

Dual-Channel Information Topology in the Chrysene Formalism: Noncommutative Transport, Latent-State Dynamics, and a Falsifiable Cognitive Interpretation

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Abstract

We develop a dual-channel information topology as a model-relative extension of the twelve-axiom Chrysene Formalism. The inherited molecular structure supplies inequivalent C3–C9 and C12–C6 directional data, but does not by itself imply Euclidean orthogonality, electronic isolation, transfer anisotropy, or cognitive function. We therefore separate four levels: the protected Chrysene foundation; definitional channel mathematics; realization-specific transport and hidden-state hypotheses; and formal cognitive/neuromorphic interpretation. The mathematical extension introduces channel-saturation topologies, a four-state Boolean channel lattice distinct from the inherited four-state cipher, a direct-sum transport model with quantitative residual leakage, and a three-sector crosspoint space comprising a C3–C9 control pathway, a persistent internal-state sector, and a C12–C6 readout pathway. Eliminating the hidden block yields an exact memory-kernel representation of the accessible dynamics; a finite-time linear observation space characterizes state discrimination, while a separately defined generated dynamical algebra records multiplicative growth. Oriented reachability yields a complete Heyting algebra and non-Boolean intuitionistic semantics. A corrected four-terminal bipartite/Hückel core model further shows that both intended terminal pairs are transmission-permitted but electronically inequivalent, with selective cross-terminal interference zeros on one delocalized chrysene π substrate. We then prove a finite neuromorphic consistency result in which unequal response scales, persistent hidden state, bounded nonzero direct cross-talk, state-mediated influence, rapid automatic processing, and later reportable accessibility can coexist without assigning consciousness to a molecular channel. Physical realization, transport-time ordering, and cognitive adequacy remain independently testable claims.

Keywords: Chrysene Formalism; dual-channel information topology; molecular quantum transport; quantum interference; latent-state dynamics; neuromorphic architecture; molecular crossbar; Heyting algebra.

1 Introduction

The revised Chrysene Formalism separates molecular characterization, tensor-network construction, noncommutative sequence algebra, spectral geometry, and physical operator realizations rather than identifying them by definition [Sch26d]. The present work preserves that separation and asks a model-existence question over the 3,6,9,12-substituted chrysene framework. The inherited C3–C9 and C12–C6 directions are geometrically inequivalent, but geometric inequivalence alone does not establish unequal electronic transfer, channel isolation, transport speed, memory, or cognitive function. We therefore ask whether one and the same molecular substrate can support a mathematically consistent and non-vacuous information architecture containing distinguishable directional pathways, a persistent internal state, bounded but nonzero cross-influence, and an accessible

readout whose later evolution can depend on information that was initially hidden. Physical anisotropy, leakage, retention, and response-time ordering enter only through explicitly declared realization hypotheses or through separately identified model calculations.

The resulting architecture is organized as

$$\text{C3–C9} \longrightarrow \mathcal{K}_m \longrightarrow \text{C12–C6},$$

where $\mathcal{K}_m$ denotes a persistent internal-state sector, with a separately parameterized direct shortcut between the two directional pathways. The terms *hidden*, *accessible*, *control*, and *readout* are initially operational: they refer to preparation, state retention, observability, and declared measurement interfaces, not to consciousness. The later neuromorphic interpretation asks a further consistency question: whether the same architecture

can support, without contradiction, a rapid propagation process, a slower state-dependent integrative process, persistent latent state, and finite cross-talk on a common physical substrate. In that interpretation, conscious and nonconscious processing are treated as functional analogues of the combined dynamical organization rather than as intrinsic properties of either molecular direction individually.

1.1 Motivating problem

A dual-pathway physical system can fail to be a dual-information system in several distinct ways. Two geometric directions may correspond to the same effective electronic channel. Two electronic channels may exist but couple so strongly that no useful information separation remains. A hidden internal degree of freedom may exist but fail to influence any later readout. Conversely, the absence of direct carrier transport between two pathways does not prevent one pathway from modulating the other through an intermediate state variable. These distinctions are blurred when geometric, Hamiltonian, dynamical, and informational language is treated as interchangeable.

The present construction therefore separates four questions:

- (i) whether two directional structures can be distinguished mathematically on the inherited lattice;
- (ii) whether a physical realization supports bounded but nonzero inter-channel leakage;
- (iii) whether a persistent hidden subsystem can modify the future accessible dynamics; and
- (iv) whether the resulting accessible–hidden architecture admits a useful cognitive interpretation.

Only the first three questions are mathematical or physical within the DCIT model itself. The fourth is an external interpretation and remains conditional throughout.

1.2 Relation to the revised Chrysene Formalism

Article 39 reconstructs the Chrysene Formalism so that the canonical molecular framework, the $\mathbb{Z}^3$ -fibrated tensor network, the crossed-product sequence algebra, the dual cipher, the spectral triple, and the physical response layers have distinct logical status [Sch26d]. That reconstruction is especially important here for three reasons.

First, the physical motif Hilbert spaces of Axiom A_{10} are not identified with the GNS Hilbert space of the sequence algebra. Accordingly, the physical channel spaces used below are introduced as realization-level spaces and are never silently equated with the spectral representation. Second, Article 39 explicitly states that geometric directional inequivalence does not imply directional transfer anisotropy. The present paper therefore treats the C3–C9 and C12–C6 roles as realization hypotheses rather than consequences of symmetry alone. Third, the native four-state Chrysene alphabet is a dual-cipher/stacking object; its

cardinality does not produce a field multiplication automatically. The new four-state channel lattice introduced below is therefore kept algebraically distinct from both the inherited alphabet and any possible $GF(4)$ realization.

The central inferential discipline is consequently

fixed foundation
+ explicit realization hypotheses
 $\implies$ derived consequences,

not the collapsed inference

geometric resemblance $\implies$
physical or cognitive identity.

1.3 Physical motivation and its limits

The physical motivation is the CohBit crossbar program, which distinguishes a longitudinal data/readout direction and a transverse control direction and proposes Stark modulation, retained gate state, and crossbar scaling [Sch26c]. That source is useful as a realization hypothesis rather than as an inherited theorem. Its own later analysis acknowledges that finite cross-hopping generates residual coupling and that device-level coherence, retention, and material parameters require first-principles or experimental determination. The present paper therefore replaces exact physical zero-coupling by an explicit residual-coupling parameter and treats memory as an independently represented hidden-state sector.

This leads to a three-sector rather than a two-wire ontology:

$$\underbrace{\mathcal{K}_x}_{\text{control path}} \otimes \underbrace{\mathcal{K}_m}_{\text{persistent state}} \otimes \underbrace{\mathcal{K}_y}_{\text{readout path}}.$$

The C3–C9 pathway may prepare or address the hidden state without being identical to that state. The C12–C6 pathway may reveal the hidden state’s consequences without receiving direct carrier transfer from C3–C9. Both named pathways remain terminal-defined responses of one delocalized chrysene π substrate rather than two physically disjoint molecular wires. Appendix D supplies an ideal bipartite/Hückel analysis in which both named terminal pairs are transmission-permitted, electronically inequivalent, and accompanied by selective cross-terminal interference zeros together with finite residual cross-terminal amplitudes. This distinction will be central to the memory theorem, the neuromorphic consistency result, and the falsification protocol.

1.4 Logical discipline and review architecture

The logical discipline follows the certified-growth framework: a protected foundation may admit strict mathematical growth, while realization premises and external-domain interpretations

remain separately typed [Sch26b]. In particular,

$$\begin{array}{c} \text{mathematical theorem transfer} \\ \Rightarrow \\ \text{empirical domain adequacy.} \end{array}$$

Accordingly, a successful crossbar experiment would test the physical realization layer; it would not by itself establish a mechanism of human cognition. Likewise, a formal cognitive interpretation can preserve mathematical structure without establishing that a biological system implements the same physical mechanism.

The manuscript is developed under the orthogonal review architecture used elsewhere in the journal program: mathematical validity, framework concordance, and chemistry/physical-domain admissibility are treated as non-substitutable constraints, followed by targeted verification of repairs and a final meta-review [Sch26a]. This review architecture is incorporated into the paper itself in Section 12 and Appendix F.

1.5 Principal mathematical contributions

The paper makes six primary mathematical constructions.

First, the inherited x - and y -directions generate channel subgroups of $\mathbb{Z}^3$, corresponding saturation topologies, quotient spaces, and a four-element Boolean lattice. This supplies a literal mathematical meaning for *dual-channel information topology* without appealing to Euclidean orthogonality.

Second, the directional physical motif spaces are assembled into an aggregate direct-sum model in which residual inter-channel transfer is controlled by an explicit bounded operator. Duhamel and resolvent estimates give finite-time and energy-domain leakage bounds.

Third, a distinct tensor-product model introduces a persistent hidden-state factor. Eliminating the hidden block by a projection-operator calculation produces an exact memory kernel in the accessible sector. The resulting reduced dynamics can exhibit explicit history dependence even when the complete declared dynamics is Markovian or Hamiltonian; no specific quantum non-Markovianity criterion is inferred from the kernel alone.

Fourth, finite-time observability gives an operational criterion for hidden information. A complete state distinction can be invisible to the instantaneous readout yet become distinguishable at later times if the interaction enlarges the accessible observable algebra.

Fifth, positive channel translations induce a partial order on the lattice. The associated Alexandrov opens form complete Heyting algebras, providing a non-Boolean object-level semantics for forward-reachable information. The manuscript's proof calculus remains classical; the intuitionistic structure belongs to the modeled information domain.

Sixth, the paper proves a finite neuromorphic consistency theorem: two distinguishable pathways supported by one physical substrate, unequal response scales, persistent hidden state,

bounded nonzero direct cross-talk, state-mediated later influence, and delayed emergence into accessibility can coexist in a single finite realization. This is a model-existence result, not an empirical biological claim.

1.6 Cognitive interpretation and falsifiability

Only after the mathematical and physical structures have been fixed is a cognitive interpretation introduced. The primary mapping is between hidden/state-dependent processing and an accessible readout interface. The neuromorphic refinement introduced later may, conditional on an independently verified speed ordering, associate C12–C6 with rapid automatic/feedforward processing and the C3–C9 plus persistent-state architecture with slower state-dependent integration. Conscious or reportable access is assigned to the integrated dynamical emergence of a previously latent distinction, not to either molecular channel in isolation. The interpretation is phrased in terms of reportability, observational equivalence, delayed causal influence, and contextual propositions rather than in terms of a specific neuroanatomical or phenomenal theory.

Three independent failure modes are therefore maintained:

$$F_{\text{math}}, \quad F_{\text{phys}}, \quad F_{\text{cog}}.$$

A mathematical theorem can fail without deciding the chemistry. A physical crossbar realization can fail without invalidating the abstract topology. A cognitive interpretation can fail for a biological target even if the physical device works exactly as designed. This localization of failure is part of the theory rather than an after-the-fact disclaimer.

1.7 Organization of the paper

Section 2 fixes the protected-reduct and statement-typing conventions. Section 3 constructs the channel topologies and four-state Boolean lattice. Section 4 develops the direct-sum transport model and leakage bounds. Sections 5 and 6 construct the hidden-state memory and finite-time observability theories. Section 7 gives the oriented Heyting semantics. Section 8 introduces the cognitive interpretation only after the mathematical core is complete. Section 9 then establishes the finite dual-process neuromorphic consistency result. Sections 10 and 11 state the candidate crossbar realization and its prospective tests. Section 12 records the certification frontier, Section 13 discusses scope and limits, and Section 14 concludes. The appendices give a nontrivial finite witness, analytic proof details, an experimental protocol, the bipartite/Hückel four-terminal core analysis, and the review ledger.

2 Logical Architecture and Concor- dance

2.1 Protected foundation

Let

$$\text{CF}_{12} = (\Sigma_{\text{CF}_{12}}, \mathcal{A}_{\text{CF}_{12}})$$

denote the complete twelve-axiom Chrysene presentation, and fix a full realization

$$\mathfrak{R} \in \text{Ext}_{12}(\text{C}_{\text{chr}}).$$

No definition or theorem of the present paper modifies the interpretation of any symbol in $\Sigma_{\text{CF}_{12}}$.

The controlling formalism is deliberately molecularly specific while permitting realization-level plurality. The canonical chrysene framework, lattice architecture, crossed-product algebra, dual cipher, spectral triple, and physical motif layer are inherited. Downstream couplings, hidden-state factors, measurement algebras, and cognitive interpretations are not.

Remark 2.1 (Standard numbered reference frame). Whenever the present paper writes C3–C9 or C12–C6, the locant labels are used in the inherited standard numbered chrysene reference frame (equivalently, the appropriate standard-locant realization class). Their use is therefore a coordinate convention inherited from the controlling framework, not a new theorem that the twelve axioms uniquely generate chemical nomenclature [Sch26d].

Convention 2.2 (Protected reduct). Every enlarged DCIT realization $\hat{\mathfrak{R}}$ carries the inherited reduct

$$\text{Red}_{\Sigma_{\text{CF}_{12}}}(\hat{\mathfrak{R}}) = \mathfrak{R}.$$

This model-relative rule is weaker than an unrestricted proof-theoretic conservativity theorem, but it is sufficient to prevent the new channel construction from silently redefining the molecular node, sequence algebra, spectral triple, physical motif operators, or admissibility geometry. The need for such separation is explicit in the controlling formalism, which permits multiple downstream physical completions over the same canonical chrysene framework [Sch26d].

2.2 Protected and extension signatures

Write

$$\Sigma_{\text{DCIT}} = \Sigma_{\text{CF}_{12}} \sqcup \Sigma_{\text{top}} \sqcup \Sigma_{\text{dyn}} \sqcup \Sigma_{\text{obs}} \sqcup \Sigma_{\text{int}},$$

where the additional signature components denote, respectively, channel-topological symbols, dynamical overlay symbols, observability symbols, and interpretation symbols. The disjoint-union notation is syntactic: it records intended typing, not a claim

that every semantic realization decomposes into independent physical subsystems.

The inclusion

$$\iota : \Sigma_{\text{CF}_{12}} \hookrightarrow \Sigma_{\text{DCIT}}$$

is fixed once and for all. Any permitted DCIT structure must restrict along ι to the controlling realization $\mathfrak{R}$.

Proposition 2.3 (Protected-symbol rigidity). *Let $s \in \Sigma_{\text{CF}_{12}}$. If $s^{\mathfrak{R}}$ denotes its interpretation in the controlling Chrysene realization and $s^{\hat{\mathfrak{R}}}$ its interpretation in a DCIT enlargement, then*

$$s^{\hat{\mathfrak{R}}} = s^{\mathfrak{R}}.$$

Proof. This is exactly the protected-reduct convention applied symbolwise. $\square$

2.3 Statement types

Convention 2.4 (Logical typing). Every assertion is assigned one of the following statuses:

- (i) inherited Chrysene statement;
- (ii) explicit mathematical definition;
- (iii) theorem derived from inherited and defined structures;
- (iv) realization-overlay hypothesis;
- (v) theorem conditional on realization hypotheses;
- (vi) formal cognitive interpretation hypothesis; or
- (vii) empirical assertion.

A later type is never promoted to an earlier type merely by appearing in a proof downstream.

The distinction is intended to be operational during review. A theorem whose proof uses only inherited structures and explicit definitions is reviewed as mathematics. A statement involving a material coupling, retained charge state, electrode geometry, or measured leakage threshold is a realization claim. A statement mapping the accessible–hidden architecture to cognition belongs to the interpretation layer. The same sentence should not move between these categories merely because it becomes rhetorically convenient.

Definition 2.5 (Status map). Let $\mathcal{K}$ denote the collection of principal manuscript assertions. Define the status map

$$\text{Stat} : \mathcal{K} \longrightarrow \{\text{inh, def, thm, real, cond, int, emp}\}$$

according to the preceding convention.

Remark 2.6. The status map is editorial metadata, not an additional object of the Chrysene formalism. Its role is to prevent a downstream physical or cognitive statement from being cited later as though it had been an axiom or an unconditional theorem.

2.4 Definitional layer and realization overlay

Definition 2.7 (Definitional DCIT layer). The definitional layer consists of objects explicitly constructed from the inherited realization without adjoining a new physical premise. Examples include channel subgroups, saturation topologies, quotient spaces, the channel Boolean lattice, and oriented reachability orders.

Definition 2.8 (DCIT realization overlay). A realization overlay is a tuple of additional physical or informational data, schematically

$$\rho_{\text{DCIT}} = (K, \mathcal{K}_m, V, \mathcal{L}_{\text{hid}}, \mathcal{L}_y, \sigma_{\text{hid}}, \mathcal{M}, \dots),$$

whose entries are not required by Axioms A_1 – A_{12} .

This is the same logical role assigned to contextual realization premises in the certified-growth framework [Sch26b]. Such premises may support conditional theorems but are not thereby protected axioms.

Proposition 2.9 (Protected-reduct preservation). *For every admissible DCIT overlay over $\mathfrak{R}$, the resulting enlarged structure has Chrysene reduct $\mathfrak{R}$.*

Proof. By construction, the overlay adjoins symbols and interpretations while leaving every symbol in $\Sigma_{\text{CF}_{12}}$ unchanged. Taking the reduct discards the added data and recovers $\mathfrak{R}$. $\square$

2.5 Definitional conservativity

The pure topological portion of DCIT is intended as an explicit definitional extension. In a standard proof calculus admitting elimination of explicit definitions, old-language consequences are therefore unchanged.

Hypothesis 2.10 (Definition elimination). Every proof of a sentence in $\text{Sent}(\Sigma_{\text{CF}_{12}})$ from an explicit definitional extension can be translated into a proof using only the original Chrysene signature and axioms.

