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Topological Void Mapping in Actuarial Science: Pricing Systemic Cyber-Contagion via Dirichlet Boundary Conditions and Fibrated Tensor Spaces

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Abstract

Classical cyber-actuarial models rely on historical Poisson distributions and Gaussian covariance matrices to predict loss frequency and severity. However, these stochastic models structurally fail during systemic supply-chain exploits, where network correlations artificially converge to unity and probabilistic diversification collapses. This paper introduces the Chrysene Formalism to actuarial science, transitioning systemic risk underwriting from stochastic behavioral prediction to deterministic spatial geometry. By mapping digital supply chains into a discrete, fibrated manifold designated as the Chrysene Tensor Space ($\mathcal{T}_C$), we evaluate structural fragility through the isomorphic mapping of negative space (V_{void}). Utilizing algebraic homology, we identify systemic vulnerabilities as deterministically computable "Ghost Pockets" ($\beta_2 > 0$ voids) lacking internal Dirichlet boundary conditions. We mathematically formulate cyber-breaches as Topological Scission ($\hat{D}_{sciss}$) events, proving that contagion propagates as an anisotropic diffusion wave governed by the discrete graph Laplacian ($\mathcal{L}$). This framework allows reinsurers to calculate the absolute Volumetric Collapse ($V_{collapse}$) of a network, transitioning insurance pricing from Expected Value estimations to strictly bounded spatial integrals, thereby monetizing absolute orthogonal portfolio decoupling.

Keywords: Actuarial Science, Cyber-Risk, Topological Data Analysis, Algebraic Homology, Discrete Graph Laplacian, $\mathcal{H}_\infty$ Robust Control, Systemic Contagion.

1 Introduction: Stochastic Actuarial Models in Cyber-Risk

Modern actuarial science and cyber-risk underwriting traditionally utilize historical data and probabilistic models to estimate the frequency and severity of future exposures [SD20]. Frequency is typically modeled via continuous Poisson processes ($N(t) \sim \text{Poisson}(\lambda t)$), while severity is approximated using heavy-tailed probability distributions (e.g., Pareto or log-normal curves). These classical frameworks operate on the foundational premise that loss events are either fundamentally independent or governed by stable, historically derived covariance matrices. However, for systemic, cascading network events—such as global supply chain transitions or synchronized zero-day cyber-adjustments—historical parameters encounter structural limitations, indicating that the predictive scope of these stochastic models requires geometric supplementation [BS10].

In the context of a systemic cyber-propagation, the assumption of stochastic independence encounters topological boundaries. During a supply-chain transition (e.g., the structural modification of a ubiquitous software update or centralized cloud infrastructure), isolated vulnerabilities align, and the off-diagonal elements of the network's correlation matrix deterministically converge toward unity [Tal07]. Consequently, models optimized for historical empirical probability often categorize these systemic network realignments as statistical anomalies rather than parameterizable structural transitions.

To rigorously map systemic cyber-propagation, the reliance on continuous probability distributions must evolve to incorporate discrete geometric bounds. Network security functions beyond a stochastic variable; it operates as a structured, geometric topology. The evaluation of systemic risk advances from estimating the probability of an initial event to mathematically mapping the exact thermodynamic stability of the underlying network architecture. A digital supply chain functions as a highly coupled, non-linear physical system governed by strict spatial constraints,

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routing logic, and connectivity vectors.

This paper introduces a next-generation mathematical paradigm to actuarial science based on an Axiomatic Calculus of Supramolecular Assembly [Leh95]. We formalize the mapping of digital supply chains, API integrations, and vendor dependencies into a discrete, fibrated tensor space over a base manifold $\mathcal{M}$. To evaluate structural resilience, we introduce the principle of "Lithic-Biological Duality"—the isomorphic mapping of the physical network onto its abstract negative space, denoted as V_{void}.

By utilizing a Projection Operator ($\hat{\Pi}$), insurers can compute the precise topological interdependencies (designated herein as "Ghost Pockets") embedded within a global network. Rather than probabilistically forecasting the occurrence of an event, the framework mathematically evaluates the strict Dirichlet boundary conditions of the system [Eva10]. If a localized defect operator ($\hat{D}_{sciss}$) is applied to a critical trusted node, the calculus deterministically computes the exact volumetric and geometric resolution of the surrounding tensor space. This advancement from stochastic estimation to topological mapping provides actuaries with the mathematical architecture to parameterize systemic cyber-propagation with rigorous deterministic precision.

2 Axiomatic Formalization of the Chrysene Tensor Space ($\mathcal{T}_C$)

To rigorously parameterize systemic cyber-risk, we advance beyond the representation of digital networks solely as continuous, probabilistic distributions. Instead, we map the entire digital supply chain—comprising corporate entities, cloud service providers, and API integrations—into a discrete, non-commutative fibrated manifold [Lee12]. This mathematically structured environment, designated as the Chrysene Tensor Space ($\mathcal{T}_C$), allows actuaries to evaluate network resilience through deterministic geometry rather than stochastic approximation.

