollateralized semantic mass that spans across institutional intersections structurally manifests as a non-trivial Čech 1-cocycle ($\tilde{H}^1 \neq 0$). This topological obstruction physically prevents the network's local ledgers from patching into a globally unified, valid section, making systemic insolvency geometrically computable prior to actual execution.

A.4 The Fredholm Index and Autonomic Scission

To explicitly quantify the algebraic imbalance between trapped liquidity and unmet market obligations, we formally elevate the financial transition matrix T_Q to a Fredholm operator. The Financial Fredholm Index, denoted $\text{ind}(T_Q)$, is defined as the analytical difference between the geometric dimensions of the kernel (trapped liquidity) and the cokernel (unclearable liabilities): $\text{ind}(T_Q) = \dim(\ker T_Q) - \dim(\text{coker } T_Q)$. A global financial network resides in absolute harmonic equilibrium if and only if the Fredholm Index evaluates strictly to zero ($\text{ind}(T_Q) = 0$).

If the index diverges ($\text{ind}(T_Q) \neq 0$) or a non-trivial Čech cocycle appears, the Topological Scission Operator ($\hat{D}_{sciss}$) autonomically triggers. This operator immediately projects all incoming and outgoing transition morphisms (liquidity conduits) connected to the toxic node to the absolute null set ($\emptyset$), mathematically severing it from the healthy network before supercritical percolation can initiate.

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