Proposition 2.11 (Conservativity of the pure definitional layer). *Let P_{def} be the presentation obtained from CF_{12} by adjoining only the explicit channel-topological definitions of Sections 3 and 7. Under the definition-elimination hypothesis, for every old-signature sentence φ ,*

$$P_{\text{def}} \vdash \varphi \iff \text{CF}_{12} \vdash \varphi.$$

Proof. The reverse implication follows from inclusion of the old axioms. For the forward implication, eliminate each newly introduced symbol by its defining formula. Since φ contains no new symbol, the resulting proof belongs entirely to the old presentation. $\square$

Remark 2.12. The proposition applies only to the explicit definitional layer. It does not claim conservativity after adding physical realization hypotheses, because such hypotheses are intentionally capable of selecting a proper subclass of Chrysene realizations.

2.6 Article-41 protected concordance certificate

Let

$$P_{\text{CF}_{12}} := (\Sigma_{\text{CF}_{12}}, \mathcal{A}_{\text{CF}_{12}})$$

be the controlling Chrysene presentation. Let

$$\Sigma_{\text{prot}} \subseteq \Sigma_{\text{CF}_{12}}, \quad C_{\text{ref}}, \quad L_{\text{id}}$$

denote, respectively, the protected subsignature, Canonical Molecular Anchor, and protected identity fragment used by the certified-growth framework [Sch26b]. Let

$$P_{\text{def}} = (\Sigma_{\text{def}}, \mathcal{A}_{\text{def}})$$

be the presentation obtained by adjoining only the explicit DCIT definitional layer, and let

$$\iota : \Sigma_{\text{CF}_{12}} \hookrightarrow \Sigma_{\text{def}}$$

be the inherited signature embedding.

Proposition 2.13 (Ordinary protected concordance of the definitional layer). *Under the definition-elimination hypothesis, P_{def} is an ordinary concordant extension of $P_{\text{CF}_{12}}$ in the sense of the certified-growth framework. More precisely,*

$$\iota_*(\mathcal{A}_{\text{CF}_{12}}) \subseteq \mathcal{A}_{\text{def}},$$

$$\text{Mod}_{\text{anc}}(P_{\text{def}}) \neq \emptyset,$$

and

$$\text{IdTh}(P_{\text{def}}) = \text{IdTh}(P_{\text{CF}_{12}})$$

on the protected identity fragment L_{id} .

Proof. The first relation follows from construction: the definitional presentation contains the embedded Article-39 axioms together with explicit definitions of the new channel objects. For anchor-compatible non-vacuity, expand the fixed Article-39 realization $\mathfrak{R}$ by those explicitly defined objects. Its reduct on $\Sigma_{\text{CF}_{12}}$ is unchanged, so its protected anchor remains C_{ref} . Finally, Proposition 2.11 gives conservativity for every sentence in the old signature under definition elimination; equality of the protected identity theory follows a fortiori on L_{id} . $\square$

Remark 2.14 (Model-relative specialization does not enlarge the protected fragment). Fixing one full realization $\mathfrak{R}$ is a model-relative specialization used to formulate the present DCIT constructions. It does not enlarge L_{id} to include every realization-specific Hamiltonian, response operator, admissibility datum, or other downstream choice. Distinct full Chrysene realizations may therefore support distinct DCIT overlays while preserving the same protected molecular identity.

2.7 Dependency structure

The new objects are organized by definitional prerequisite rather than by a claim of physical causation. A convenient dependency chain is

$$\mathfrak{R} \prec \{L_S, \tau_S, \mathbb{B}_{\text{ch}}\} \prec \{\mathcal{K}_{\text{tr}}, \Pi_x, \Pi_y\} \prec \{K, H_{\varepsilon}^{\text{tr}}\}$$

for the direct-leakage branch, and

$$\begin{aligned} \mathfrak{R} \prec \{\mathcal{K}_x, \mathcal{K}_y\} \prec \{\mathcal{K}_m, \mathcal{K}_{\text{XB}}\} \\ \prec \{\mathcal{L}, P_{\text{acc}}, Q_{\text{hid}}\} \prec \{\mathcal{K}(t), \mathfrak{D}_T\}. \end{aligned}$$

for the hidden-state branch. Appendix D adds a separately typed ideal core-graph/Hückel branch: its graph and Green-function statements are conditional on that declared model and do not modify the protected Chrysene presentation. The neuromorphic existence construction of Section 9 is downstream of the channel, hidden-state, and ideal core-graph structures; the speed-ordered interpretation additionally depends on the open realization hypothesis H_{τ} . Cognitive and biological-domain interpretations remain downstream of all of these structures.

Proposition 2.15 (Acyclicity of the DCIT prerequisite graph). *The dependency graph generated by the preceding relations contains no directed cycle.*

Proof. Assign ranks

$$0, 1, 2, 3, 4$$

to the inherited realization, definitional topology, aggregate representations, dynamics, and interpretation layers, respectively. Every declared prerequisite arrow increases rank strictly. Therefore a directed cycle is impossible. $\square$

Remark 2.16. As in Article 39, definitional prerequisite should not be read as automatic physical causation. For example, the existence of $\mathbb{B}_{\text{ch}}$ does not cause a four-level molecular device, and the existence of a hidden-state projection does not physically perform a measurement or erasure.

2.8 Non-vacuity of the overlay class

Proposition 2.17 (Overlay non-vacuity). *Assume the inherited directional motif spaces $\mathcal{K}_x$ and $\mathcal{K}_y$ are nonzero. Then at least one DCIT overlay exists.*

Proof. Take $K = 0$, choose $\mathcal{K}_m = \mathbb{C}$, set all added interactions and bounded generators to zero, and choose any normalized hidden reference state. These data satisfy the formal overlay requirements. Appendix A gives a nontrivial finite witness with nonzero hidden-state memory. $\square$

Remark 2.18 (Logical strength of the witness). Non-vacuity does not establish a chemically realistic device, a nonzero modulation mechanism, or cognitive adequacy. It establishes only that the enlarged mathematical language has models over the protected Chrysene realization. Appendix A strengthens the trivial existence argument by exhibiting a finite model with a nonzero memory kernel and nonzero hidden-state influence.

2.9 No inference collapse

The main logical safeguards of the paper can be summarized as

$$\begin{aligned} \text{directional inequivalence} &\Rightarrow \text{electronic isolation,} \\ \text{electronic isolation} &\Rightarrow \text{persistent memory,} \\ \text{persistent memory} &\Rightarrow \text{reportability structure,} \\ \text{reportability structure} &\Rightarrow \text{cognitive adequacy.} \end{aligned}$$

Each implication may become valid only after the additional hypotheses required for its own proof have been supplied.

3 Dual-Channel Information Topology

3.1 Inherited directional data

The inherited spatial base is equipped with a fixed lattice-coordinate identification

$$\chi_C : X_C \xrightarrow{\sim} \mathbb{Z}^3.$$

Throughout this paper we transport the additive group law through χ_C and, in these fixed lattice coordinates, identify

$$X_C = \mathbb{Z}^3.$$

Thus subset addition, subgroup cosets, and coordinate differences below are literal operations in $\mathbb{Z}^3$. With this convention the standard generators are

$$e_x = (1, 0, 0), \quad e_y = (0, 1, 0), \quad e_z = (0, 0, 1).$$

The primary molecular directions are

$$d_x = r_9 - r_3, \quad d_y = r_6 - r_{12},$$

with $d_x \neq 0$, $d_y \neq 0$, and $d_x \times d_y \neq 0$. No Euclidean orthogonality condition $d_x \cdot d_y = 0$ is imposed. This distinction is essential: the channel topology introduced below depends on the two distinguished lattice generators, not on an assertion that the embedded molecular displacement vectors meet at a right angle.

The third lattice generator e_z remains the inherited stacking direction. The present dual-channel theory initially concerns the planar information pair $\{x, y\}$; the z -fiber remains available for later physical realizations but is not folded into the two-channel alphabet.

3.2 Channel subgroups and saturation

Let

$$\text{Ch} = \{x, y\}.$$

For $S \subseteq \text{Ch}$, define

$$L_S := \bigoplus_{\alpha \in S} \mathbb{Z}e_\alpha \subseteq \mathbb{Z}^3.$$

Thus

$$L_\emptyset = \{0\}, \quad L_x = \mathbb{Z}e_x, \quad L_y = \mathbb{Z}e_y, \quad L_{xy} = \mathbb{Z}e_x \oplus \mathbb{Z}e_y.$$

Definition 3.1 (Channel saturation). For $A \subseteq X_C$, set

$$\text{Sat}_S(A) := A + L_S = \{a + \ell : a \in A, \ell \in L_S\}.$$

Proposition 3.2 (Saturation closure). For each $S \subseteq \text{Ch}$, Sat_S is a Kuratowski closure operator. Explicitly,

$$\text{Sat}_S(\emptyset) = \emptyset,$$

$$A \subseteq \text{Sat}_S(A),$$

$$\text{Sat}_S(\text{Sat}_S(A)) = \text{Sat}_S(A),$$

and

$$\text{Sat}_S(A \cup B) = \text{Sat}_S(A) \cup \text{Sat}_S(B).$$

Proof. The empty-set property is immediate. Extensivity follows from $0 \in L_S$. Because L_S is a subgroup,

$$L_S + L_S = L_S,$$

which gives idempotence. Finally, Minkowski addition distributes over unions. $\square$

Definition 3.3 (Channel-saturation topology). Define

$$\tau_S := \{U \subseteq X_C : U + L_S = U\}.$$

Every member of τ_S is a union of L_S -cosets. Because the complement of a union of cosets is again a union of cosets, each τ_S -open set is also τ_S -closed.

Proposition 3.4 (Topology and closure). The family τ_S is a topology and

$$\overline{A}^{\tau_S} = \text{Sat}_S(A).$$

Moreover, every connected component of (X_C, τ_S) is a single coset of L_S .

Proof. The L_S -saturated subsets contain $\emptyset$ and X_C , are closed under arbitrary unions, and are closed under finite intersections. The smallest saturated set containing A is $A + L_S$. Since the cosets are pairwise disjoint and each is clopen, no connected subset can meet two distinct cosets. $\square$

3.3 Channel equivalence and quotient spaces

Definition 3.5 (Channel equivalence). Write

$$k \sim_S \ell \iff k - \ell \in L_S.$$

The equivalence classes are precisely the cosets $k + L_S$.

Proposition 3.6 (Quotient characterization). Let

$$q_S : X_C \longrightarrow X_C/L_S$$

be the quotient map and equip the quotient set with the discrete topology. Then

$$\tau_S = \{q_S^{-1}(B) : B \subseteq X_C/L_S\}.$$

Two lattice sites have the same τ_S -neighborhoods if and only if they are $\sim_S$ -equivalent.

Proof. A subset of X_C is L_S -saturated exactly when it is a union of equivalence classes, hence exactly when it is the inverse image of a subset of the quotient. Sites in the same class belong to precisely the same saturated sets. Distinct classes are separated by the clopen coset of either point. $\square$

Example 3.7 (Single-site saturation). For the origin $0 \in \mathbb{Z}^3$,

$$\begin{aligned}\text{Sat}_x(\{0\}) &= L_x, \\ \text{Sat}_y(\{0\}) &= L_y, \\ \text{Sat}_{xy}(\{0\}) &= L_{xy}.\end{aligned}$$

Thus the three closures distinguish one-dimensional x -reachability, one-dimensional y -reachability, and the complete planar subgroup.

3.4 Order among the channel topologies

If $S \subseteq T \subseteq \text{Ch}$, then $L_S \subseteq L_T$. Saturation by the larger subgroup produces larger closure classes and therefore a coarser topology.

Proposition 3.8 (Topology order reversal). *If $S \subseteq T$, then*

$$\tau_T \subseteq \tau_S.$$

In particular,

$$\tau_{xy} \subseteq \tau_x, \quad \tau_{xy} \subseteq \tau_y,$$

and

$$\tau_\emptyset = \mathcal{P}(X_C)$$

is the discrete topology.

Proof. If $U + L_T = U$ and $L_S \subseteq L_T$, then

$$U + L_S \subseteq U + L_T = U.$$

The reverse inclusion follows from $0 \in L_S$, hence $U + L_S = U$. $\square$

The pair (τ_x, τ_y) may therefore be viewed as a bitopological structure on the same lattice. We do not require the general theory of bitopological spaces in what follows; the term simply records that two distinct saturation topologies coexist on a common set and are compared through the common refinement $\tau_\emptyset$ and common coarsening τ_{xy} .

3.5 Four-state channel lattice

Let

$$\mathfrak{L}_{\text{ch}} := \{L_\emptyset, L_x, L_y, L_{xy}\}.$$

Theorem 3.9 (Four-state channel lattice). *The poset $(\mathfrak{L}_{\text{ch}}, \subseteq)$ is a four-element Boolean lattice canonically isomorphic to $\mathcal{P}(\{x, y\})$. In particular,*

$$L_S \wedge L_T = L_{S \cap T}, \quad L_S \vee L_T = L_{S \cup T}.$$

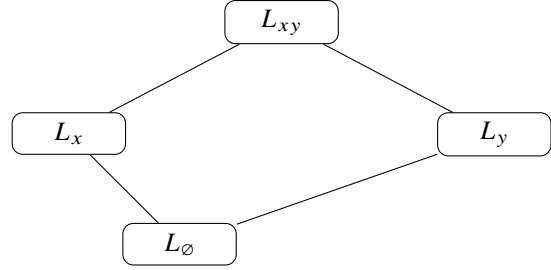


Figure 1: Hasse diagram of the four-state channel lattice. The order is subgroup inclusion. The diagram records channel-regime structure only; it is distinct from the inherited dual-cipher/registry alphabet.

The Boolean complements are

$$\begin{aligned}\neg_{\text{ch}} L_\emptyset &= L_{xy}, & \neg_{\text{ch}} L_{xy} &= L_\emptyset, \\ \neg_{\text{ch}} L_x &= L_y, & \neg_{\text{ch}} L_y &= L_x.\end{aligned}$$

Proof. The map $S \mapsto L_S$ is bijective. Linear independence of e_x, e_y gives

$$L_x \cap L_y = \{0\}$$

and

$$L_x + L_y = L_{xy}.$$

Thus meet and join agree with intersection and union of subsets of $\{x, y\}$, and the complement operation is transported from the power-set Boolean algebra. $\square$

Definition 3.10 (Channel alphabet). Write

$$\mathbb{B}_{\text{ch}} := \{0, x, y, xy\}$$

for the four symbols corresponding to

$$L_\emptyset, L_x, L_y, L_{xy}.$$

Remark 3.11 (Three different four-state objects). The new Boolean lattice $\mathbb{B}_{\text{ch}}$ is not the inherited structural alphabet

$$\Sigma_C = \mathbb{Z}_2^{dc} \times \mathbb{Z}_2^z,$$

and neither cardinality-four structure is automatically a field. A $GF(4)$ interpretation requires an independently defined multiplication, exactly as emphasized in the revised formalism [Sch26d; LN97].

The distinction is algebraic, not merely terminological. $\mathbb{B}_{\text{ch}}$ is canonically a Boolean lattice because its operations are inherited from subsets of the channel index set. The structural alphabet Σ_C carries its inherited additive dual-cipher/registry law. A field

structure would require a multiplication satisfying the finite-field axioms. Equality of cardinality among these three objects does not identify their operations.

3.6 Commuting closures

Proposition 3.12 (Commuting channel saturation). *For all $A \subseteq X_C$,*

$$\text{Sat}_x(\text{Sat}_y(A)) = \text{Sat}_y(\text{Sat}_x(A)) = \text{Sat}_{xy}(A).$$

Proof. Using associativity and commutativity of subgroup addition,

$$\text{Sat}_x(\text{Sat}_y(A)) = (A + L_y) + L_x = A + (L_x + L_y) = A + L_{xy}.$$

The reverse order is identical. $\square$

Remark 3.13 (Topological commutativity versus sequence noncommutativity). The preceding proposition concerns closure operators on the lattice index set. It does not imply commutativity of the inherited crossed-product sequence algebra

$$\mathfrak{A}_C = \mathfrak{Q}_C \rtimes_{\alpha} \mathbb{Z}^3.$$

The latter remains noncommutative because translation changes localization. Thus the same theory contains commuting channel-saturation operators and noncommuting localization/translation sequences without contradiction.

3.7 Channel-index path geometries

For $\alpha \in \{x, y\}$, define the graph

$$G_{\alpha}^{\text{ch}} = (X_C, E_{\alpha}^{\text{ch}})$$

with

$$E_{\alpha}^{\text{ch}} := \{\{k, k + e_{\alpha}\} : k \in \mathbb{Z}^3\},$$

and let $\delta_{\alpha}^{\text{ch}}$ be its extended unit-weight distance. Define δ_{xy}^{ch} analogously using both edge families.

Proposition 3.14 (Explicit channel distances). *For $k = (m, n, p)$, $\ell = (m', n', p')$,*

$$\delta_x^{\text{ch}}(k, \ell) = \begin{cases} |m' - m|, & n' = n, p' = p, \\ +\infty, & \text{otherwise,} \end{cases}$$

$$\delta_y^{\text{ch}}(k, \ell) = \begin{cases} |n' - n|, & m' = m, p' = p, \\ +\infty, & \text{otherwise,} \end{cases}$$

and

$$\delta_{xy}^{\text{ch}}(k, \ell) = \begin{cases} |m' - m| + |n' - n|, & p' = p, \\ +\infty, & p' \neq p. \end{cases}$$

Proof. An x -edge changes only the first coordinate by one unit, so an x -path exists exactly when the second and third coordinates agree. The shortest such path has length $|m' - m|$. The y -case is identical. In the combined graph the first two coordinates may be changed independently while the third is fixed, giving the planar Manhattan distance. $\square$

Corollary 3.15 (Metric connected components). *The finite-distance components of δ_x^{ch} are the cosets of L_x ; those of δ_y^{ch} are the cosets of L_y ; and those of δ_{xy}^{ch} are the cosets of L_{xy} .*

Thus the saturation topology and the extended channel metric encode the same channel-equivalence partition in two different mathematical languages: topologically through clopen cosets and metrically through finite-distance components.

Remark 3.16 (Not the inherited admissible metric). These are auxiliary lattice-index metrics. They are not the Axiom- A_{12} admissible structural-state distance and are not identified with the Connes metric of the spectral triple. No exact equality, bi-Lipschitz relation, or disconnectivity equivalence among these metric constructions is assumed.