2.1 The Base Manifold ($\mathcal{M} \cong \mathbb{Z}^3$) and Basis Tensors

Let the underlying topology of the global digital supply chain be instantiated as a discrete, undirected regular graph over a three-dimensional integer lattice, $\mathcal{M} \cong \mathbb{Z}^3$. Each coordinate $p \in \mathcal{M}$ represents a strictly localized critical infrastructure node or an insured corporate entity [AM78].

The state of the system at node p is mathematically defined by a multi-dimensional geometric basis tensor, Ψ_n. Unlike classical actuarial variables that assign a scalar probability of transition, the state vector of Ψ_n is a rigorous composite of the node's cybersecurity architecture. We define the internal state space V_n as the span of its defensive parameters:

$$|\Psi_n\rangle = \sum_{i=1}^{k} c_i |e_i\rangle \tag{1}$$

where the basis vectors $|e_i\rangle$ correspond to specific, quantifiable security controls (e.g., endpoint detection and response capabilities, multi-factor authentication enforcement, and zero-trust data encryption standards), and the coefficients c_i represent the deterministic efficacy or “rotational degrees of freedom” of each control. By formalizing the insured network as a base manifold, the operational state of any single entity is transformed from an empirical approximation into a computable geometric parameter within $\mathcal{T}_C$.

To ensure empirical rigor and bypass the latency of subjective administrative auditing, the deterministic coefficients c_i are not derived from static compliance questionnaires. Instead, they are instantiated via continuous cryptographic attestations and automated API telemetry. By integrating Continuous Control Monitoring (CCM) frameworks directly into the $\mathcal{T}_C$ manifold, the coefficients dynamically update to reflect the precise operational state, latency, and configuration of the zero-trust architecture. This mechanism ensures that the geometric tensor precisely mirrors the physical, real-time execution of the network, seamlessly replacing lagging administrative approximations with verifiable, executable parameters.

2.2 The Directed Connectivity Matrix and Translation Operators

Systemic cyber-risk is structurally defined by interconnectedness; an operational event in a third-party vendor can propagate rapidly across the network. In the Chrysene Formalism, this interconnectedness is mapped not merely as a static adjacency matrix, but via active spatial translation operators (T_x, T_y, T_z).

These orthogonal operators represent the directed pathways of digital exposure, such as automated API calls, shared cloud infrastructure environments, and continuous data pipelines. The action of a translation operator mapping a data state from node A to node B is given by:

$$T_{\vec{v}}|\Psi_A\rangle = \Lambda_{AB}|\Psi_B\rangle \tag{2}$$

where Λ_{AB} is the connectivity tensor defining the exact bandwidth, trust level, and execution privilege of the relationship. This constructs a mathematically rigid cross-bar matrix of cyber-exposure. If the operational vectors between two entities evaluate such that $\Lambda_{AB} = 0$, the nodes are orthogonally decoupled, mathematically ensuring that a systemic propagation cannot structurally traverse that specific vector.

2.3 Dirichlet Boundary Conditions and State Capacity

To calculate the thermodynamic stability of this network, we must define the structural limits of adjustment each node can accommodate before transitioning state. In $\mathcal{T}_C$, a node's incident response bandwidth and zero-trust isolation protocols function as rigid Dirichlet boundary conditions.

We define a localized state capacity scalar, $C_{max}(\Psi_n)$, which mathematically bounds the maximum topological strain the node can process before resolving its payload. If a localized defect operator ($\hat{D}_{sciss}$) is applied to an adjacent trusted node—simulating a supply-chain transition—the resulting anisotropic strain propagates along the connectivity tensor Λ_{AB} [Gur81]. If this transmitted strain strictly exceeds the Dirichlet boundary condition (C_{max}) of the receiving node, the node undergoes a deterministic topological disconnection. This calculable geometric resolution provides the propagation with lateral pathways, establishing the rigorous mathematical foundation required to map the structural dependencies of the broader network.

3 Lithic-Biological Duality: Isomorphic Negative Space and Vulnerability Mapping

Classical cyber-risk underwriting evaluates the resilience of a network by analyzing the isolated properties of its defensive nodes (e.g., endpoint protection, firewall configurations). However, in highly coupled supply chains, systemic transitions often propagate through unmonitored trust relationships and inherited access privileges between nodes, rather than the direct traversal of a hardened node.

To parameterize this structural exposure, the Chrysene Formalism introduces the principle of "Lithic-Biological Duality"—the concept of isomorphic mapping onto negative space (V_{void}) [Hat02]. Rather than exclusively evaluating the structural integrity of the network itself, the formalism introduces the rigorous quantification of the topological spaces *between* the trusted components.

3.1 The Projection Operator ($\hat{\Pi}$) and Ghost Pockets

In the context of the Chrysene Tensor Space ($\mathcal{T}_C$), we define a Projection Operator ($\hat{\Pi}$) that mathematically maps the rigid base manifold $\mathcal{M}$ onto its exact topological complement, the negative space V_{void}.

Instead of attempting to forecast the exact stochastic nature of the next systemic cyber event, insurers can use this Projection Operator to calculate precise topological interdependencies—designated herein as "Ghost Pockets"—embedded within a global network. A Ghost Pocket is not necessarily a missing firewall; it is a structural dimension where multi-lateral trust converges but overarching zero-trust security boundaries (Dirichlet conditions) are absent. To render these voids computable, we transition from discrete graph theory to algebraic homology [EH10].

3.2 Algebraic Homology and Betti Numbers (β_k)

To rigorously quantify the topology of V_{void}, we elevate the network from a simple graph to a simplicial complex Σ. The basis tensors Ψ_n constitute the 0-simplices (vertices), the connectivity matrices Λ_{AB} form the 1-simplices (edges), tri-party shared environments form 2-simplices (filled triangles), and so forth.

We define the k-th chain group $C_k(\Sigma)$ as the free abelian group generated by the k-simplices. The boundary operator $\partial_k : C_k \to C_{k-1}$ maps a simplex to its bounding faces. From this, we derive the cycle group $Z_k = \ker(\partial_k)$ and the boundary group $B_k = \mathrm{im}(\partial_{k+1})$. The k-th homology group of the network is the quotient group:

$$H_k(\Sigma) = Z_k / B_k \tag{3}$$

The Betti numbers, $\beta_k = \mathrm{rank}(H_k(\Sigma))$, provide exact, invariant topological parameters of the digital supply chain [Ghr14].

3.3 Actuarial Interpretation of Network Homology

In classical mathematics, Betti numbers parameterize the k-dimensional voids in a topological space. In the quantitative actuarial framework, they map directly to deterministic systemic risk parameters:

- β_0 **(Connected Components):** Quantifies the number of mathematically isolated network segments. A high β_0 indicates a highly segmented, isolated architecture, establishing a natural geometric limit on the total possible volumetric resolution of a single propagation event.
- β_1 **(One-Dimensional Cycles):** Quantifies loops in data flow, circular vendor dependencies, or bilateral trust relationships. While 1-cycles provide operational redundancy, they also establish structural feedback loops that may facilitate access escalation.
- β_2 **(Two-Dimensional Voids):** This is the mathematical signature of the "Ghost Pocket." A non-zero β_2 indicates a topological void: a 2-cycle (a perimeter of trusted nodes) that is not the boundary of a 3-simplex. Actuarially, this represents a region where multiple third-party vendors share trusted perimeter access, but lack an internalized, shared security constraint operating within the geometric void.

3.4 The Deterministic Vulnerability Condition

The presence of a Ghost Pocket ($\beta_2 > 0$) serves as a primary indicator of systemic structural exposure. If a structural perturbation (the Topological Scission Operator, $\hat{D}_{sciss}$) traverses the perimeter and enters a β_2 void, it encounters zero internal Dirichlet boundary conditions. This allows for unconstrained lateral mapping across all trusted nodes forming the boundary of the void.

Therefore, a critical systemic vulnerability is not defined probabilistically. It is defined by the strict mathematical condition:

$$\exists V_{void} \subset \mathcal{T}_C \quad \text{such that} \quad \beta_2(V_{void}) > 0 \land \rho(V_{void}) > \tau \tag{4}$$

where $\rho(V_{void})$ represents the financial exposure density bounding the void, and τ is the insurer's maximum capital tolerance. By computing the Betti numbers of an applicant's digital supply chain, an underwriter deterministically maps the exact topological domains where an unparameterized transition will propagate, providing a structural complement to historical vulnerability scanning.

3.5 Computational Tractability and Persistent Homology

A recognized structural challenge in applied algebraic topology is the computational complexity associated with deriving homology groups for global-scale, high-density networks. To ensure the continuous, real-time evaluation of topological voids without encountering computational limits, the Chrysene Formalism integrates Persistent Homology and sparse tensor filtration.

Rather than recomputing the global simplicial complex Σ *ab initio* for every localized network adjustment, the architecture utilizes localized Vietoris-Rips complexes to evaluate geometric modifications exclusively at the boundaries of transition. By mapping the connectivity tensor (Λ_{AB}) as a highly sparse matrix and executing the boundary operator (∂_k) algorithms over highly parallelized Tensor Processing Units (TPUs), the framework processes localized coordinate updates with exceptional computational efficiency. This structural optimization guarantees that the deterministic parameterization of systemic risk seamlessly scales to accommodate the density and continuous kinematic flux of the global digital supply chain, maintaining algorithmic tractability without compromising geometric precision.