3.8 Channel grading

Let the inherited Axiom- A_{10} exact-closure motif spaces be $\mathcal{K}_x$ and $\mathcal{K}_y$. Define

$$\mathcal{K}_{\text{tr}} := \mathcal{K}_x \oplus \mathcal{K}_y$$

and

$$\Gamma_{\text{ch}} = \begin{pmatrix} I_{\mathcal{K}_x} & 0 \\ 0 & -I_{\mathcal{K}_y} \end{pmatrix}.$$

Then

$$\Gamma_{\text{ch}}^* = \Gamma_{\text{ch}}, \quad \Gamma_{\text{ch}}^2 = I.$$

Set

$$\Pi_x = \frac{I + \Gamma_{\text{ch}}}{2}, \quad \Pi_y = \frac{I - \Gamma_{\text{ch}}}{2}.$$

Proposition 3.17 (Complementary channel projections). *The operators Π_x, Π_y are complementary orthogonal projections onto $\mathcal{K}_x$ and $\mathcal{K}_y$:*

$$\begin{aligned} \Pi_x^2 &= \Pi_x = \Pi_x^*, & \Pi_y^2 &= \Pi_y = \Pi_y^*, \\ \Pi_x \Pi_y &= 0, & \Pi_x + \Pi_y &= I. \end{aligned}$$

Proof. The identities follow directly from $\Gamma_{\text{ch}}^2 = I$ and self-adjointness of the grading. $\square$

Remark 3.18 (Meaning of channel orthogonality). Projection orthogonality in the aggregate Hilbert space does not assert Euclidean orthogonality of d_x, d_y , does not prove vanishing electronic coupling, and does not imply equality or inequality of the inherited directional transfer functions. It is an algebraic decomposition introduced so that residual physical coupling can be represented explicitly rather than hidden inside the definition of a channel.

3.9 Summary of the topological layer

The dual-channel topology therefore supplies four mutually consistent descriptions of the same two-direction organization:

- (i) subgroup data L_x, L_y, L_{xy} ;
- (ii) saturation topologies $\tau_x, \tau_y, \tau_{xy}$;
- (iii) extended channel-index metrics $\delta_x^{\text{ch}}, \delta_y^{\text{ch}}, \delta_{xy}^{\text{ch}}$; and
- (iv) a $\mathbb{Z}_2$ -graded aggregate transport space with projections Π_x, Π_y .

None of these structures, by itself, assigns data/control, accessible/hidden, or cognitive meaning to either direction. Those roles enter only in later realization layers.

4 Channel Operator Algebras and Coupled Transport

4.1 Directional physical realizations

For the fixed realization, let

$$H_x = H_x^* \in B(\mathcal{K}_x), \quad H_y = H_y^* \in B(\mathcal{K}_y)$$

be the bounded motif Hamiltonians supplied by the physical layer. Article 39 explicitly permits the two motif spaces and Hamiltonians to be independent of the GNS spectral representation and does not impose equality or inequality of the x - and y -transfer operators [Sch26d]. The present section therefore starts from two separately specified physical motif spaces and asks how a controlled amount of cross-channel coupling may be adjoined.

Define the uncoupled transport Hamiltonian

$$H_0^{\text{tr}} := H_x \oplus H_y$$

on

$$\mathcal{K}_{\text{tr}} = \mathcal{K}_x \oplus \mathcal{K}_y.$$

The grading projections Π_x and Π_y from Section 3 reduce H_0^{tr} .

Proposition 4.1 (Ideal sector reduction).

$$[H_0^{\text{tr}}, \Pi_x] = [H_0^{\text{tr}}, \Pi_y] = 0.$$

Consequently,

$$e^{-itH_0^{\text{tr}}/\hbar} = e^{-itH_x/\hbar} \oplus e^{-itH_y/\hbar}.$$

Proof. Both projections are block diagonal in the direct-sum decomposition, as is H_0^{tr} . Functional calculus for direct sums gives the propagator identity. $\square$

4.2 Residual direct coupling

Let

$$K : \mathcal{K}_y \longrightarrow \mathcal{K}_x$$

be bounded and define the self-adjoint off-diagonal operator

$$V_K := \begin{pmatrix} 0 & K \\ K^* & 0 \end{pmatrix}.$$

For $\varepsilon \in \mathbb{R}$, set

$$H_\varepsilon^{\text{tr}} := H_0^{\text{tr}} + \varepsilon V_K = \begin{pmatrix} H_x & \varepsilon K \\ \varepsilon K^* & H_y \end{pmatrix}.$$

Proposition 4.2 (Self-adjoint coupled transport). *For every real ε , $H_\varepsilon^{\text{tr}}$ is bounded and self-adjoint.*

Proof. The diagonal direct sum is bounded and self-adjoint and $V_K = V_K^*$. Their real linear combination is therefore bounded and self-adjoint. $\square$

Definition 4.3 (Ideal transport separation). The direct-sum transport model is ideally separated when

$$\varepsilon = 0.$$

The ideal limit is a reference point, not a claim that a real molecular junction has exactly zero cross-coupling. The physically relevant quantities are the norm of the off-diagonal transfer and the resulting finite-time or energy-domain leakage.

4.3 Finite-time leakage

Let

$$U_\varepsilon(t) := e^{-itH_\varepsilon^{\text{tr}}/\hbar}.$$

Definition 4.4 (Leakage amplitude). Define

$$\Lambda_{x \rightarrow y}(t) := \|\Pi_y U_\varepsilon(t) \Pi_x\|, \quad \Lambda_{y \rightarrow x}(t) := \|\Pi_x U_\varepsilon(t) \Pi_y\|.$$

Theorem 4.5 (Finite-time leakage bound). *For every $t \in \mathbb{R}$,*

$$\Lambda_{x \rightarrow y}(t) \leq \frac{|\varepsilon| |t| \|K\|}{\hbar},$$

and symmetrically

$$\Lambda_{y \rightarrow x}(t) \leq \frac{|\varepsilon| |t| \|K\|}{\hbar}.$$

Proof. Duhamel's formula and $\|V_K\| = \|K\|$ give the estimate. The full norm argument is in Appendix B. $\square$

Definition 4.6 (Worst-case leakage probability). For a normalized x -sector state, define

$$\mathcal{P}_{x \rightarrow y}(t) := \sup_{\substack{\psi \in \mathcal{K}_x \\ \|\psi\|=1}} \|\Pi_y U_\varepsilon(t) \psi\|^2.$$

Corollary 4.7 (Worst-case leakage probability).

$$\mathcal{P}_{x \rightarrow y}(t) \leq \min \left\{ 1, \left(\frac{|\varepsilon| |t| \|K\|}{\hbar} \right)^2 \right\}.$$

The $\min\{1, \cdot\}$ is important: the perturbative bound may become numerically larger than one for long times even though a transition probability cannot.

4.4 Energy-domain leakage

For $z \in \rho(H_0^{\text{tr}})$, set

$$\delta(z) := \text{dist}(z, \sigma(H_0^{\text{tr}})).$$

Define the resolvents

$$R_0(z) := (zI - H_0^{\text{tr}})^{-1}, \quad R_\varepsilon(z) := (zI - H_\varepsilon^{\text{tr}})^{-1}.$$

Theorem 4.8 (Cross-channel resolvent estimate). *If*

$$|\varepsilon| \|K\| < \delta(z),$$

then $z \in \rho(H_\varepsilon^{\text{tr}})$ *and*

$$\|\Pi_y R_\varepsilon(z) \Pi_x\| \leq \frac{|\varepsilon| \|K\|}{\delta(z) [\delta(z) - |\varepsilon| \|K\|]}.$$

Proof. Use the Neumann criterion and the resolvent identity; see Appendix B. $\square$

Remark 4.9 (Relation to inherited directional transfer). The block

$$\Pi_y R_\varepsilon(z) \Pi_x$$

is a cross-channel propagator in the new aggregate direct-sum model. It is not the inherited directional Schur-complement transfer $T_\alpha^{(N)}(z)$, which is defined within a

single node–bridge–node motif. The two constructions have different domains, codomains, and physical roles.

4.5 Comparison with Axiom- A_{10} transfer operators

The inherited directional transfer operator may be written schematically as

$$T_\alpha^{(N)}(z) = J_\alpha^{(N)} + (W_{L,\alpha}^{(N)})^* R_{B,\alpha}^{(N)}(z) W_{R,\alpha}^{(N)}.$$

Its role is intra-directional: it integrates out bridge degrees of freedom and returns an effective terminal coupling. The new operator K instead links the x - and y -sector Hilbert spaces. Thus one may have

$$T_x^{(N_x)}(z) \neq 0, \quad T_y^{(N_y)}(z) \neq 0, \quad K = 0,$$

or, conversely, a realization with nonzero K even if one directional effective transfer is weak.

Proposition 4.10 (Logical independence of intra- and inter-channel transfer). *The existence of nonzero inherited directional transfer operators does not imply $K \neq 0$, and $K \neq 0$ does not imply directional transfer anisotropy.*

Proof. The operators are distinct entries of distinct effective models. No definition or axiom identifies their values. Concrete relations require a further realization map between the aggregate channel model and the motif Hamiltonians. $\square$

4.6 Joint-channel composite

The direct sum above measures alternative transport sectors. Simultaneously represented channel and memory degrees of freedom require a different construction. Introduce a nonzero Hilbert space $\mathcal{K}_m$ and define

$$\mathcal{K}_{\text{XB}} := \mathcal{K}_x \otimes \mathcal{K}_m \otimes \mathcal{K}_y.$$

Set

$$\mathcal{K}_{\text{hid}} := \mathcal{K}_x \otimes \mathcal{K}_m, \quad \mathcal{K}_{\text{acc}} := \mathcal{K}_y.$$

Convention 4.11 (Two effective representations). The direct sum $\mathcal{K}_{\text{tr}}$ is used for direct channel leakage. The tensor product $\mathcal{K}_{\text{XB}}$ is used for simultaneous control, persistent hidden state, and readout. No natural identification between the two is assumed.

This distinction is not cosmetic. A direct sum describes mutually exclusive sector labels for one aggregate amplitude, whereas a tensor product permits simultaneously present degrees of freedom and correlations. The two constructions answer different questions and should not be converted into one another by notation alone.

4.7 Local channel algebras

On $\mathcal{K}_{XB}$, define

$$\mathfrak{A}_x^{\text{loc}} := B(\mathcal{K}_x) \bar{\otimes} I_{\mathcal{K}_m} \bar{\otimes} I_{\mathcal{K}_y},$$

$$\mathfrak{A}_m^{\text{loc}} := I_{\mathcal{K}_x} \bar{\otimes} B(\mathcal{K}_m) \bar{\otimes} I_{\mathcal{K}_y},$$

and

$$\mathfrak{A}_{\text{acc}} := I_{\mathcal{K}_{\text{hid}}} \bar{\otimes} B(\mathcal{K}_y).$$

Proposition 4.12 (Tensor-factor commutativity). *Observables supported on distinct tensor factors commute. In particular,*

$$[\mathfrak{A}_x^{\text{loc}}, \mathfrak{A}_{\text{acc}}] = 0, \quad [\mathfrak{A}_m^{\text{loc}}, \mathfrak{A}_{\text{acc}}] = 0.$$

Remark 4.13 (Static commutativity versus dynamical influence). The preceding commutators concern the instantaneous tensor-factor algebras. Once an interaction Hamiltonian is introduced, the Heisenberg evolution of an accessible observable need not remain inside $\mathfrak{A}_{\text{acc}}$. Thus static algebraic commutativity is compatible with later dynamical observability of hidden variables.

4.8 Hidden-readout interaction

Let

$$H_{\text{hid}} = H_{\text{hid}}^* \in B(\mathcal{K}_{\text{hid}}), \quad H_y = H_y^* \in B(\mathcal{K}_y),$$

and let

$$V_{\text{hy}} = V_{\text{hy}}^* \in B(\mathcal{K}_{XB})$$

be a bounded hidden-readout interaction. Define

$$H_{\lambda}^{\text{joint}} := H_{\text{hid}} \otimes I_y + I_{\text{hid}} \otimes H_y + \lambda V_{\text{hy}}.$$

Proposition 4.14 (Self-adjoint joint generator). *For every real λ , $H_{\lambda}^{\text{joint}}$ is bounded and self-adjoint.*

The scalar coefficients ε and λ are representation-dependent and need not be dimensionless. We therefore compare effective interaction strengths

$$g_{\text{leak}} := |\varepsilon| \|K\|, \quad g_{\text{hy}} := |\lambda| \|V_{\text{hy}}\|.$$

A central regime of interest is

$$g_{\text{leak}} \ll g_{\text{hy}},$$

which represents weak unwanted direct transport in the direct-sum model together with appreciable hidden-readout interaction in the joint model. No numerical comparison of the bare coefficients alone is used.

4.9 Compatibility with inherited noncommutativity

Let $(\pi_{\omega}, \mathcal{H}_{\omega}, \Omega_{\omega})$ be the inherited GNS representation. On

$$\widehat{\mathcal{H}} = \mathcal{H}_{\omega} \otimes \mathcal{K}_{XB},$$

represent structural operators by

$$\widehat{\pi}(a) = \pi_{\omega}(a) \otimes I_{\mathcal{K}_{XB}}$$

and device operators by

$$I_{\mathcal{H}_{\omega}} \otimes B, \quad B \in B(\mathcal{K}_{XB}).$$

Theorem 4.15 (Local device algebra with global sequence noncommutativity). *The device-factor observables commute with represented Chrysen structural observables, while the inherited sequence commutators remain nonzero whenever*

$$[\pi_{\omega}(u_m), \pi_{\omega}(P_k)] \neq 0.$$

Hence local tensor-factor commutativity is compatible with global crossed-product noncommutativity.

Proof. The factors act on distinct tensor components, so

$$[\pi_{\omega}(a) \otimes I, I \otimes B] = 0.$$

For the inherited sequence commutator,

$$[\widehat{\pi}(u_m), \widehat{\pi}(P_k)] = [\pi_{\omega}(u_m), \pi_{\omega}(P_k)] \otimes I,$$

which is nonzero whenever the inherited commutator is nonzero. $\square$

Remark 4.16 (External tensor-factor commutativity is not the dual-cipher commutant). The commutation relation

$$[\pi_{\omega}(a) \otimes I, I \otimes B] = 0$$

arises solely because the operators act on different factors of the enlarged external tensor product. It is not the Axiom- A_8 primary-dual commutant construction of the Chrysen Formalism. The latter is tied specifically to the inherited primary/dual cipher representation and retains its own domain and logical role [Sch26d].

Remark 4.17 (Three notions of noncommutativity). It is useful to distinguish:

- (i) sequence noncommutativity in $\mathfrak{A}_C$, generated by translation acting on localized observables;
- (ii) noncommutation of an interaction Hamiltonian with a local device observable; and

- (iii) noncommutation internal to any particular finite-dimensional matrix model.

The title *Noncommutative Transport* refers to a theory in which the inherited noncommutative sequence structure coexists with explicitly modeled physical transport. It does not assert that every pair of channel observables must fail to commute.

5 Latent-State Dynamics and Memory

5.1 Three-sector architecture

The DCIT crosspoint decomposes as

$$\mathcal{K}_{\text{XB}} = \mathcal{K}_{\text{hid}} \otimes \mathcal{K}_{\text{acc}} = (\mathcal{K}_x \otimes \mathcal{K}_m) \otimes \mathcal{K}_y.$$

The elimination procedure used below is a bounded, self-contained specialization of the projection-operator strategy associated with the Nakajima–Zwanzig formalism [Nak58; Zwa60; BP07]. The citations provide conventional context only; the derivation needed here is given explicitly in the main text and Appendix B. The C3–C9 factor $\mathcal{K}_x$ is a control-access pathway; $\mathcal{K}_m$ is a persistent internal-state factor; $\mathcal{K}_y$ is the directly interrogated readout. The hidden state is therefore not identified with the C3–C9 path alone. The combined hidden block is

$$\mathcal{K}_{\text{hid}} := \mathcal{K}_x \otimes \mathcal{K}_m.$$

Let $\mathfrak{T}_1(\mathcal{H})$ denote trace-class operators on a Hilbert space $\mathcal{H}$, with norm

$$\|\rho\|_1 := \text{Tr} |\rho|.$$

Let

$$\mathcal{S}(\mathcal{H}) := \{\rho \in \mathfrak{T}_1(\mathcal{H}) : \rho \geq 0, \text{Tr} \rho = 1\}$$

denote the normal state space.

Assume $\mathcal{L}_{\text{hid}}$ and $\mathcal{L}_y$ are bounded generators of uniformly continuous completely positive trace-preserving semigroups on $\mathfrak{T}_1(\mathcal{K}_{\text{hid}})$ and $\mathfrak{T}_1(\mathcal{K}_y)$, respectively. Choose a stationary hidden reference state

$$\sigma_{\text{hid}} \in \mathcal{S}(\mathcal{K}_{\text{hid}})$$

with

$$\mathcal{L}_{\text{hid}}(\sigma_{\text{hid}}) = 0.$$

Define the uncoupled generator

$$\mathcal{L}_0 := \mathcal{L}_{\text{hid}} \otimes \text{id} + \text{id} \otimes \mathcal{L}_y.$$

Assume that the canonical ampliations

$$\mathcal{L}_{\text{hid}} \otimes \text{id}, \quad \text{id} \otimes \mathcal{L}_y$$

extend to bounded operators on

$$\mathfrak{T}_1(\mathcal{K}_{\text{XB}}).$$

This assumption is automatic for the finite-dimensional witness of Appendix A. In the general bounded setting it is stated explicitly so that $\mathcal{L}_0$ is a well-typed bounded generator on the joint trace-class space.