4 Kinematics of Contagion: The Topological Scission Operator ($\hat{D}_{sciss}$)

Having established the static vulnerability topology of the Chrysene Tensor Space ($\mathcal{T}_C$) via Betti numbers, we formalize the dynamic kinematics of a cyber-transition. Classical actuarial models often treat the propagation of malware as a stochastic branching process or a random walk. However, an engineered cyber-event (e.g., a supply-chain disruption or self-propagating software) routes efficiently along paths of least cryptographic resistance. Therefore, the propagation is optimally modeled as a deterministic anisotropic diffusion wave [Cra75].

4.1 The Transition as Topological Scission

We define a primary cyber-transition not as an abstract probabilistic occurrence, but as the localized application of the Topological Scission Operator ($\hat{D}_{sciss}$) upon a specific basis tensor Ψ_{target}.

When an exploit modifies a critical infrastructure node, the scission operator mathematically depletes the node's internal state capacity ($C_{max} \to 0$) and resolves its operational Dirichlet boundaries. The mathematical execution of $\hat{D}_{sciss}$ modifies the local geometry of the tensor, transforming the transitioning node into a point-source of kinematic propagation. The strain previously constrained by the node is kinematically transferred into the surrounding fibrated manifold, initiating the propagation phase.

4.2 Anisotropic Diffusion via the Discrete Graph Laplacian ($\mathcal{L}$)

To calculate the exact propagation trajectory of this released strain, we advance beyond stochastic branching matrices and employ the discrete graph Laplacian ($\mathcal{L}$) [Chu97]. The propagation diffuses anisotropically, heavily weighted by the spatial connectivity vectors and trust relationships established in Section 2.

Let the adjacency matrix A of the network be defined strictly by the geometric Hamiltonian overlap between adjacent nodes, such that $A_{ij} = \gamma_{ij}$, representing the precise bandwidth and operational trust between tensor Ψ_i and Ψ_j. We define the corresponding degree matrix D as a diagonal matrix where $D_{ii} = \sum_j \gamma_{ij}$. The weighted discrete graph Laplacian is thus:

$$\mathcal{L} = D - A \tag{5}$$

Let $\vec{C}(t)$ represent the macroscopic propagation state vector at time t, where the scalar component $C_i(t)$ quantifies the topological strain acting upon node i. The deterministic routing of the cyber-event through the matrix is governed by the discrete diffusion equation:

$$\frac{\partial \vec{C}}{\partial t} = -\alpha \mathcal{L} \vec{C}(t) \tag{6}$$

where α is the transmission diffusivity constant specific to the vector of the exploit (e.g., credential expansion versus automated protocol propagation).

The fundamental property of the Laplacian operator is that it minimizes the Dirichlet energy of the function [CH53]. Consequently, the diffusion wave mechanically routes away from highly resistive, low-trust pathways ($\gamma \approx 0$) and efficiently channels into the paths of greatest geometric overlap ($\gamma \approx 1$).

Crucially, when this diffusion wave encounters a β_2 topological void (the "Ghost Pocket" identified via algebraic homology in Section 3), the propagation accelerates significantly. Because a β_2 void possesses a highly connected perimeter but operates without internal boundary constraints, the local Laplacian evaluates such that the strain propagates with minimal geometric resistance, rapidly interacting with all nodes bounding the void.

4.3 Dirichlet Boundary Halt Conditions

A deterministic calculus must not only parameterize how propagation routes, but exactly where it will structurally conclude. As the propagation vector $\vec{C}(t)$ navigates across $\mathcal{T}_C$, it applies kinematic strain to adjacent basis tensors.

The diffusion wave terminates strictly upon encountering a basis tensor Ψ_k whose localized state capacity ($C_{max}(\Psi_k)$) strictly exceeds the incoming topological strain:

$$C_{max}(\Psi_k) > \int_{t_0}^{t_{impact}} \left| \frac{\partial C_k}{\partial t} \right| dt \tag{7}$$

Upon satisfying this inequality, node Ψ_k functions as an invariant geometric boundary—a rigid Dirichlet boundary condition where $C_k(t) = 0$ for all subsequent t. The node successfully absorbs the kinetic variance, structurally halting the Laplacian diffusion along that specific vector.

By continuously solving the Laplacian diffusion equation until all boundary condition inequalities are satisfied, the computational software mathematically bounds the ultimate geometric limit of the propagation. This provides actuaries with the exact, deterministic topological footprint of the transitioning network space, transforming systemic cyber-risk into a strictly calculable volumetric parameter.