Let $V_{\text{hy}} = V_{\text{hy}}^* \in B(\mathcal{K}_{\text{XB}})$ and define the bounded commutator superoperator

$$\mathcal{V}(\rho) := -\frac{i}{\hbar} [V_{\text{hy}}, \rho].$$

Then

$$\|\mathcal{V}\| \leq \frac{2\|V_{\text{hy}}\|}{\hbar}.$$

For $\lambda \in \mathbb{R}$, set

$$\mathcal{L}_\lambda := \mathcal{L}_0 + \lambda \mathcal{V}.$$

Remark 5.1 (Why bounded generators are used). The bounded setting is chosen to keep the projection argument domain-free and transparent. It includes every finite-dimensional model and uniformly continuous open-system dynamics. Unbounded Lindblad generators and more general quantum dynamical semigroups are natural extensions, but they require additional domain-invariance hypotheses and are not needed for the present existence, memory, or observability results.

5.2 Accessible projection

Definition 5.2 (Accessible reduction). Define

$$\mathbf{P}_{\text{acc}} \rho := \sigma_{\text{hid}} \otimes \text{Tr}_{\text{hid}} \rho, \quad \mathbf{Q}_{\text{hid}} := I - \mathbf{P}_{\text{acc}}.$$

The reduced accessible state is

$$\rho_y := \text{Tr}_{\text{hid}} \rho.$$

Thus the projection does not merely trace out the persistent-state factor $\mathcal{K}_m$; it traces out the complete hidden block $\mathcal{K}_x \otimes \mathcal{K}_m$. This is required if the declared readout is the C12–C6 sector alone.

Lemma 5.3 (Norm-one accessible projection). $\mathbf{P}_{\text{acc}}$ is a norm-one bounded projection with range

$$\text{Ran}(\mathbf{P}_{\text{acc}}) = \{\sigma_{\text{hid}} \otimes \eta : \eta \in \mathfrak{T}_1(\mathcal{K}_y)\}.$$

Proof. Idempotence uses $\text{Tr} \sigma_{\text{hid}} = 1$; trace-norm contractivity of the partial trace gives $\|\mathbf{P}_{\text{acc}}\| \leq 1$, and equality follows because the projection is the identity on its nonzero range. Details appear in Appendix B. $\square$

Remark 5.4 (The hidden complement is linear, not a state space). The range of $\mathbf{Q}_{\text{hid}}$ is a closed linear subspace of $\mathfrak{T}_1(\mathcal{K}_{\text{XB}})$, but its elements need not be positive or normal-

ized. Accordingly, the $\mathbf{Q}_{\text{hid}}$ -sector is a hidden-information subspace in the projection-operator calculation; it is not itself a second convex state space.

Proposition 5.5 (Decoupled reduction).

$$\mathbf{Q}_{\text{hid}} \mathcal{L}_0 \mathbf{P}_{\text{acc}} = 0, \quad \mathbf{P}_{\text{acc}} \mathcal{L}_0 \mathbf{Q}_{\text{hid}} = 0.$$

Proof. Stationarity of σ_{hid} makes $\text{Ran}(\mathbf{P}_{\text{acc}})$ invariant under $\mathcal{L}_0$; trace preservation keeps $\text{Ker } \mathbf{P}_{\text{acc}}$ invariant. See Appendix B. $\square$

5.3 Exact hidden-state memory equation

Let $\rho(t)$ solve

$$\dot{\rho}(t) = \mathcal{L}_\lambda \rho(t),$$

and put

$$p(t) = \mathbf{P}_{\text{acc}} \rho(t), \quad q(t) = \mathbf{Q}_{\text{hid}} \rho(t).$$

Then

$$\rho(t) = p(t) + q(t).$$

Theorem 5.6 (Hidden-state memory theorem). *The accessible projection satisfies*

$$\frac{dp}{dt} = \mathbf{P}_{\text{acc}} \mathcal{L}_\lambda \mathbf{P}_{\text{acc}} p(t) + \int_0^t \mathcal{K}_\lambda(t-s) p(s) ds + \mathcal{I}_\lambda(t),$$

where

$$\mathcal{K}_\lambda(t) = \mathbf{P}_{\text{acc}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}} e^{t \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}}} \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{P}_{\text{acc}}$$

and

$$\mathcal{I}_\lambda(t) = \mathbf{P}_{\text{acc}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}} e^{t \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}}} \mathbf{Q}_{\text{hid}} \rho(0).$$

Proof. Project the full equation with $\mathbf{P}_{\text{acc}}$ and $\mathbf{Q}_{\text{hid}}$. The hidden component satisfies the inhomogeneous linear equation

$$\dot{q} = \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{P}_{\text{acc}} p + \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}} q.$$

Variation of constants gives

$$q(t) = e^{t \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}}} q(0) + \int_0^t e^{(t-s) \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}}} \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{P}_{\text{acc}} p(s) ds.$$

Substitution into the projected p -equation yields the result. The complete norm-typed calculation is repeated in Appendix B. $\square$

Remark 5.7 (Memory kernel versus non-Markovianity). The exact projected equation is a memory-kernel representa-

tion of the accessible dynamics. The presence of a nonzero kernel is not adopted here as a definition of quantum non-Markovianity and does not, by itself, establish failure of CP divisibility or any other specific non-Markovianity criterion.

Corollary 5.8 (Factorized initialization). *If*

$$\rho(0) = \sigma_{\text{hid}} \otimes \rho_y(0),$$

then

$$\mathbf{Q}_{\text{hid}} \rho(0) = 0$$

and therefore

$$\mathcal{I}_\lambda(t) = 0.$$

5.4 Second-order origin of the memory term

Theorem 5.9 (Second-order interaction structure). *Under the decoupled-reduction hypotheses,*

$$\mathcal{K}_\lambda(t) = \lambda^2 \mathbf{P}_{\text{acc}} \mathcal{V} \mathbf{Q}_{\text{hid}} e^{t \mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}}} \mathbf{Q}_{\text{hid}} \mathcal{V} \mathbf{P}_{\text{acc}}.$$

In particular,

$$\mathcal{K}_0(t) = 0.$$

Proof. The cross terms of $\mathcal{L}_0$ vanish by the preceding proposition, leaving one factor of $\lambda \mathcal{V}$ at each entrance to and exit from the hidden sector. $\square$

Corollary 5.10 (Interaction is necessary for reduced memory). *Under factorized initialization, if $\lambda = 0$ then*

$$\frac{dp}{dt} = \mathbf{P}_{\text{acc}} \mathcal{L}_0 \mathbf{P}_{\text{acc}} p(t),$$

so the reduced accessible dynamics is autonomous.

The result is conceptually important. Merely having a hidden tensor factor does not create memory. History dependence in the readout arises only when the declared dynamics moves information between the accessible range and its hidden complement.

5.5 Kernel norm and finite-horizon memory

Let

$$a_\lambda := \|\mathbf{Q}_{\text{hid}} \mathcal{L}_\lambda \mathbf{Q}_{\text{hid}}\|.$$

By submultiplicativity,

$$\|\mathcal{K}_\lambda(t)\| \leq |\lambda|^2 \|\mathbf{P}_{\text{acc}} \mathcal{V} \mathbf{Q}_{\text{hid}}\| \|\mathbf{Q}_{\text{hid}} \mathcal{V} \mathbf{P}_{\text{acc}}\| e^{a_\lambda t}.$$

Definition 5.11 (Finite-horizon memory magnitude). For

$T > 0$, define

$$\mathfrak{M}(T) := \int_0^T \|\mathcal{K}_\lambda(t)\| dt.$$

Proposition 5.12 (Finite-horizon memory bound). *If $a_\lambda > 0$, then*

$$\mathfrak{M}(T) \leq |\lambda|^2 \|\mathbf{P}_{\text{acc}} \mathcal{V} \mathbf{Q}_{\text{hid}}\| \|\mathbf{Q}_{\text{hid}} \mathcal{V} \mathbf{P}_{\text{acc}}\| \frac{e^{a_\lambda T} - 1}{a_\lambda}.$$

If $a_\lambda = 0$, then

$$\mathfrak{M}(T) \leq |\lambda|^2 T \|\mathbf{P}_{\text{acc}} \mathcal{V} \mathbf{Q}_{\text{hid}}\| \|\mathbf{Q}_{\text{hid}} \mathcal{V} \mathbf{P}_{\text{acc}}\|.$$

Proof. Integrate the preceding kernel-norm bound over $[0, T]$. $\square$

Remark 5.13. The quantity $\mathfrak{M}(T)$ is a model-dependent mathematical measure of the strength of the memory term over a finite horizon. It is not the same as an experimentally measured retention time, which is defined later from readout discrimination under matched present conditions.

5.6 Initial hidden correlations

When $\mathbf{Q}_{\text{hid}}\rho(0) \neq 0$, the inhomogeneous term

$$\mathcal{I}_\lambda(t)$$

carries information about hidden components already present at the start of the readout window. Thus two mechanisms can alter the accessible trajectory:

- (i) a memory kernel generated by repeated hidden-accessible exchange; and
- (ii) an initial hidden-state term generated by unresolved preparation history.

The preparation-readout protocol in Section 11 is designed to make this distinction experimentally meaningful by separating the state-preparation interval from the subsequent matched-condition readout interval.

5.7 Indirect modulation and the ideal leakage reference

Consider a joint Hamiltonian

$$H = H_{\text{hid}} \otimes I_y + I_{\text{hid}} \otimes H_y + \lambda V_{\text{hy}},$$

while the separate direct-sum leakage model is evaluated at its ideal reference value $\varepsilon = 0$. Let

$$\rho_{\text{hid}}^{(0)}, \quad \rho_{\text{hid}}^{(1)}$$

be two hidden states and let the readout state ρ_y be the same in both cases.

Theorem 5.14 (Indirect hidden-state modulation). *Assume*

$$\lambda \neq 0.$$

If there exists $A_y = A_y^* \in \mathcal{B}(\mathcal{H}_y)$ such that

$$\text{Tr}[(\Delta\rho_{\text{hid}} \otimes \rho_y)[V_{\text{hy}}, I_{\text{hid}} \otimes A_y]] \neq 0,$$

where

$$\Delta\rho_{\text{hid}} = \rho_{\text{hid}}^{(1)} - \rho_{\text{hid}}^{(0)},$$

then the two readout trajectories differ for all sufficiently small nonzero times of one sign.

Proof. The initial readout expectations agree. Their derivative difference at zero is

$$\frac{i\lambda}{\hbar} \text{Tr}[(\Delta\rho_{\text{hid}} \otimes \rho_y)[V_{\text{hy}}, I \otimes A_y]],$$

which is nonzero by hypothesis. Continuity gives the claim. $\square$

Corollary 5.15 (Compatibility of idealized leakage separation with hidden modulation). *The direct-sum model may be evaluated at the ideal reference value*

$$\varepsilon = 0$$

while the joint tensor-product model, with $\lambda \neq 0$, exhibits distinct readout trajectories generated by different hidden states.

The corollary is a compatibility statement between two effective models. Because no intertwining map between them is assumed, $\varepsilon = 0$ in the direct-sum model is not, by itself, a theorem that every microscopic direct x - y process vanishes in the joint device model. The stronger physical statement requires an additional compatibility hypothesis.

5.8 Preparation and readout as separate dynamical windows

The memory theorem is stated for an autonomous readout generator, but the control history that prepares the hidden state may be time dependent. To avoid unnecessary time-dependent projection machinery in the present paper, we divide an experiment into two stages.

Definition 5.16 (Preparation–readout decomposition). A preparation–readout experiment consists of:

- (i) a preparation interval $[-T_p, 0]$ in which a control history acts on the hidden block; and
- (ii) a readout interval $[0, T_r]$ in which the contemporaneous control setting is fixed to a common declared value and the resulting hidden state is interrogated through the accessible sector.

The state at $t = 0$ therefore contains the preparation history. The exact memory equation governs the subsequent readout window. This separation is mathematically economical and experimentally useful: history dependence is tested while present external conditions are matched.

5.9 Interpretive boundary

Nothing in the memory theorem asserts that $\mathcal{K}_{\text{hid}}$ is biological subconscious processing. The theorem states only that elimination of a dynamically coupled hidden sector produces a memory kernel in the accessible reduced dynamics. The cognitive interpretation of that result is introduced separately in Section 8.

6 Observability and Information Separation

6.1 Accessible distinguishability

For states $\rho, \sigma \in \mathcal{S}(\mathcal{K}_{\text{XB}})$, define the full trace distinguishability

$$D_{\text{full}}(\rho, \sigma) := \frac{1}{2} \|\rho - \sigma\|_1$$

and the accessible distinguishability

$$D_{\text{acc}}(\rho, \sigma) := \frac{1}{2} \|\text{Tr}_{\text{hid}} \rho - \text{Tr}_{\text{hid}} \sigma\|_1.$$

Proposition 6.1 (Information contraction).

$$0 \leq D_{\text{acc}}(\rho, \sigma) \leq D_{\text{full}}(\rho, \sigma) \leq 1.$$

Proof. The partial trace is completely positive and trace preserving, and trace distance is contractive under CPTP maps [NC10]. The final inequality is the standard bound for density operators. $\square$

Definition 6.2 (Instantaneously hidden distinction). A pair (ρ, σ) is instantaneously hidden when

$$D_{\text{acc}}(\rho, \sigma) = 0 \quad \text{but} \quad D_{\text{full}}(\rho, \sigma) > 0.$$

Thus two complete states may differ even though no measurement confined to the declared readout sector can distinguish them at that instant.

Definition 6.3 (Accessible equivalence). Write

$$\rho \sim_{\text{acc}} \sigma \iff \text{Tr}_{\text{hid}} \rho = \text{Tr}_{\text{hid}} \sigma.$$

The equivalence classes of $\sim_{\text{acc}}$ are the fibers later used in the cognitive interpretation.

6.2 Measurement interfaces

Let $\mathcal{Y}$ be a finite or countable outcome space and let

$$\{M_a\}_{a \in \mathcal{Y}} \subset B(\mathcal{K}_y)$$

be a POVM satisfying

$$M_a \geq 0, \quad \sum_{a \in \mathcal{Y}} M_a = I_y.$$

For a joint state ρ , define the declared readout probabilities

$$p_\rho(a) := \text{Tr}[\rho(I_{\text{hid}} \otimes M_a)].$$

By the defining property of partial trace,

$$p_\rho(a) = \text{Tr}[(\text{Tr}_{\text{hid}} \rho) M_a].$$

Hence every such readout depends only on the accessible reduced state.

Definition 6.4 (Informationally complete readout). In finite dimension, the POVM is informationally complete when

$$\text{span}_{\mathbb{R}}\{M_a : a \in \mathcal{Y}\} = B(\mathcal{K}_y)_{\text{sa}}.$$

Proposition 6.5 (POVM characterization of accessible equivalence). For an informationally complete readout POVM,

$$p_\rho(a) = p_\sigma(a) \quad \text{for every } a \in \mathcal{Y}$$

if and only if

$$\rho \sim_{\text{acc}} \sigma.$$

Proof. If the reduced states agree, all readout probabilities agree. Conversely, equality of all POVM statistics gives equality of expectation values on a spanning set of self-adjoint readout operators, hence equality of the reduced density operators. $\square$

This proposition connects the algebraic definition of accessibility to an operational measurement statement. An instantaneously hidden distinction is precisely a full-state distinction erased by the declared reduction map.

6.3 Finite-horizon revelation

For unitary joint dynamics $U(t)$, write

$$\rho(t) = U(t)\rho U(t)^*.$$

Define

$$\mathcal{R}_T(\rho, \sigma) := \sup_{0 \leq t \leq T} D_{\text{acc}}(\rho(t), \sigma(t)).$$

Proposition 6.6 (Revelation bound).

$$\mathcal{R}_T(\rho, \sigma) \leq D_{\text{full}}(\rho, \sigma).$$

Proof. Unitary evolution preserves full trace distance and partial trace is contractive at every time. $\square$

Definition 6.7 (Dynamically revealed hidden distinction). An instantaneously hidden pair is revealed within horizon T if

$$\mathcal{R}_T(\rho, \sigma) > 0.$$

The quantity $\mathcal{R}_T$ is deliberately distinct from the memory magnitude $\mathfrak{M}(T)$ of Section 5. The former measures actual distinguishability at the readout; the latter measures the norm of the projected memory kernel. A model may possess a nonzero kernel yet fail to reveal a particular pair of states through a particular observable.

6.4 Readout observable algebra

The finite-time state-determination viewpoint used below is consistent with standard quantum-control treatments of observability and state determination [DAI07]. The present formulation is specialized to the declared readout algebra and is proved directly.

The immediately accessible von Neumann algebra is

$$\mathfrak{A}_{\text{acc}} = I_{\text{hid}} \bar{\otimes} B(\mathcal{K}_Y) \subseteq B(\mathcal{K}_{\text{XB}}).$$

Let

$$\tau_t(A) = U(t)^* A U(t)$$

be the Heisenberg evolution.

Definition 6.8 (Finite-time observation space). For $T \geq 0$, define the real linear observation space

$$\mathfrak{B}_T := \text{span}_{\mathbb{R}} \left\{ \tau_t(A) : \begin{array}{l} A = A^* \in \mathfrak{A}_{\text{acc}}, \\ 0 \leq t \leq T \end{array} \right\},$$

with

$$\mathfrak{B}_T \subseteq B(\mathcal{K}_{\text{XB}})_{\text{sa}}.$$

The space $\mathfrak{B}_T$ records exactly the self-adjoint observables whose expectation values are linearly available from the declared

time-evolved readout family. It is the object used below for state discrimination and tomography.

Definition 6.9 (Finite-time generated dynamical algebra). Define

$$\mathfrak{D}_T := W^*(\{\tau_t(A) : A \in \mathfrak{A}_{\text{acc}}, 0 \leq t \leq T\}).$$

The algebra $\mathfrak{D}_T$ records multiplicative algebraic closure of the accessible Heisenberg orbit. In general, expectation values on the generating family do not by themselves determine expectation values of arbitrary products in $\mathfrak{D}_T$; therefore $\mathfrak{D}_T$ is used for algebra-growth statements, while $\mathfrak{B}_T$ is used for operational state separation.