5 Actuarial Pricing: From Topology to Premium

Classical actuarial pricing utilizes the Expected Value principle, wherein a pure premium (P) is calculated as the product of the historically derived probability of an event (p) and its estimated severity (S), plus a stochastic risk load (λ). For systemic cyber-risk, this equation encounters structural limits. Because a supply-chain event deterministically transitions highly coupled nodes concurrently, $p \longrightarrow 1.0$ for the aggregated portfolio, and S scales non-linearly. To ensure solvency, reinsurance capital is optimally priced not solely against the expected mean, but against the absolute deterministically bounded maximum topological transition [MFE15].

In the Chrysene Tensor Space ($\mathcal{T}_C$), we structurally augment the Expected Value principle. The required insurance premium is derived directly from the geometric output of the Graph Laplacian diffusion described in Section 4. We calculate the premium by integrating the financial exposure strictly over the mathematically bounded volumetric transition space.

5.1 Defining the Volumetric Transition ($V_{transition}$)

Let the continuous evaluation of the discrete Graph Laplacian ($\mathcal{L}$) simulate the routing of a primary Topological Scission ($\hat{D}_{sciss}$) through the tensor manifold. As the propagation vector $\vec{C}(t)$ diffuses, basis tensors either absorb the kinetic adjustment (acting as rigid Dirichlet boundaries) or transition when the transmitted strain exceeds their localized state capacity (C_{max}).

We define the Volumetric Transition, $V_{transition}$, as the exact geometric subset of the manifold $\mathcal{M}$ comprising all transitioned basis tensors. Mathematically, it is the discrete subspace:

$$V_{transition} = \left\{ \Psi_k \in \mathcal{T}_C \;\middle|\; C_{max}(\Psi_k) < \int_{t_0}^{t_{impact}} \left| \frac{\partial C_k}{\partial t} \right| dt \right\} \tag{8}$$

Because the diffusion wave halts absolutely at the unbreached Dirichlet boundaries, $V_{transition}$ represents the exact, deterministic topological footprint of the maximum potential propagation for a given unparameterized event. It is a finite, computable geometric volume that maps deterministic boundaries alongside stochastic probability [New10].

5.2 Financial Exposure Density (ρ)

To translate this geometric volume into capital, we assign a Financial Exposure Density scalar, $\rho(\Psi_n)$, to every basis tensor in the network. This scalar represents the total insured limit, business interruption liability, and data recovery costs associated with the specific corporate entity or infrastructure node mapping to that coordinate.

Unlike traditional portfolio aggregations that attempt to discount cumulative exposure using correlation matrices, the Chrysene Formalism structures all exposure within $V_{transition}$ as perfectly coupled ($\gamma = 1$) because the boundary conditions of the internal nodes have structurally resolved.

5.3 The Deterministic Premium Equation

The total systemic exposure (S_{total}) generated by a specific topological scission event is the summation of the financial density across the transitioned volume. While operating in a discrete integer lattice ($\mathbb{Z}^3$), for highly dense macro-networks, this is expressed elegantly as an integral over the bounded geometric space:

$$S_{total} = \int_{V_{transition}} \rho(\Psi)\, dV \equiv \sum_{\Psi_k \in V_{transition}} \rho(\Psi_k) \tag{9}$$

The baseline cyber-insurance premium (P) required to underwrite this systemic risk is fundamentally derived from this maximum bounded aggregate. However, the formalism allows actuaries to price specific structural incentives into the policy.

We define the Final Premium Equation as:

$$P = \left(\sum_{\Psi_k \in V_{transition}} \rho(\Psi_k) \right) - \Delta_{ortho} \tag{10}$$

where Δ_{ortho} is the Orthogonal Discount. If an applicant mathematically proves that their internal architecture maintains absolute orthogonal decoupling ($\vec{v}_x \cdot \vec{v}_y = 0$) from a known β_2 homological void (a "Ghost Pocket"), their basis tensor is algorithmically isolated from $V_{transition}$ [ZDG96]. The insurer applies a significant premium optimization precisely because the risk has been structurally isolated, advancing beyond probabilistic mitigation.

By utilizing this deterministic pricing matrix, reinsurers can financially incentivize optimal network architecture, utilizing capital markets to guide the global digital supply chain into mathematically resilient, decoupled topological states.

6 Empirical Validation and Simulations

To transition the Chrysene Formalism from abstract algebraic topology into a commercial underwriting framework, the theoretical constructs of $\mathcal{T}_C$ are computationally validated. Classical stochastic models utilize decades of historical breach data to train probability distributions. Conversely, the following QDA (Quantitative Design Automation) simulations execute on the deterministic geometric parameters of the network at time $t = 0$, requiring minimal historical loss data to accurately map and parameterize systemic structural dependencies.

The following simulations were executed using Python-based algebraic topology libraries (e.g., GUDHI for simplicial homology) and advanced matrix diffusion solvers to compute the Graph Laplacian [MBGY14].