Proposition 6.10 (Monotonicity in horizon). If $0 \leq T_1 \leq T_2$, then

$$\mathfrak{B}_{T_1} \subseteq \mathfrak{B}_{T_2}, \quad \mathfrak{D}_{T_1} \subseteq \mathfrak{D}_{T_2}.$$

Proof. Both generating families for horizon T_1 are contained in the corresponding generating families for T_2 . $\square$

Proposition 6.11 (No growth without hidden–readout interaction). If the joint Hamiltonian is a tensor sum

$$H = H_{\text{hid}} \otimes I_Y + I_{\text{hid}} \otimes H_Y$$

with no hidden–readout interaction, then

$$\mathfrak{D}_T = \mathfrak{A}_{\text{acc}}$$

for every $T \geq 0$, and

$$\mathfrak{B}_T = (\mathfrak{A}_{\text{acc}})_{\text{sa}}.$$

Proof. For $A = I_{\text{hid}} \otimes B$,

$$\tau_t(A) = I_{\text{hid}} \otimes (e^{itH_Y/\hbar} B e^{-itH_Y/\hbar}),$$

which remains in $\mathfrak{A}_{\text{acc}}$. Since $t = 0$ is included, equality follows. $\square$

6.5 Interaction-induced observability growth

Theorem 6.12 (Infinitesimal algebraic observability growth). Suppose there exists a self-adjoint $A \in \mathfrak{A}_{\text{acc}}$ such that

$$[H, A] \notin \mathfrak{A}_{\text{acc}}.$$

Then for every $T > 0$,

$$\mathfrak{D}_T \supsetneq \mathfrak{A}_{\text{acc}}.$$

Proof. The Heisenberg orbit is norm differentiable and

$$\left. \frac{d}{dt} \tau_t(A) \right|_{t=0} = \frac{i}{\hbar} [H, A].$$

If $\tau_t(A)$ remained in the norm-closed algebra $\mathfrak{A}_{\text{acc}}$ for a nontrivial interval, every difference quotient and its norm limit would belong to $\mathfrak{A}_{\text{acc}}$, contradicting the stated commutator condition. $\square$

Corollary 6.13 (Interaction-driven growth). *For*

$$H = H_{\text{hid}} \otimes I_y + I_{\text{hid}} \otimes H_y + \lambda V_{\text{hy}},$$

if $\lambda \neq 0$ and there exists self-adjoint $A \in \mathfrak{A}_{\text{acc}}$ with

$$[V_{\text{hy}}, A] \notin \mathfrak{A}_{\text{acc}},$$

then $\mathfrak{D}_T \supsetneq \mathfrak{A}_{\text{acc}}$ for every $T > 0$.

Remark 6.14. The theorem is qualitative: it proves strict algebraic growth, not complete observability. The newly generated observables may still span only a small proper subalgebra of $B(\mathcal{K}_{\text{XB}})$.

6.6 Finite-dimensional state separation

Assume

$$d := \dim \mathcal{K}_{\text{XB}} < \infty.$$

Definition 6.15 (Observational equivalence). Write

$$\rho \sim_T \sigma$$

when

$$\text{Tr}[(\rho - \sigma)A] = 0$$

for every $A \in \mathfrak{B}_T$.

Theorem 6.16 (Finite-time state-separation criterion). *The following are equivalent:*

$$\mathfrak{B}_T = B(\mathcal{K}_{\text{XB}})_{\text{sa}},$$

and

$$\rho \sim_T \sigma \implies \rho = \sigma$$

for all density operators ρ, σ . If

$$\mathfrak{B}_T \subsetneq B(\mathcal{K}_{\text{XB}})_{\text{sa}},$$

then distinct observationally equivalent density matrices exist.

Proof. If $\mathfrak{B}_T = B(\mathcal{K}_{\text{XB}})_{\text{sa}}$, equality of expectations on $\mathfrak{B}_T$ gives equality on every self-adjoint operator and hence equality of density matrices. Conversely choose nonzero self-adjoint

$$X \in \mathfrak{B}_T^\perp$$

with respect to the real Hilbert–Schmidt pairing. Since $I \in \mathfrak{B}_T$, one has $\text{Tr } X = 0$. For

$$0 < \epsilon < (d\|X\|)^{-1},$$

set

$$\rho_\pm = I/d \pm \epsilon X.$$

These are distinct density matrices with identical expectations on $\mathfrak{B}_T$. Full details are in Appendix B. $\square$

Corollary 6.17 (Hidden subspace characterization). *In finite dimension, a traceless self-adjoint perturbation X is invisible throughout the horizon $[0, T]$ exactly when*

$$X \in \mathfrak{B}_T^\perp.$$

This provides a compact mathematical characterization of hidden information at a fixed horizon.

6.7 Observation-space fraction

Definition 6.18 (Observation-space fraction). In finite dimension define

$$\mathfrak{o}(T) := \frac{\dim_{\mathbb{R}} \mathfrak{B}_T}{d^2}.$$

Proposition 6.19.

$$0 < \mathfrak{o}(T) \leq 1,$$

and if $T_1 \leq T_2$, then

$$\mathfrak{o}(T_1) \leq \mathfrak{o}(T_2).$$

Moreover,

$$\mathfrak{o}(T) = 1 \iff \mathfrak{B}_T = B(\mathcal{K}_{\text{XB}})_{\text{sa}}.$$

The fraction is algebraic rather than statistical: it measures the real dimension of the directly available observation space relative to the self-adjoint part of the full matrix algebra. It does not describe the signal-to-noise ratio of a laboratory measurement. The generated algebra $\mathfrak{D}_T$ may be larger than the linear observation space $\mathfrak{B}_T$.

6.8 Example: hidden phase revealed by interaction

Suppose two initial states share the same readout density matrix but differ in the expectation of a hidden observable A_{hid} . If

$$V_{\text{hy}} = A_{\text{hid}} \otimes B_y$$

and there exists a readout observable C_y with

$$[B_y, C_y] \neq 0,$$

then

$$[V_{\text{hy}}, I \otimes C_y] = A_{\text{hid}} \otimes [B_y, C_y],$$

which lies outside $\mathfrak{A}_{\text{acc}}$ whenever A_{hid} is non-scalar. Thus the interaction-driven algebra-growth criterion applies whenever the interaction coefficient is nonzero. Appendix A gives an explicit finite instance in which different hidden states produce different readout derivatives at $t = 0$.

6.9 Information hierarchy

The operational hierarchy can now be written as

$$\begin{array}{c} \text{complete state distinction} \\ \supseteq \\ \text{instantaneously accessible distinction} \\ \supseteq \\ \text{declared measurement distinction.} \end{array}$$

Interaction can partially reverse the informational loss over time:

$$\begin{array}{c} \text{initially hidden distinction} \\ \Downarrow \\ \text{interaction-generated observation/algebra growth} \\ \Downarrow \\ \text{later accessible distinction.} \end{array}$$

The arrow denotes a possibility supported by the dynamics, not an inevitability for every hidden state.

6.10 Boundary before interpretation

Nothing in this section defines consciousness. The phrases *accessible*, *hidden*, *revealed*, and *observable* refer only to a specified reduction map, observable algebra, measurement interface, and time horizon. Section 8 will ask whether these operational notions admit a useful cognitive interpretation, but that interpretation is not required for any theorem above.

7 Oriented Reachability and Intuitionistic Semantics

The proof calculus of the manuscript remains classical. The intuitionistic structure introduced here is an object-level semantics generated by oriented channel reachability. The use of upper-set Alexandrov topology follows the classical discrete-space tradition [Ale37], while the Heyting/Kripke interpretation is placed in the standard intuitionistic-semantic setting [Kri65; MM92]. Its role is to formalize persistent, context-dependent information on a partially ordered lattice without asserting that human reasoning or physical time is itself intuitionistic.

7.1 Positive channel semigroups

For $S \subseteq \{x, y\}$, define the additive semigroup

$$L_S^+ := \bigoplus_{\alpha \in S} \mathbb{N}_0 e_\alpha.$$

For $k, \ell \in X_C$, write

$$k \preceq_S \ell \iff \ell - k \in L_S^+.$$

Proposition 7.1 (Channel reachability order). *For every $S \subseteq \{x, y\}$, $\preceq_S$ is a partial order on X_C .*

Proof. Reflexivity follows from $0 \in L_S^+$. Transitivity follows from closure of the semigroup under addition. If both $k \preceq_S \ell$ and $\ell \preceq_S k$, then $\ell - k$ and its negative both lie in the pointed semigroup L_S^+ , so $k = \ell$. $\square$

Remark 7.2 (Reachability is not physical time). The order $\preceq_S$ is induced by positive lattice translation. It is not a temporal order and does not identify a lattice coordinate with time. A temporal interpretation requires a separately specified dynamical law, in keeping with the inherited distinction between algebraic sequence order and physical evolution [Sch26d].

Definition 7.3 (Forward cone). For $k \in X_C$, define

$$\uparrow_S k := \{\ell \in X_C : k \preceq_S \ell\}.$$

For example, in the two-channel order,

$$\uparrow_{xy}(m, n, p) = \{(m + a, n + b, p) : a, b \in \mathbb{N}_0\}.$$

Thus the forward cone is a planar quadrant at fixed z -coordinate.

7.2 Alexandrov information topology

Definition 7.4 (Oriented information topology). Define

$$O_S^+ := \{U \subseteq X_C : k \in U, k \preceq_S \ell \implies \ell \in U\}.$$

Thus O_S^+ is the family of upper sets of the poset $(X_C, \preceq_S)$.

Proposition 7.5 (Alexandrov property). *The family O_S^+ is a topology closed under arbitrary intersections as well as arbitrary unions. Hence it is an Alexandrov topology.*

Proof. If a point belongs to a union of upper sets, it belongs to one of them and every forward-reachable point remains in that set. If a point belongs to an intersection of upper sets, every forward-reachable point remains in every member of the intersection. Thus both unions and arbitrary intersections are upper. $\square$

Remark 7.6 (Two topologies on the same channel data). The saturation topology τ_S of Section 3 is generated by the group L_S and therefore ignores orientation. The Alexandrov topology O_S^+ is generated by the positive semigroup L_S^+ and records orientation. The former identifies complete two-sided channel cosets; the latter records persistent forward reachability. These are distinct constructions on the same lattice.

7.3 Heyting operations

The lattice of open sets of any topological space is a complete Heyting algebra. In the present Alexandrov setting the implication operation has an explicit reachability form.

For $U, V \in O_S^+$, define

$$U \wedge V := U \cap V, \quad U \vee V := U \cup V,$$

and

$$U \Rightarrow_S V := \{k \in X_C : (\uparrow_S k) \cap U \subseteq V\}.$$

Define the pseudocomplement

$$\neg_S U := U \Rightarrow_S \emptyset.$$

Theorem 7.7 (Reachability Heyting algebra). *For every $S \subseteq \{x, y\}$, the ordered set*

$$(O_S^+, \subseteq)$$

is a complete Heyting algebra. Its implication is

$$U \Rightarrow_S V = \{k : (\uparrow_S k) \cap U \subseteq V\},$$

and its pseudocomplement is

$$\neg_S U = \{k : (\uparrow_S k) \cap U = \emptyset\}.$$

Proof. The complete lattice structure follows from arbitrary unions and intersections. For implication, let $W \in O_S^+$. If

$$W \subseteq U \Rightarrow_S V,$$

then for every $k \in W \cap U$, the point k itself belongs to $\uparrow_S k \cap U$, hence $k \in V$. Thus

$$W \cap U \subseteq V.$$

Conversely, suppose $W \cap U \subseteq V$. If $k \in W$, then W is upper, so $\uparrow_S k \subseteq W$. Therefore

$$(\uparrow_S k) \cap U \subseteq W \cap U \subseteq V,$$

so $k \in U \Rightarrow_S V$. Hence $U \Rightarrow_S V$ is the largest open W satisfying $W \cap U \subseteq V$, which is the defining Heyting adjunction. The pseudocomplement is the special case $V = \emptyset$. A fuller verification is included in Appendix B. $\square$

7.4 Directional persistence

Write

$$O_x^+ := O_{\{x\}}^+, \quad O_y^+ := O_{\{y\}}^+, \quad O_{xy}^+ := O_{\{x,y\}}^+.$$

Theorem 7.8 (Intersection of directional persistence conditions). *As families of subsets of X_C ,*

$$O_{xy}^+ = O_x^+ \cap O_y^+.$$

Proof. Every set upper for the combined order is upper for each coordinate order separately. Conversely, if U is both x -upper and y -upper and

$$\ell - k = ae_x + be_y, \quad a, b \in \mathbb{N}_0,$$

then repeated x -persistence followed by repeated y -persistence carries $k \in U$ to $\ell \in U$. Thus U is upper for the combined order. $\square$

The theorem makes precise a useful semantic statement: persistence under both primitive channel directions is equivalent to persistence under every finite positive composition of the two.

7.5 Kripke forcing

For $U \in O_S^+$, define

$$k \Vdash_S U \iff k \in U.$$

Proposition 7.9 (Persistence of forcing). *If*

$$k \Vdash_S U$$

and

$$k \preceq_S \ell,$$

then

$$\ell \Vdash_S U.$$

Proof. This is exactly the upper-set condition. $\square$

Under this forcing semantics,

$$k \Vdash_S (U \Rightarrow_S V)$$

means that every forward-reachable $\ell \succeq_S k$ that forces U also forces V . Thus implication is local to the reachability context determined by the current lattice point.

7.6 Explicit failure of excluded middle

The Heyting algebra is generally non-Boolean.

Theorem 7.10 (Failure of excluded middle). *Let*

$$U_+ := \{(m, n, p) \in \mathbb{Z}^3 : m \geq 0\}.$$

Then

$$U_+ \in \mathcal{O}_{xy}^+, \quad \neg_{xy} U_+ = \emptyset,$$

and therefore

$$U_+ \vee \neg_{xy} U_+ = U_+ \neq X_C.$$

Hence $\mathcal{O}_{xy}^+$ is not Boolean.

Proof. If $m \geq 0$ and $a, b \geq 0$, then $m + a \geq 0$, so U_+ is upper. Now fix any $k = (m, n, p)$. If $m \geq 0$, then $k \in U_+$. If $m < 0$, choose $a = -m$; then

$$k + ae_x = (0, n, p) \in U_+.$$

Thus every forward cone intersects U_+ , so no point lies in its Heyting pseudocomplement. Since U_+ is a proper subset of X_C , excluded middle fails for this proposition. $\square$

Remark 7.11 (Meaning of the failure). The equation

$$U \vee \neg U \neq X_C$$

does not say that ordinary classical logic is invalid. It says that the internal algebra of persistent open propositions generated by the reachability order is non-Boolean. The manuscript continues to reason classically about that algebra from the outside.

7.7 Channel propositions and contextual information

Definition 7.12 (Reachability proposition). An oriented information proposition is an element

$$U \in \mathcal{O}_{xy}^+.$$

Its mathematical content is persistence: once a lattice point lies in U , every point forward-reachable by the declared positive channel translations also lies in U .

A proposition may therefore have nontrivial persistent content even when its Heyting negation is empty. This supplies a rigorous sense in which information can be contextually available without participating in a global Boolean partition of the state space.

7.8 Relation to the channel Boolean lattice

The four-element Boolean lattice $\mathbb{B}_{\text{ch}}$ of Section 3 and the Heyting algebra $\mathcal{O}_{xy}^+$ are different mathematical levels. The first classifies which channel subgroup is activated in a coarse structural sense. The second is an infinite lattice of persistent propositions on the oriented state space.

Proposition 7.13 (Boolean channel labels do not force Boolean proposition logic). *The Boolean character of $\mathbb{B}_{\text{ch}}$ does not imply that $\mathcal{O}_{xy}^+$ is Boolean.*

Proof. The preceding failure-of-excluded-middle theorem supplies a counterexample inside $\mathcal{O}_{xy}^+$, while $\mathbb{B}_{\text{ch}}$ is Boolean by construction. The two lattices encode different objects and operations. $\square$

Remark 7.14 (No operator-to-Heyting identification). The present paper does not construct a valuation from operator-algebraic propositions, density-state distinctions, the accessible projection $\mathbf{P}_{\text{acc}}$, or the finite-time observation spaces $\mathfrak{B}_{\mathcal{T}}$ into the reachability Heyting algebra $\mathcal{O}_{xy}^+$. Accordingly, the intuitionistic layer is an auxiliary contextual semantics on the oriented lattice rather than a theorem-level reconstruction of the operator observability theory. Constructing such a valuation is left to subsequent work.

7.9 Preparation for the cognitive interpretation

The intuitionistic layer will later be combined with a reportability nucleus

$$j_{\text{rep}} : \mathcal{O}_{xy}^+ \rightarrow \mathcal{O}_{xy}^+.$$

That additional map is not needed for the present mathematical construction. It belongs to the cognitive interpretation layer

because it specifies which persistent propositions are treated as globally stable under the declared reportability semantics.

The mathematical core therefore ends with a strict separation:

oriented reachability $\implies$ Heyting proposition algebra,
while

Heyting proposition algebra $\Rightarrow$ consciousness.

8 A Falsifiable Cognitive Interpretation

No theorem in the preceding sections concerns human consciousness. The mathematical core establishes channel topology, hidden-state dynamics, observability, and reachability semantics. This section introduces a separate formal interpretation and retains the certified-growth distinction between theorem transfer and empirical domain adequacy [Sch26b]. The interpretation is deliberately narrower than a theory of phenomenal consciousness: it concerns latent or state-dependent information, an accessible readout interface, and the dynamical emergence of previously hidden distinctions into reportability.

8.1 Epistemic status and terminology

Convention 8.1 (Cognitive terminology). The primary terms of this section are *accessible/reportable*, *latent/hidden*, *automatic/feedforward*, and *state-dependent/integrative*. The words *conscious*, *subconscious*, and *nonconscious* are secondary interpretive labels. In particular, no molecular channel is conscious or subconscious by definition. A conscious-access analogue, when invoked, refers to an integrated dynamical process in which a previously latent distinction becomes reportably accessible.

The distinction is therefore information-theoretic and operational rather than phenomenological. This terminology is chosen so that the mathematical content survives unchanged if the biological labels are removed.

8.2 Formal cognitive domain

Definition 8.2 (Formal cognitive system). A formal cognitive system is a tuple

$$\mathfrak{C} = (\mathcal{C}_{\text{hid}}, \mathcal{C}_{\text{rep}}, \mathfrak{B}_{\text{rep}}, \mathcal{G}, \mathcal{R}, \mathcal{K}_{\text{cog}}),$$

where:

- (i) $\mathcal{C}_{\text{hid}}$ is a state space of latent internal variables;
- (ii) $\mathcal{C}_{\text{rep}}$ is a state space of immediately reportable variables;
- (iii) $\mathfrak{B}_{\text{rep}}$ is an algebra of reportable observables;

- (iv) $\mathcal{G}$ is a declared cognitive evolution law;
- (v) $\mathcal{R}$ is a report map assigning observable output statistics to states; and
- (vi) $\mathcal{K}_{\text{cog}}$ is an optional memory kernel by which latent history modifies reportable dynamics.

The tuple is intentionally abstract. It does not assume that a cognitive system is quantum, molecular, or even Hilbert-space valued. The interpretation below is defined only on a declared fragment for which corresponding structure can be mapped coherently.

For theorem transfer we additionally require formal presentations. Let

$$P_{\text{flow}} = (\Sigma_{\text{flow}}, A_{\text{flow}})$$

denote the declared DCIT information presentation on the fragment used below, and let

$$P_{\text{cog}} = (\Sigma_{\text{cog}}, A_{\text{cog}})$$

be a formal cognitive presentation whose sorts and symbols include the corresponding hidden-state, reportable-state, observable, evolution, and memory objects.

Definition 8.3 (Proof-preserving cognitive interpretation). A cognitive interpretation is a typed signature interpretation

$$\tau_{\text{cog}} : P_{\text{flow}} \rightsquigarrow P_{\text{cog}}$$

on a declared fragment such that, for every sentence φ in that fragment,

$$P_{\text{flow}} \vdash \varphi \implies P_{\text{cog}} \vdash \tau_{\text{cog}}(\varphi).$$

The interpretation is required to map the designated hidden sort to $\mathcal{C}_{\text{hid}}$, the accessible/readout sort to $\mathcal{C}_{\text{rep}}$, the accessible observable symbols to $\mathfrak{B}_{\text{rep}}$, and the memory-kernel symbol to $\mathcal{K}_{\text{cog}}$.

Definition 8.4 (Report equivalence). For formal cognitive states c_1, c_2 , write

$$c_1 \sim_{\text{rep}} c_2 \iff \mathcal{R}(c_1) = \mathcal{R}(c_2).$$

Definition 8.5 (Reportable and latent distinctions). A pair (c_1, c_2) contains an immediately reportable distinction when

$$c_1 \not\sim_{\text{rep}} c_2.$$

It contains a latent distinction at an interrogation time when

$$c_1 \sim_{\text{rep}} c_2$$

but the complete hidden states differ.

8.3 Cognitive Dual-Channel Interpretation Hypothesis

Let $\mathfrak{D}_{\text{DCIT}}$ denote the mathematical realization of the declared flow presentation together with a specified realization overlay.

Hypothesis 8.6 (Cognitive Dual-Channel Interpretation). There exists a proof-preserving cognitive interpretation

$$\tau_{\text{cog}} : P_{\text{flow}} \rightsquigarrow P_{\text{cog}}$$

on the declared fragment such that, at the level of interpreted structures,

$$\begin{aligned} \tau_{\text{cog}}(\mathcal{K}_{\text{hid}}) &= \mathcal{C}_{\text{hid}}, & \tau_{\text{cog}}(\mathcal{K}_{\text{y}}) &= \mathcal{C}_{\text{rep}}, \\ \tau_{\text{cog}}(\mathfrak{A}_{\text{acc}}) &= \mathfrak{B}_{\text{rep}}, & \tau_{\text{cog}}(\mathcal{K}) &= \mathcal{K}_{\text{cog}}. \end{aligned}$$

Under this interpretation, $\mathcal{K}_{\text{hid}}$ models latent or state-dependent internal processing, while $\mathcal{K}_{\text{y}}$ supplies the declared accessible readout interface. Reportable or conscious access is associated with the coupled dynamical process by which an initially hidden distinction becomes accessible; it is not assigned to $\mathcal{K}_{\text{y}}$ in isolation.

Warning 8.7 (Interpretation is not identification). The hypothesis does not establish that human consciousness is a chrysenes-like quantum process, that biological latent processing resides on a molecular crossbar, or that a successful DCIT device constitutes a model of the brain. Those are external empirical questions.

Figure 2 summarizes the principal discipline of the paper. The upper row can, in principle, be tested in a device. The lower row is an interpretation. Under the optional speed-ordering hypothesis introduced in Section 9, the C12–C6 pathway may be used as a rapid automatic/feedforward analogue and the C3–C9 plus persistent-state architecture as a slower integrative analogue. A successful upper-row experiment does not validate that biological mapping automatically.

8.4 Accessible quotient and hidden fiber

Define

$$\rho \sim_{\text{acc}} \sigma \iff \text{Tr}_{\text{hid}} \rho = \text{Tr}_{\text{hid}} \sigma.$$

Definition 8.8 (Accessible quotient). The accessible quotient is

$$Q_{\text{acc}} := \mathcal{S}(\mathcal{K}_{\text{XB}}) / \sim_{\text{acc}}.$$

Under the cognitive interpretation, an element of this quotient represents a formal reportable state.

Each quotient point corresponds to many complete states that differ only in information discarded by the accessible reduction.

Definition 8.9 (Hidden fiber). For a reduced readout state ρ_{y} , define

$$\mathcal{F}_{\rho_{\text{y}}} := \{\rho \in \mathcal{S}(\mathcal{K}_{\text{XB}}) : \text{Tr}_{\text{hid}} \rho = \rho_{\text{y}}\}.$$

Hidden variation consists of distinctions among complete states within the same fiber.

This gives a useful geometric metaphor that remains mathematically precise:

$$\text{reportable state} \leftrightarrow \text{quotient point},$$

while

$$\text{latent variation} \leftrightarrow \text{movement within its fiber}.$$

The metaphor does not identify the quotient with subjective experience; it only describes the information discarded by the declared report map.

8.5 Emergence into accessibility

Let

$$\rho, \sigma \in \mathcal{F}_{\rho_{\text{y}}}$$

initially. Define

$$D_{\text{acc}}(t) := \frac{1}{2} \|\text{Tr}_{\text{hid}} \rho(t) - \text{Tr}_{\text{hid}} \sigma(t)\|_1.$$

Then

$$D_{\text{acc}}(0) = 0.$$

Definition 8.10 (Thresholded accessibility-emergence time). For a declared discrimination threshold $\delta > 0$, define

$$t_{\delta} := \inf\{t \geq 0 : D_{\text{acc}}(t) \geq \delta\},$$

with $t_{\delta} = +\infty$ if the set is empty.

Under the cognitive interpretation, a finite t_{δ} describes a latent distinction becoming reportably accessible at the declared discrimination scale δ . This is a formal mapping of the observability theory, not a neuroscientific mechanism claim.

Remark 8.11. The thresholded emergence time depends on the chosen pair of states, the declared dynamics, the accessible reduction, and δ . It is not a universal time scale of consciousness and should not be compared directly with behavioral reaction times without a separately specified empirical model.

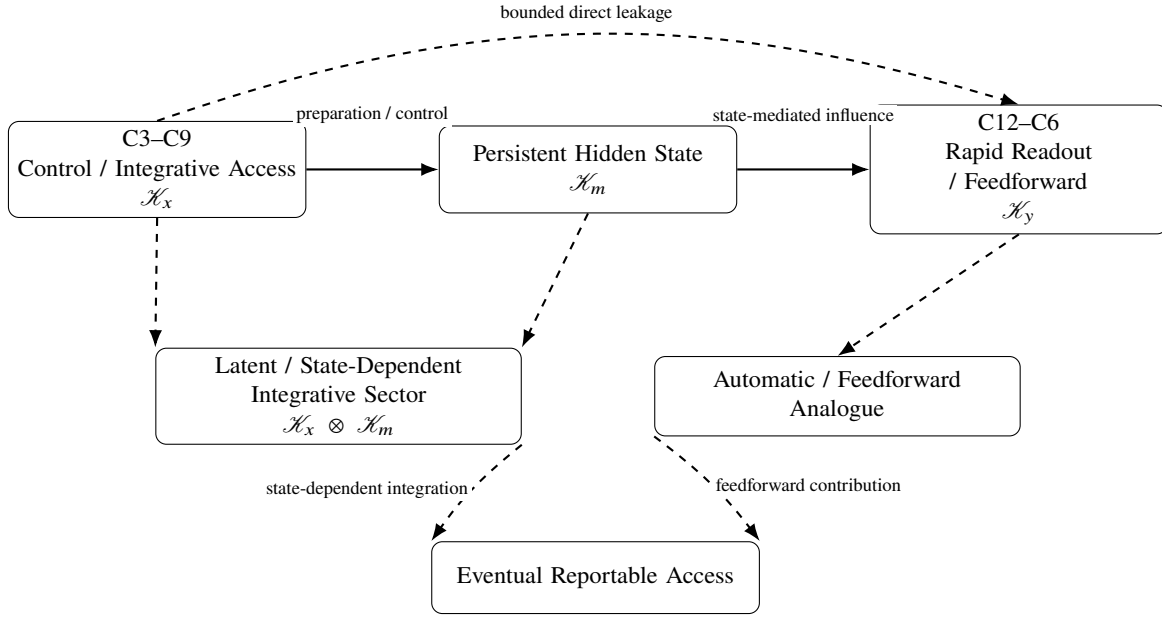


Figure 2: Physical and interpretive roles after the neuromorphic refinement. The upper row is a candidate device architecture. The lower construction separates state-dependent integration, a rapid automatic/feedforward analogue, and eventual reportable access. No molecular channel is identified with consciousness or nonconsciousness in isolation.

8.6 Translated hidden-state influence

Proposition 8.12 (Conditional hidden cognitive influence). *Assume the cognitive interpretation and the hypotheses of the Indirect Hidden-State Modulation Theorem. Then distinct interpreted hidden states with the same initial reportable state generate distinct subsequent reportable trajectories.*

Proof. The DCIT theorem gives distinct readout trajectories despite equality of the initial accessible state. The proof-preservation property of τ_{cog} transfers the corresponding sentence on the declared fragment to P_{cog} . Empirical adequacy of that formal presentation for any biological target is not part of this proof. $\square$

The proposition formalizes one possible meaning of latent state-mediated influence: two systems can be indistinguishable under immediate report yet diverge later because of a state variable outside the initial reportable quotient.

8.7 Cognitive memory kernel

Under the interpretation hypothesis, define formally

$$\mathcal{K}_{\text{cog}}(t) := \tau_{\text{cog}}(\mathcal{K}(t)).$$

The translated reportable dynamics has the schematic form

$$\frac{dc_{\text{rep}}}{dt} = \mathcal{L}_{\text{rep}}c_{\text{rep}}(t) + \int_0^t \mathcal{K}_{\text{cog}}(t-s)c_{\text{rep}}(s) ds + \mathcal{I}_{\text{cog}}(t).$$

Definition 8.13 (Formal latent-memory contribution). The convolution term

$$\int_0^t \mathcal{K}_{\text{cog}}(t-s)c_{\text{rep}}(s) ds$$

is called the latent-memory contribution of the interpreted model.

The terminology belongs to the interpretation layer. Mathematically it is simply the translated hidden-state memory kernel.

8.8 Intuitionistic reportability

The reachability semantics of Section 7 can be enriched by a reportability closure.

Hypothesis 8.14 (Reportability nucleus). The formal cognitive interpretation is equipped with a nucleus

$$j_{\text{rep}} : \mathcal{O}_{xy}^+ \rightarrow \mathcal{O}_{xy}^+$$

satisfying

$$U \subseteq j_{\text{rep}}(U),$$

$$j_{\text{rep}}(j_{\text{rep}}(U)) = j_{\text{rep}}(U),$$

and

$$j_{\text{rep}}(U \cap V) = j_{\text{rep}}(U) \cap j_{\text{rep}}(V).$$

Definition 8.15 (Report-stable proposition). A proposition $U \in \mathcal{O}_{xy}^+$ is report-stable when

$$j_{\text{rep}}(U) = U.$$

Definition 8.16 (Latent contextual proposition). A proposition is latent contextual when

$$j_{\text{rep}}(U) \neq U.$$

Under the cognitive interpretation, report-stable propositions are assigned to the reportable fragment, while latent contextual propositions are assigned to the latent fragment. This assignment concerns semantic accessibility; it does not assert that the biological brain literally implements a Grothendieck topos.

8.9 Non-complementarity of latent and reportable content

A Boolean two-box model would suggest that latent content is simply the logical complement of reportable content. The Heyting semantics does not require such a complementarity relation.

Proposition 8.17 (Non-complementarity). *Latent contextual content need not be the Heyting negation of reportable content. In particular, because excluded middle can fail in $\mathcal{O}_{xy}^+$, the formal latent sector is not identified with the logical complement of the formal reportable sector.*

This is one reason for retaining intuitionistic semantics in the paper. Latent content can be contextually meaningful and dynamically effective without being equivalent to the negation of whatever is currently reportable.

8.10 Two routes to accessibility

The formalism distinguishes two different ways in which latent information can become accessible.

Definition 8.18 (Dynamical access). A hidden distinction becomes dynamically accessible when its time evolution produces

$$D_{\text{acc}}(t) > 0$$

for some later time.

Definition 8.19 (Semantic access). A contextual proposition becomes semantically accessible when, under the declared contextual update, its image is fixed by the reportability nucleus.

These mechanisms need not coincide. Dynamical access concerns state evolution and observation. Semantic access concerns the internal logic of the interpreted proposition space. A future empirical theory might relate them, but the present paper keeps them distinct.

8.11 Non-uniqueness of interpretation

Remark 8.20 (No uniqueness theorem). The mathematics does not prove that τ_{cog} is unique. Different formal cognitive domains may admit structure-preserving interpretations of the same DCIT architecture. Accordingly, the present work proposes a disciplined cognitive interpretation; it does not establish categorical uniqueness of that interpretation.

This non-uniqueness is not a defect in the mathematical core. It is an explicit statement that the same abstract information architecture may support several external semantic models until empirical evidence distinguishes among them.

8.12 Neuromorphic refinement

The existence result of Section 9 introduces a second, more specific interpretation that remains conditional on the empirical speed-ordering hypothesis H_τ . Under that refinement,

$$\text{C12-C6} \mapsto \begin{array}{l} \text{rapid automatic/feedforward} \\ \text{processing analogue,} \end{array}$$

while

$$\begin{array}{l} \text{C3-C9} + \mathcal{K}_m + \text{state-dependent interaction} \\ \mapsto \text{slower integrative access analogue.} \end{array}$$

The word *conscious* in the second line refers to the integrated process culminating in accessibility, not to the C3–C9 molecular pathway by itself. Static transmission magnitude and transport time remain logically distinct, so this refinement is unavailable unless H_τ is independently supported.

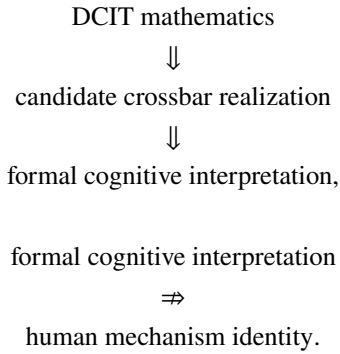
8.13 Cognitive domain-evidence boundary

Definition 8.21 (Cognitive domain-evidence certificate). A cognitive domain-evidence certificate is an external evidentiary object supporting the claim that a specified biological or behavioral system is adequately represented by the formal cognitive domain on a declared observable fragment.

A mathematical theorem, a successful crossbar realization, and a cognitive domain-evidence certificate are therefore three different evidentiary objects. The broader Chrysene literature provides theoretical motivation for relating retained state and

gating to cognition [Sch26e], but that motivation does not supply such a certificate.

The required hierarchy is



8.14 Interpretation-level falsification

Let $\mathcal{T}_{\text{cog}}$ be a declared family of external tests asking whether a target domain exhibits:

- (i) immediately reportable state distinctions;
- (ii) latent distinctions invisible to immediate report;
- (iii) measurable effects of latent state on subsequent reportable behavior;
- (iv) history dependence after present external conditions are matched; and
- (v) a reproducible separation between direct readout and latent modulation.

Proposition 8.22 (Interpretation-localized falsification). *Suppose a target cognitive domain is claimed to satisfy the Cognitive Dual-Channel Interpretation. If the declared experiments reveal no reproducible latent state distinction capable of changing later reportable behavior under matched present conditions, then the proposed accessible–hidden interpretation fails for that domain. Such failure does not invalidate the DCIT mathematical theorems or a separately successful crossbar realization.*

The cognitive interpretation is therefore falsifiable at its own layer rather than insulated by the success of the mathematics below it.

9 Neuromorphic Consistency of Dual-Pathway Dynamics

9.1 Purpose and epistemic status

The preceding sections distinguish three logically separate objects: the C3–C9 control-access sector, the persistent internal-state sector, and the C12–C6 readout sector. Appendix D further

establishes, within the ideal bipartite core-graph model, that the two terminal pairs are physically co-supported by a single chrysenes π network while nevertheless possessing inequivalent terminal-response structure and selective, rather than complete, cross-channel suppression.

The present section asks a narrower mathematical question. It does not ask whether consciousness is implemented by chrysenes, nor whether a biological nervous system possesses the molecular architecture described here. Instead it asks whether the DCIT structures admit a nonempty class of realizations having the following neuromorphic features simultaneously:

1. two distinguishable information pathways supported by one physical substrate;
2. unequal effective dynamical response scales;
3. a persistent state capable of storing preparation history;
4. nonzero but bounded interaction between the pathways;
5. initially latent information capable of altering later accessible output; and
6. an interpretation in which rapid automatic processing and slower state-dependent integrative processing remain distinct but causally coupled.