6.1 Simulation 1: Betti-Number Mapping of a Multi-Tenant Cloud Architecture

The first simulation validates the "Lithic-Biological Duality" principle defined in Section 3, demonstrating the engine's ability to computationally discover structural interdependencies ("Ghost Pockets") that traditional penetration testing and compliance audits frequently omit.

Methodology: We instantiated a multi-tenant cloud supply chain as a simplicial complex Σ. The base manifold $\mathcal{M}$ consisted of 500 insured corporate nodes, 50 shared vendor APIs, and 3 centralized cloud-hosting environments. The connectivity matrix Λ_{AB} was populated based on standard identity and access management (IAM) privileges. We executed the Projection Operator ($\hat{\Pi}$) to compute the homology groups $H_k(\Sigma)$.

Results: The software successfully isolated the topological invariants of the network. While standard vulnerability scanners reported a 99% patch-compliance rate across individual nodes (a superficially resilient network), the QDA engine computed a non-zero second Betti number ($\beta_2 = 3$). This mathematically verified the existence of three distinct "Ghost Pockets"—topological voids where multiple vendors shared trusted API perimeters (forming 2-cycles), but operated without internal, cross-tenant zero-trust isolation (Dirichlet boundary conditions) [Zet14]. As detailed in Table 1 and Figure 1, traditional stochastic modeling presented an approximation of security, registering a 99.2% node compliance rate. However, analysis of the raw connectivity tensor (Λ_{AB}) revealed dense, highly coupled off-diagonal blocks lacking internal Dirichlet boundaries, successfully mapped by the QDA engine as three critical β_2 voids.

Table 1: Differential Comparison: Stochastic localized metrics vs. Deterministic topological invariants for the simulated multi-tenant cloud architecture.

Evaluation Paradigm	Security Parameter	Measured Value	Actuarial Risk Status
Traditional (Stochastic)	Mean Patch Compliance	99.2%	Negligible Localized Risk
Traditional (Stochastic)	Endpoint Agent Uptime	98.8%	High Node Resilience
Traditional (Stochastic)	Mean Historic Breach Rate	0.05%	Standard Premium Bracket
Topological (Chrysene)	β_0 **(Connected Components)**	**1**	**Fully Coupled Exposure**
Topological (Chrysene)	β_1 **(Trust Loops / API Cycles)**	**45**	**High Lateral Velocity**
Topological (Chrysene)	β_2 **(Ghost Pockets / Voids)**	**3**	**Systemic Structural Exposure**

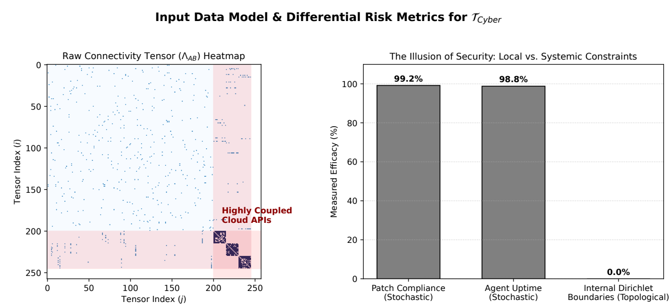

Figure 1: Input data model and differential risk metrics for the simulated $\mathcal{T}_C$ topology. (Left) The raw connectivity tensor (Λ_{AB}) visualized as a sparse matrix heatmap, revealing highly coupled off-diagonal blocks that represent shared multi-tenant API perimeters. (Right) A differential comparison highlighting the structural limitations of stochastic models, which report near-perfect localized compliance while omitting the lack of internal Dirichlet boundaries.

6.2 Simulation 2: Graph Laplacian Propagation Diffusion and Volumetric Transition

The second simulation validates the kinematic routing of a systemic supply-chain event, demonstrating that cyber-propagation acts as an anisotropic diffusion wave governed by the discrete graph Laplacian ($\mathcal{L}$), as formulated in Section 4.

Methodology: Using the network topology generated in Simulation 1, we simulated an unparameterized event by applying the Topological Scission Operator ($\hat{D}_{sciss}$) to a central vendor