The result below is therefore an *existence and consistency statement*. It establishes that the declared neuromorphic interpretation is compatible with the DCIT architecture. It does not establish empirical identity between a molecular pathway and a biological conscious or nonconscious process.

9.2 Neuromorphic admissibility data

Let

$$\mathcal{K}_x, \quad \mathcal{K}_m, \quad \mathcal{K}_y$$

denote, respectively, the C3–C9 directional sector, the persistent-state sector, and the C12–C6 directional sector, and write

$$\mathcal{K}_{\text{XB}} = \mathcal{K}_x \otimes \mathcal{K}_m \otimes \mathcal{K}_y.$$

For direct inter-channel transfer, retain the effective aggregate space

$$\mathcal{K}_{\text{tr}} = \mathcal{K}_x \oplus \mathcal{K}_y$$

with effective transport generator

$$H_{\varepsilon}^{\text{tr}} = \begin{pmatrix} H_x & \varepsilon K \\ \varepsilon K^* & H_y \end{pmatrix}.$$

The parameter

$$g_{\text{leak}} := |\varepsilon| \|K\|$$

measures the effective strength of direct channel leakage.

On the simultaneous three-sector realization, let

$$V_{xm}$$

couple the C3–C9 sector to the persistent state and let

$$V_{my}$$

couple the persistent state to the C12–C6 sector. Their corresponding effective strengths are denoted

$$g_{xm}, \quad g_{my}.$$

Definition 9.1 (Neuromorphically admissible realization). A finite DCIT realization is called *neuromorphically admissible* when the following conditions hold.

(N1) The two terminal sectors are distinct and nontrivial:

$$\mathcal{K}_x \neq \{0\}, \quad \mathcal{K}_y \neq \{0\}.$$

(N2) Both sectors arise from a common physical realization of the chrysene core rather than from disjoint molecular substrates.

(N3) Direct cross-channel transfer is nonzero but bounded:

$$0 < g_{\text{leak}} < g_*$$

for some prospectively declared tolerance $g_* > 0$.

(N4) The persistent state is dynamically nontrivial and is coupled both to preparation and readout:

$$g_{xm} > 0, \quad g_{my} > 0.$$

(N5) There exist two complete states

$$\rho_0, \rho_1 \in \mathcal{S}(\mathcal{K}_{\text{XB}})$$

whose initial accessible reductions agree,

$$\text{Tr}_{x,m} \rho_0 = \text{Tr}_{x,m} \rho_1,$$

but for which some later accessible readout distinguishes the corresponding trajectories.

(N6) The characteristic response scales

$$\tau_x, \tau_y > 0$$

are independently defined realization parameters. No ordering is included in the definition.

Condition (N3) is deliberately stronger than complete decoupling and weaker than unrestricted cross-talk. The admissible regime is therefore neither

$$g_{\text{leak}} = 0$$

nor

$$g_{\text{leak}} \sim g_{xm}, g_{my}$$

by definition. It permits the two nominal pathways to remain distinguishable while retaining causal communication through the common physical system.

9.3 Model-consistency theorem

Let

$$\mathcal{P}_{\text{DCIT}}$$

denote the DCIT presentation restricted to the channel, hidden-state, dynamical, and accessible-readout structures used above.

Let

$$\mathcal{P}_{\text{NM}}$$

be the enlargement obtained by adjoining the predicates (N1)–(N6) and the interpretation symbols introduced below.

Theorem 9.2 (Neuromorphic dual-pathway consistency). Assume the finite core-graph realization of Appendix D and the finite hidden-state realization of the DCIT model. Then there exists a finite model

$$\mathfrak{M}_{\text{NM}} \models \mathcal{P}_{\text{NM}}$$

whose reduct to the DCIT language is an admissible DCIT realization. In particular,

$$\text{Mod}(\mathcal{P}_{\text{NM}}) \neq \emptyset.$$

The same physical realization may therefore contain simultaneously:

1. two distinguishable terminal-defined pathways;
2. common underlying molecular matter;
3. finite nonzero cross-channel coupling;
4. a persistent hidden degree of freedom;
5. hidden-state-dependent later readout; and
6. independently selectable unequal characteristic response times.

Proof. Appendix D supplies a finite chrysene core-graph witness in which the C3–C9 and C12–C6 terminal responses are both nontrivial but inequivalent. Because both responses are matrix elements of one and the same molecular core operator, they are supported by a common physical substrate. Thus (N1) and (N2) are simultaneously satisfiable.

The same appendix also shows that the four-terminal core is not equivalent to the orthogonal direct sum of two completely isolated molecular wires: selected cross-terminal amplitudes are suppressed, whereas other cross-terminal amplitudes remain nonzero. Hence the common core admits nonvanishing inter-channel transfer.

Choose a nonzero finite-dimensional effective cross-channel

operator K_0 compatible with the nonvanishing cross-terminal amplitudes of the common-core realization. This is an overlay choice: no canonical identification of K_0 with the core Green-function matrix is asserted. For arbitrary $g_* > 0$, choose

$$0 < |\varepsilon| < \frac{g_*}{\|K_0\|}.$$

Setting

$$K = K_0$$

gives

$$0 < |\varepsilon| \|K\| < g_*.$$

Therefore (N3) is satisfiable with arbitrarily small but strictly nonzero direct leakage. Complete physical isolation is not required.

For the persistent state choose a nontrivial finite-dimensional factor, for example

$$\mathcal{K}_m \simeq \mathbb{C}^2.$$

The finite DCIT witness supplies bounded interactions

$$V_{xm} \neq 0, \quad V_{my} \neq 0.$$

Multiplying these by arbitrary nonzero finite coupling constants gives

$$g_{xm} > 0, \quad g_{my} > 0,$$

and hence (N4).

The hidden-state modulation theorem of the main text gives states whose initial accessible reductions coincide but whose later accessible trajectories differ whenever the declared hidden-readout commutator condition is nonzero. The explicit finite DCIT witness contains such a pair. Hence there exist

$$\rho_0 \neq \rho_1$$

such that

$$\text{Tr}_{x,m} \rho_0 = \text{Tr}_{x,m} \rho_1$$

at the initial interrogation time while, for some $t > 0$ and some accessible observable A_y ,

$$\text{Tr}[\rho_0(t)(I_{x,m} \otimes A_y)] \neq \text{Tr}[\rho_1(t)(I_{x,m} \otimes A_y)].$$

Thus (N5) holds.

It remains only to verify compatibility with unequal response scales. In finite dimension choose bounded completely positive trace-preserving semigroup generators

$$\mathcal{L}_x, \quad \mathcal{L}_y$$

with positive characteristic rates

$$\gamma_x, \gamma_y > 0.$$

Define

$$\tau_x = \gamma_x^{-1}, \quad \tau_y = \gamma_y^{-1}.$$

Since positive finite rates may be chosen independently, any strict ordering

$$\tau_x > \tau_y$$

or

$$\tau_y > \tau_x$$

is mathematically realizable without modifying the preceding channel, hidden-state, or leakage construction. Therefore (N6) is compatible with (N1)–(N5).

All constituent spaces and operators are finite dimensional and bounded. Their product construction therefore yields a finite model of the enlarged presentation. Hence

$$\text{Mod}(\mathbf{P}_{\text{NM}}) \neq \emptyset.$$

□

Remark 9.3 (Strength of the theorem). Theorem 9.2 is a model-existence statement. It proves that dual pathways, shared physical substrate, persistent state, finite cross-talk, delayed hidden-state influence, and unequal time scales are mutually compatible. It does not prove that any particular synthesized chrysen junction realizes the chosen coupling constants or time-scale ordering.

9.4 Speed-ordered refinement

The core-graph analysis of Appendix D motivates a distinction between the two directional responses, but static transmission amplitude is not identical to propagation time. Accordingly, introduce the separate realization hypothesis

$$H_\tau : \quad \tau_y < \tau_x,$$

where

$$x = C3 - C9, \quad y = C12 - C6.$$

Corollary 9.4 (Fast–slow dual-pathway realization). *The theory*

$$\mathbf{P}_{\text{NM}} + H_\tau$$

is satisfiable.

Proof. In the construction of Theorem 9.2, choose

$$\gamma_y > \gamma_x > 0.$$

Then

$$\tau_y = \gamma_y^{-1} < \gamma_x^{-1} = \tau_x.$$

No other condition in the construction is altered. □

Thus the proposed ordering

$$C12 - C6 \text{ faster than } C3 - C9$$

is mathematically compatible with the DCIT architecture. Its physical truth remains a realization question requiring a declared transport-time observable and independent calculation or experiment.

9.5 Neuromorphic interpretation map

The empirical neuroscience literature distinguishes, with important theory-dependent qualifications, between processing that can occur without immediate awareness and later processing associated with recurrent amplification, integration, or broad reportability. Masked-priming experiments establish that information not available to immediate report can nevertheless alter later behavior [VVR09]. Reviews of processing without awareness likewise support the existence of substantial nonreportable sensory processing while emphasizing methodological and theoretical limits on stronger claims [MD22]. Within the Global Neuronal Workspace framework, conscious access is associated with recurrent amplification and sustained global availability rather than with the initial feedforward sweep alone [Mas+20]. These observations motivate an analogy but do not determine a unique biological architecture.

Let

$$\mathcal{N}_{\text{aut}}$$

denote a formal class of rapid automatic or nonconscious processing variables,

$$\mathcal{N}_{\text{int}}$$

a formal class of state-dependent integrative variables, and

$$\mathcal{N}_{\text{rep}}$$

a formal class of reportable output variables.

Definition 9.5 (Neuromorphic interpretation). A neuromorphic interpretation of a speed-ordered DCIT realization is a typed map

$$\Phi_{\text{NM}}$$

on the declared fragment satisfying

$$\Phi_{\text{NM}}(\mathcal{K}_y) = \mathcal{N}_{\text{aut}},$$

$$\Phi_{\text{NM}}(\mathcal{K}_x \otimes \mathcal{K}_m) = \mathcal{N}_{\text{int}},$$

and mapping the state-dependent evolution from the hidden block into the accessible readout to the transition

$$\mathcal{N}_{\text{int}} \longrightarrow \mathcal{N}_{\text{rep}}.$$

The interpretation does not identify

$$C3 - C9$$

with consciousness as a substance or state, nor

$$C12 - C6$$

with the biological unconscious. Rather, it assigns functional roles:

$$\begin{array}{c} C12 - C6 \\ \mapsto \\ \text{rapid automatic/feedforward} \\ \text{processing analogue} \end{array}$$

and

$$\begin{array}{c} C3 - C9 + \mathcal{K}_m \\ + \text{state-dependent interaction} \\ \mapsto \\ \text{integrative access analogue} \end{array}$$

In this formulation, reportability is a property of the complete state-dependent process and its declared observation map. It is not a property of a molecular wire in isolation.

Proposition 9.6 (Consistency of the neuromorphic interpretation). *Under Theorem 9.2 and H_τ , a neuromorphic interpretation Φ_{NM} exists on the declared fragment.*

Proof. The theorem supplies distinct fast and slow sectors, a persistent state, and a nonzero interaction by which an initially inaccessible state difference can alter a later accessible observable. Assign the faster y sector to $\mathcal{N}_{\text{aut}}$, assign the state-dependent $x \otimes m$ block to $\mathcal{N}_{\text{int}}$, and assign the later accessible observable algebra to $\mathcal{N}_{\text{rep}}$. These assignments preserve the required distinctions among immediate propagation, persistent internal state, and later reportability. Hence the map is well typed on the declared fragment. $\square$

9.6 Cross-talk as a structural feature

A strict two-box cognitive analogy would impose

$$\text{conscious processing} \perp \text{nonconscious processing},$$

so that no information from one sector could influence the other. Neither the biological evidence nor the present molecular model requires such exact isolation.

Define the normalized direct-coupling parameter

$$\chi_{\text{dir}} := \frac{g_{\text{leak}}}{g_{\text{ref}}},$$

where $g_{\text{ref}} > 0$ is a prospectively declared within-channel reference scale. The neuromorphically relevant regime is

$$0 < \chi_{\text{dir}} \ll 1.$$

Separately define a hidden-state modulation magnitude over a finite observation horizon T by

$$\chi_{\text{state}}(T) := \sup_{\rho_0, \rho_1} \sup_{0 \leq t \leq T} D_{\text{acc}}(\rho_0(t), \rho_1(t)),$$

where the supremum is restricted to pairs having the same initial accessible reduction and distinct admissible hidden preparations.

The distinction

$$\chi_{\text{dir}} \neq \chi_{\text{state}}$$

is essential. The first measures unwanted direct feedthrough between declared channels; the second measures the later accessible influence of information that was initially hidden.

Proposition 9.7 (Non-isolated dual processing). *There exist neuromorphically admissible realizations satisfying*

$$0 < \chi_{\text{dir}} < \delta$$

for every prescribed $\delta > 0$, while simultaneously

$$\chi_{\text{state}}(T) > 0$$

for some finite $T > 0$.

Proof. By Theorem 9.2, choose the direct cross-channel coefficient ε sufficiently small that

$$0 < |\varepsilon| \|K\| < \delta g_{\text{ref}}.$$

Then

$$0 < \chi_{\text{dir}} < \delta.$$

Independently, retain the nonzero hidden-readout interaction V_{my} and choose the hidden states appearing in condition (N5). Their initial accessible reductions agree, but a later accessible observable distinguishes the two trajectories. Therefore

$$D_{\text{acc}}(\rho_0(t), \rho_1(t)) > 0$$

for some $t > 0$, and hence

$$\chi_{\text{state}}(T) > 0$$

for every T containing such a time. $\square$

The proposition formalizes the regime

weak direct cross-talk + nonzero state-mediated influence

This is structurally compatible with experimental observations that stimuli lacking immediate conscious report can nevertheless modify later reaction, decision, or response variables [VVR09; MD22]. The proposition does not identify a behavioral priming effect with a molecular transmission coefficient; it asserts only that both domains admit the same qualitative separation between direct accessibility and indirect causal influence.

9.7 Thresholded conscious-access analogue

Let

$$D_{\text{acc}}(t)$$

be the accessible distinguishability introduced previously and let $\Delta_{\text{rep}} > 0$ denote a declared reportability threshold. Define

$$t_{\text{rep}} = \inf \{t \geq 0 : D_{\text{acc}}(t) \geq \Delta_{\text{rep}}\}.$$

A hidden distinction may therefore satisfy

$$D_{\text{acc}}(0) = 0$$

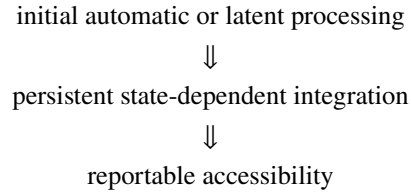
while

$$D_{\text{acc}}(t) > 0$$

at later times and eventually

$$D_{\text{acc}}(t_{\text{rep}}) \geq \Delta_{\text{rep}}.$$

This yields the neuromorphic sequence



No physical channel is assigned consciousness. The conscious-access analogue is the composite event in which a previously latent state distinction becomes accessible through the coupled control–memory–readout dynamics.

Corollary 9.8 (Composite conscious-access analogue). *A neuromorphically admissible DCIT realization may possess a rapid automatic-processing analogue and a distinct slower conscious-access analogue on the same physical substrate, with nonzero causal interaction between them, without identifying either process with an independent material subsystem.*

Proof. The rapid pathway is represented by the y sector under H_τ . The slower integrative route is represented by the $x \otimes m$ preparation and state sector together with its later coupling to the accessible y observable algebra. Proposition 9.7 supplies nonzero but arbitrarily bounded direct coupling and nonzero state-mediated influence. Theorem 9.2 places all three factors in one finite physical realization. Hence the stated configuration exists. $\square$

9.8 Interpretive boundary

The preceding theorem and corollaries establish

mathematical consistency of a dual-process neuromorphic interpretation

and not

empirical identification of chrysene transport
with human consciousness

More precisely, the result is the non-vacuity statement

$$\text{Mod}(\mathbf{P}_{\text{DCIT}} + H_{\tau} + \Phi_{\text{NM}}) \neq \emptyset.$$

It shows that the following properties can coexist without mathematical contradiction:

common physical substrate,
two distinguishable directional channels,
unequal response scales,
persistent latent state,
bounded nonzero direct cross-talk,
nonzero indirect state-mediated influence,
and delayed emergence into accessibility.

These are precisely the structural features required for the proposed neuromorphic analogy. Whether the C12–C6 realization is experimentally faster than the C3–C9 realization, whether its direct leakage satisfies the required isolation threshold, and whether any biological conscious/nonconscious system is adequately represented by the interpretation remain independent empirical questions.

9.9 Stacked Dual-Channel Extension as a Neuromorphic Consistency Witness

The preceding neuromorphic interpretation requires neither a multilayer molecular architecture nor persistent quantum coherence. Its minimal structure is the coexistence, on one physical substrate, of distinguishable directional pathways, a persistent internal state, bounded direct cross-coupling, and state-dependent later accessibility. The single-core DCIT realization therefore remains logically sufficient for the results established above.

A larger realization class is nevertheless available by stacking two 3,6,9,12-substituted chrysene nodes in a cofacial geometry. Such a dimer supplies two copies of the terminal-defined directional structure together with an additional interlayer degree of freedom. One layer may remain principally transport-active while the second acts as a field-coupled or charge-retaining state layer. In this case the persistent DCIT sector $\mathcal{K}_m$ acquires a candidate physical realization as an interlayer charge or electrostatic state, and the readout transfer function becomes state dependent:

$$T_y = T_y(E; F, q_m, \omega),$$

where F denotes an applied control field, q_m a retained interlayer state, and ω the remaining operational and environmental variables.

The stacked extension is relevant to the neuromorphic analogy because it permits three functional time scales to coexist on the same molecular assembly:

rapid propagation, state-dependent control/integration,
persistent internal memory.