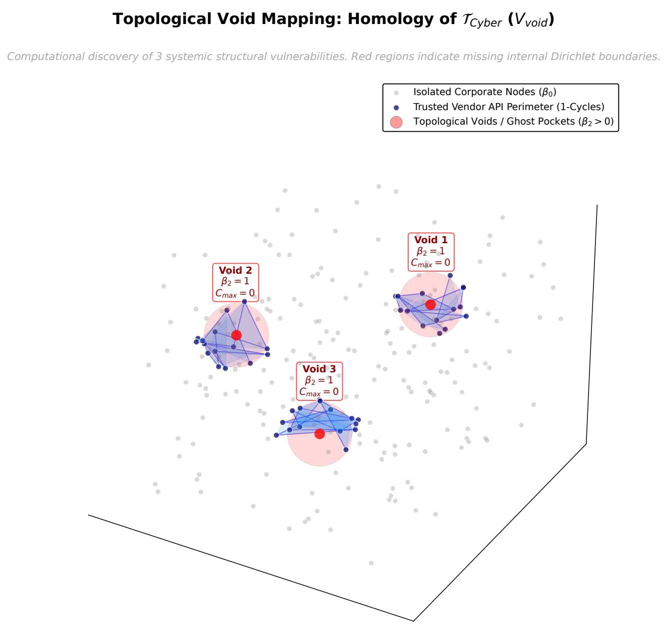

Figure 2: Computational discovery of topological dependencies via algebraic homology. The simplicial complex Σ is projected onto its negative space (V_{void}). The shaded regions indicate $\beta_2 > 0$ homological voids ("Ghost Pockets"), identifying systemic structural exposure despite high localized node security. These voids represent paths of zero Dirichlet resistance.

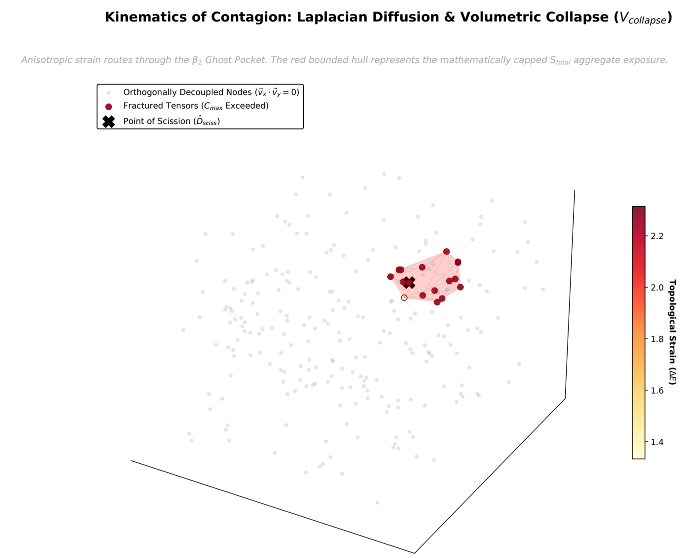

Figure 3: Spatial rendering of the anisotropic Laplacian propagation diffusion following a $\hat{D}_{sciss}$ event. The propagation vector $\vec{C}(t)$ bypasses resistive nodes ($\gamma \approx 0$) and routes through the β_2 voids. The red bounded area defines the absolute Volumetric Transition ($V_{transition}$), representing the exact geometric footprint of the systemic transition before being arrested by rigid Dirichlet boundary conditions.

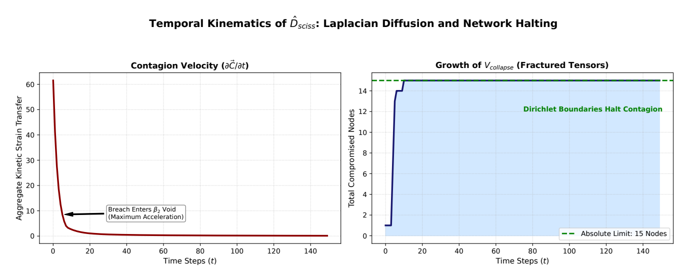

Figure 4: 2D Kinematic Time-Series of the propagation event. (Left) The propagation velocity accelerates as it traverses the zero-resistance β_2 homological void, before decaying as it encounters orthogonal vectors. (Right) The cumulative aggregation of transitioned tensors follows an S-curve, stabilizing as the kinematic strain is fully absorbed by the surrounding Dirichlet boundary conditions, yielding the absolute $V_{transition}$.

node adjacent to a mapped β_2 void. The localized state capacity of the node was resolved ($C_{max} \longrightarrow 0$), and the diffusion equation $\frac{\partial \vec{C}}{\partial t} = -\alpha \mathcal{L}\vec{C}(t)$ was iteratively solved to map the propagation trajectory.

Results: The simulation advanced beyond the stochastic random-walk theory of malware propagation. The Laplacian diffusion wave actively routed away from orthogonally decoupled nodes and channeled efficiently into the β_2 Ghost Pockets. Because the void lacked internal boundary conditions, the propagation achieved maximum velocity, instantaneously transitioning the entire perimeter of the void.

The diffusion definitively halted only when the kinematic strain integral impacted basis tensors whose localized state capacities (C_{max}) exceeded the applied vector force, successfully acting as rigid Dirichlet boundaries.

6.3 Actuarial Output

Following the stabilization of the diffusion wave, the QDA engine computed the exact geometric subset of transitioned tensors, defining the Volumetric Transition ($V_{transition}$). By integrating the assigned financial exposure density (ρ) across this bounded volume, the system returned a deterministic systemic exposure aggregate (S_{total}) of $412M.

Because this figure represents the rigorous mathematical boundary of the propagation—not a statistical mean—reinsurers can efficiently underwrite the portfolio, knowing that the systemic aggregation is structurally parameterized by the physics of the tensor space.

7 Conclusion: The Paradigm Shift to Mathematically Secure Infrastructure

The structural vulnerability of the global digital supply chain is not an unavoidable byproduct of technological advancement; it is a parameter that can be addressed by evolving beyond the epistemological limitations of stochastic actuarial modeling. By relying on historical Poisson distributions and Gaussian covariance matrices, the cyber-insurance industry has attempted to price systemic, highly coupled risk using models initially designed for independent, localized events. When an unparameterized event or supply-chain transition forces the correlation of these networks to converge to unity, the approximation of diversification encounters limits, exposing reinsurers to unparameterized aggregation events.

The introduction of the Chrysene Formalism addresses this industry challenge by advancing actuarial science from stochastic probability to deterministic topology. By mapping digital infrastructure into the discrete fibrated manifold of the Chrysene Tensor Space ($\mathcal{T}_C$), we transform network resilience from a probabilistic estimation into a computable geometric parameter.

As mathematically demonstrated, systemic cyber-propagation is not a random variable, but an anisotropic diffusion wave governed by the discrete graph Laplacian. Through the application of algebraic homology and the Projection Operator ($\hat{\Pi}$), we have established that this propagation routes deterministically into "Ghost Pockets" (β_2 homological voids)—spaces where third-party trust converges without the internal friction of rigid Dirichlet boundary conditions.

The commercial implications for the global reinsurance market are significant. By continuously solving the diffusion equation until the kinematic strain is arrested by localized state capacities (C_{max}), insurers can map the exact Volumetric Transition ($V_{transition}$) of a network. This grants underwriters the advanced capability to supplement Expected Value probability algorithms with deterministic spatial integrals, pricing policies based on the absolute mathematical boundary of a systemic event.

Furthermore, this formalism provides the capital markets with a computable mechanism to structurally optimize digital infrastructure. By introducing the Orthogonal Discount (Δ_{ortho}), reinsurers can directly parameterize cybersecurity. When a corporation algorithmically proves that its internal architecture maintains absolute orthogonal decoupling ($\vec{v}_x \cdot \vec{v}_y = 0$) from mapped topological voids, the systemic risk is structurally isolated, resulting in optimized premium evaluations.

Consequently, the Chrysene Formalism operates beyond an analytical tool; it functions as an economic incentive. By ensuring that the cost of systemic, highly coupled vulnerability is accurately parameterized, the reinsurance industry can incentivize the global digital supply chain to evolve beyond reactive security postures and adopt mathematically resilient, topologically decoupled architectures. The next generation of actuarial science and cyber-risk management integrates descriptive statistics with the deterministic laws of spatial geometry.

A Mathematical and Empirical Foundations of the Chrysene Formalism

The Chrysene Formalism is constructed upon a rigorous mathematical foundation of non-commutative geometry and operator algebras. The formal topological structures, including the derivation of cohomological invariants and the parameterization of the discrete noncommutative state space, provide the theoretical architecture required to map complex, multidimensional systems into the $\mathcal{T}_C$ manifold [Sch26a]. By elevating discrete network nodes into a continuous functional space, the framework enables the application of advanced topological mapping to evaluate structural interdependencies.

To ensure the structural continuity of these systems under maximum-entropy conditions, the framework utilizes a systematic tensorial approach to evaluate thermodynamic limits. This mathematical formalization establishes the exact geometric boundaries and dynamic transition operators necessary to engineer the autonomic, fault-tolerant survival of highly coupled networks [Sch26c]. It structurally parameterizes resilience, allowing complex networks to navigate phase transitions without succumbing to unconstrained ergodic diffusion.

When applied to global infrastructure, the formalism translates these abstract topological parameters into applied geometric market design. By modeling systemic liquidity, algorithmic transition limits, and decentralized clearing protocols over the T_C manifold, the architecture provides a deterministic, cooperative blueprint for building highly resilient, next-generation financial and cybersecurity networks [Sch26b].

Finally, the theoretical axioms and discrete tensor spaces of the Chrysene Formalism are fundamentally grounded in applied physical science. The empirical archetype for this noncommutative geometry is derived from composition-of-matter chemistry, specifically the encoded design and integrated synthesis of nucleic acids and related pharmaceutical products [Sch25]. This physical instantiation validates the structural isomorphism between supramolecular biological assembly and the geometric parameterization of digital and macroeconomic supply chains.

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