Accordingly, it supplies a possible hardware witness for the distinction among automatic propagation, slower contextual integration, and retained latent state without identifying any individual molecular pathway with consciousness.

The principal logical point is existence rather than necessity:

single-layer DCIT suffices for the abstract architecture,

whereas

a stacked dimer enlarges the realizable state and control space.

The dimer does not automatically constitute an eight-channel device. It is an eight-terminal molecular system whose number of independent transport eigenchannels depends on the realized Hamiltonian, electrode couplings, and interlayer interaction. Likewise, interlayer coherence, resonant HOMO/LUMO manipulation, Coulomb blockade, and long-lived charge retention remain realization hypotheses rather than prerequisites of the DCIT neuromorphic interpretation.

Appendix E develops the resulting hierarchy of stacked performance architectures and proves that the single-layer DCIT model embeds consistently into a finite dimer extension containing field control and persistent-state modulation.

10 Crossbar Realization

The preceding theory is independent of a unique material implementation. We now state one candidate realization motivated by substituted-chrysene crossbar architectures. The role of this section is translation from abstract objects to measurable device quantities, not assertion that the device has already been synthesized.

10.1 Finite crossbar geometry

Let

$$\Lambda_{N,M} := \{1, \dots, N\} \times \{1, \dots, M\}.$$

Each crosspoint $(i, j) \in \Lambda_{N,M}$ carries a node with a C3–C9 control interface, a C12–C6 readout interface, and a persistent internal variable represented abstractly by $\mathcal{K}_m$. In the CohBit-inspired realization, the primary candidate microscopic implementation of $\mathcal{K}_m$ is a vertically coupled retained charge or electrostatic state associated with a gate subsystem [Sch26c]. Configurational, redox, or other hidden-state mechanisms are separate possible DCIT realizations and are not asserted here as

properties of substituted chrysene. Appendix D further shows that the two named directional responses arise within one delocalized chrysene π system. They are terminal-defined response functions of common molecular matter, not independent carbon wires. The ideal core graph predicts selective cross-terminal interference suppression rather than complete inter-channel isolation.

At each crosspoint,

$$\mathcal{K}_{\text{XB}}^{(i,j)} = \mathcal{K}_x^{(i,j)} \otimes \mathcal{K}_m^{(i,j)} \otimes \mathcal{K}_y^{(i,j)}.$$

The finite crossbar state space may be taken, at the simplest level, as

$$\mathcal{K}_{N,M} := \bigotimes_{(i,j) \in \Lambda_{N,M}} \mathcal{K}_{\text{XB}}^{(i,j)},$$

for finite-dimensional crosspoints. More general quasi-local limits may be developed for large arrays, but are not required for the finite falsification protocol of the present paper.

A literal device interpretation requires a separately specified representation or effective-model map from the abstract crosspoint spaces into the physical Hilbert space of the realized junctions. The tensor product records modeled simultaneous degrees of freedom; it does not assert that a microscopic many-electron Hilbert space factorizes uniquely in this manner.

10.2 Effective crosspoint Hamiltonian

Let

$$H_x = H_x^* \in B(\mathcal{K}_x),$$

$$H_m = H_m^* \in B(\mathcal{K}_m),$$

$$H_y = H_y^* \in B(\mathcal{K}_y).$$

Choose bounded self-adjoint interactions V_{xm} and V_{my} coupling control-to-memory and memory-to-readout, respectively, and a bounded parasitic term V_{xy} . With tensor identities on unmentioned factors written implicitly, define

$$H_{\text{XB}} = H_x + H_m + H_y + \lambda_{xm} V_{xm} + \lambda_{my} V_{my} + \varepsilon V_{xy}.$$

Because the interaction operators need not share a common normalization, define the effective strengths

$$g_{xy} := |\varepsilon| \|V_{xy}\|,$$

$$g_{xm} := |\lambda_{xm}| \|V_{xm}\|,$$

$$g_{my} := |\lambda_{my}| \|V_{my}\|.$$

The candidate device regime is

$$g_{xy} \ll g_{xm}, g_{my},$$

interpreted through measured transfer, modulation, and retention observables. These are effective-model quantities rather than direct geometric invariants of the chrysene core.

Remark 10.1 (Relation to the direct-sum leakage model). The parameter ε in the effective crosspoint Hamiltonian and the operator K of the direct-sum leakage model represent related but not identical descriptions. A concrete device model may provide an intertwining map between them. Until such a map is specified, the mathematical results use them in their respective effective representations only.

10.3 Physical realization hypotheses

A candidate substituted-chrysene implementation is summarized by the following realization hypotheses.

$H_{\partial\pi}$ The realization satisfies the inherited Article-39 Conjugated π -Framework Hypothesis: the relevant benzenoid carbon framework participates in the ordinary delocalized sp^2 π system. This hypothesis does not by itself assert directional conductivity or a specific device transport law [Sch26d].

H_{chem} A chemically instantiated node preserves the declared standard-locant chrysene framework and provides linker/contact chemistry at the C3–C9 and C12–C6 interfaces that is compatible with the declared propagation model without destroying the conjugated framework required by $H_{\partial\pi}$.

H_{addr} The realized contacts and linkers permit the C3–C9 and C12–C6 directional interfaces to be addressed separately over the stated operating window.

H_{prop} Given $H_{\partial\pi}$, H_{chem} , and H_{addr} , together with separately specified environment, state preparation, Hamiltonian, and transport observable, the declared directional propagation model is applicable over the stated operating window.

H_{aniso} A prospectively specified direction-resolved observable exhibits a reproducible physical distinction between the intended C12–C6 and C3–C9 responses over the declared operating window. The ideal core result in Appendix D motivates, but does not certify, the ordering

$$\mathcal{T}_{12,6} > \mathcal{T}_{3,9}.$$

H_{QI} Over a declared near-symmetry operating region, the realized four-terminal junction retains the selective interference pattern motivated by Appendix D, with C3–C12 and C9–C6 transfer suppressed relative to the intended directional responses. Exact zeros are not required in the realized junction.

H_{XB} A finite crosspoint realizes the declared control–hidden-state–readout architecture with separately identifiable C3–C9 and C12–C6 interfaces.

H_{prep} A prospectively specified C3–C9 preparation history reproducibly changes the independently calibrated hidden-state proxy S_m beyond the declared state-discrimination threshold under matched non-control conditions.

H_{ctrl} Contemporaneous perturbation through the C3–C9 interface produces a measurable change in the declared C12–C6 readout observable over the stated operating family.

H_{state} There exists an independently calibrated observable or state proxy S_m , distinct from the principal C12–C6 readout observable, that distinguishes at least two candidate retained states to a prospectively specified resolution.

H_{mem} States distinguished by S_m persist reproducibly after the preparation histories are returned to matched present external conditions over the declared retention window.

H_{read} The C12–C6 readout depends measurably on the independently distinguished retained state.

H_{iso} Direct parasitic C3–C9/C12–C6 transfer remains below a prospectively declared leakage threshold over the full declared operating-state family.

H_{array} For a claimed finite-array realization, the node/linker assembly preserves the declared row and column interfaces, permits crosspoint-selective interrogation to the stated resolution, and keeps unintended inter-node or sneak-path transfer below a prospectively specified array-level threshold.

If no independent state proxy satisfying H_{state} is available, the manuscript permits only the weaker operational history claim

$$H_{\text{hist}}$$

for reproducible preparation-history dependence in later readout under matched present conditions and does not assert separate empirical certification of H_{mem} and H_{read} .

A Stark-specific implementation may additionally introduce an optional field-window hypothesis H_{field} , requiring that the declared control-field range remain below experimentally determined irreversible-change or dielectric-failure thresholds. Stark tuning is not required by the abstract DCIT architecture.

A speed-ordered implementation may additionally introduce

$$H_{\tau} : \quad \tau_{12,6} < \tau_{3,9},$$

where each τ is defined by a prospectively declared dynamical observable appropriate to the transport regime. The ideal static transmission hierarchy of Appendix D motivates this ordering but does not imply it.

None of these hypotheses is implied merely by the C_{2h} -type molecular symmetry or the existence of the inherited tensor network. They belong to the physical realization overlay.

10.4 Pathway roles

The locant labels in this subsection are understood in the inherited standard numbered chrysene reference frame; no uniqueness of chemical nomenclature is derived here from the twelve axioms.

Hypothesis 10.2 (Crossbar pathway assignment). For the candidate physical realization,

$$x = \text{C3} \leftrightarrow \text{C9}$$

functions as a control-access pathway and

$$y = \text{C12} \leftrightarrow \text{C6}$$

functions as the principal readout pathway.

This role assignment is not entailed by the geometric inequivalence of the two directions. It is a physical realization hypothesis to be tested independently.

The persistent state is not assumed to be the C3–C9 pathway itself. Instead, C3–C9 is modeled as a pathway that prepares or addresses $\mathcal{K}_m$. In the CohBit-inspired specialization the primary candidate retained state is vertically coupled charge or electrostatic occupation; other hidden-state mechanisms require separate realization evidence.

The ideal core calculation does not replace H_{iso} . It provides one molecular-topological mechanism that may contribute to low selected cross-transmission, while finite residual coupling remains allowed and must be measured over the realized operating family. Likewise, H_{aniso} , H_{QI} , and H_{τ} remain realization-level claims even though Appendix D supplies exact conditional results for the ideal graph/Hückel model.

10.5 Array couplings

For a finite crossbar, one may adjoin nearest-neighbor row and column couplings. Let

$$H_{\text{row}} = \sum_{(i,j)} J_y^{(i,j)}$$

denote effective couplings along the C12–C6 readout rows and

$$H_{\text{col}} = \sum_{(i,j)} J_x^{(i,j)}$$

denote effective couplings along the C3–C9 control columns. The complete finite-array Hamiltonian can then be represented schematically as

$$H_{N,M} = \sum_{(i,j)} H_{\text{XB}}^{(i,j)} + H_{\text{row}} + H_{\text{col}} + H_{\text{env}}.$$

The environmental term may encode contacts, substrate, phonons, solvent, dielectric environment, or other realization-specific effects. No theorem below requires a particular microscopic choice for H_{env} ; the experimental observables are defined operationally.

10.6 Direction-resolved transfer observables

The abstract physical protocol is not restricted to a single microscopic transport regime.

Definition 10.3 (Direction-resolved transfer observable). Let

$$\mathcal{T}_{\alpha\beta}(\xi; \omega)$$

denote the realization-specific transfer or input–output observable from interface β to interface α , evaluated at operating parameter ξ and operational state/environment label ω .

In a coherent scattering or NEGF specialization one may take

$$\xi = E, \quad \mathcal{T}_{\alpha\beta}(E; \omega) = T_{\alpha\beta}(E; \omega).$$

For purposes of the leakage ratio below, $\mathcal{T}_{\alpha\beta}$ is required to be a nonnegative transfer magnitude with a common normalization or common physical units across the four directional entries being compared. If the primitive measurement is signed, such as a current or response coefficient, the realization must specify prospectively the nonnegative magnitude, norm, or other leakage functional used to construct $\mathcal{T}_{\alpha\beta}$. Other realizations may therefore use conductance magnitudes, transition rates, current-response magnitudes, or another direction-resolved transfer measure, provided its normalization and physical interpretation are specified prospectively.

Let I_ξ denote the declared operating-parameter window and let Ω_{op} denote the prospectively declared family of retained states, control settings, and environmental conditions belonging to the claimed operating regime.

Definition 10.4 (Leakage-test domain). Define

$$\Omega_{\text{leak}} := \{(\xi, \omega) \in I_\xi \times \Omega_{\text{op}} : \mathcal{T}_{xx}(\xi; \omega) + \mathcal{T}_{yy}(\xi; \omega) > 0\}.$$

A leakage-ratio claim is well posed only when $\Omega_{\text{leak}} \neq \emptyset$. Operating points outside Ω_{leak} have no defined ratio and cannot be used as evidence for H_{iso} .

Definition 10.5 (Direct leakage ratio). For $(\xi, \omega) \in \Omega_{\text{leak}}$, define

$$\eta(\xi; \omega) := \frac{\mathcal{T}_{xy}(\xi; \omega) + \mathcal{T}_{yx}(\xi; \omega)}{\mathcal{T}_{xx}(\xi; \omega) + \mathcal{T}_{yy}(\xi; \omega)}.$$

The CohBit source itself notes that finite residual cross-hopping produces nonzero residual coupling and that exact zero coupling is only an ideal limit; its numerical values are illustrative rather than first-principles [Sch26c]. The present realization therefore treats η as a measured or independently computed quantity rather than assuming $\eta = 0$.

Definition 10.6 (Operating isolation margin). Define

$$\eta_{\text{op}} := \sup_{(\xi, \omega) \in \Omega_{\text{leak}}} \eta(\xi; \omega).$$

The hypothesis H_{iso} is the prospective inequality

$$\eta_{\text{op}} < \eta_{\text{max}}.$$

For an array-level claim, let η_{array} denote a separately defined worst-case unintended inter-node or sneak-path transfer metric over the declared finite array and operating family. Hypothesis H_{array} requires

$$\eta_{\text{array}} < \eta_{\text{array, max}}.$$

10.7 Control-to-readout modulation

Let u_x denote a declared control variable applied through the C3–C9 interface. Depending on the realization, u_x may represent a terminal voltage, local electric field, chemical potential, gate occupancy control, or another externally specified input.

Definition 10.7 (Modulation gain). When differentiable, define

$$G_{x \rightarrow y}(\xi; \omega) := \frac{\partial \mathcal{T}_{yy}(\xi; u_x, \omega)}{\partial u_x}.$$

For discrete sweeps, a prospectively specified finite-difference estimator may be used instead.

The desired information-processing regime is

$$\eta_{\text{op}} \text{ small} \quad \text{while} \quad \sup_{\substack{\xi \in I_\xi \\ \omega \in \Omega_{\text{op}}}} |G_{x \rightarrow y}(\xi; \omega)| \text{ is nonzero.}$$

Thus unwanted direct transfer can be weak while useful control or hidden-state modulation remains appreciable.

10.8 Independent state proxy and retention

Let $u_x^{(0)}$ and $u_x^{(1)}$ be two preparation histories that return the external control terminal to the same present readout condition. Let S_m be the independent retained-state proxy required by H_{state} . Define

$$\Delta S_m(t) := |\langle S_m \rangle_1(t) - \langle S_m \rangle_0(t)|.$$

For a prospectively declared threshold $S_{\text{min}} > 0$, the two retained states are operationally distinguished at time t when

$$\Delta S_m(t) \geq S_{\text{min}}.$$

For a readout observable A_y , define

$$M_{A_y}(t) := |\langle A_y \rangle_1(t) - \langle A_y \rangle_0(t)|.$$

Definition 10.8 (Retention time). For a declared retention window $I_t \subseteq [0, \infty)$, define

$$\tau_{\text{ret}} := \sup(\{t \in I_t : \Delta S_m(t) \geq S_{\min}\} \cup \{0\}).$$

A nonzero τ_{ret} under matched present conditions supports H_{mem} . The separate hypothesis H_{read} requires that independently distinguished retained states alter the C12–C6 readout beyond the prospectively declared readout threshold. If no independent S_m is available, only the weaker history-dependent readout claim H_{hist} is operationally available.

10.9 Physical factorization map

The abstract crosspoint tensor product acquires literal device meaning only through an explicit realization map.

Hypothesis 10.9 (Physical composite realization). There exists a physical state space $\mathcal{H}_{\text{phys}}^{\text{XB}}$ and an isometric or effective-model map

$$W_{\text{phys}} : \mathcal{K}_{\text{XB}} \longrightarrow \mathcal{H}_{\text{phys}}^{\text{XB}}$$

that intertwines the declared control, hidden-state, and readout observables to the accuracy claimed for the realization.

The hypothesis may be satisfied approximately in an effective model; exact microscopic factorization is not assumed.

11 Experimental Operationalization and Falsification

A theory is experimentally useful only if its realization claims can fail without requiring reinterpretation of the data after the fact. The present section therefore treats threshold selection, preparation history, matched present conditions, independent hidden-state discrimination, and layer-localized failure as part of the physical architecture rather than as optional experimental details.

11.1 Prospective thresholds

Before testing a proposed device, specify an operating window I_ξ , a retention interval I_t , an operational-state family Ω_{op} , and thresholds

$$\eta_{\max} > 0, \quad G_{\min} > 0, \quad S_{\min} > 0, \quad M_{\min} > 0.$$

For an array-level claim also specify

$$\eta_{\text{array}, \max} > 0.$$

The measurement resolution, replication rule, uncertainty treatment, and statistical decision procedure should likewise be fixed prospectively.

Definition 11.1 (Operational DCIT realization window). A finite device lies in the declared operational DCIT realization window when

$$\eta_{\text{op}} < \eta_{\max},$$

$$\sup_{\substack{\xi \in I_\xi \\ \omega \in \Omega_{\text{op}}}} |G_{x \rightarrow y}(\xi; \omega)| > G_{\min},$$

and

$$\sup_{t \in I_t} M_{A_y}(t) > M_{\min}.$$

These three inequalities are deliberately non-substitutable. Excellent isolation cannot compensate for absent modulation; strong modulation cannot compensate for absent retained-history readout; and history dependence alone does not establish directional isolation.

Definition 11.2 (Mechanistically resolved three-sector realization). An operational DCIT realization is mechanistically resolved at the three-sector level when, in addition, the C3–C9 preparation history is shown to prepare the hidden state through

$$\sup_{t \in I_t} \Delta S_m(t) > S_{\min},$$

for an independently calibrated state proxy S_m , and independently distinguished retained states alter the principal C12–C6 readout beyond the declared readout threshold. This additional requirement tests H_{prep} as well as H_{state} .

Without an independent state proxy, the device may support H_{hist} but the empirical decomposition into separate H_{mem} and H_{read} claims remains unresolved.

11.2 Preparation–readout separation

A test run contains two intervals. During

$$[-T_p, 0],$$

a specified C3–C9 control history prepares the candidate hidden state. At $t = 0$, the contemporaneous control condition is returned to a common declared readout value for all comparison histories. The C12–C6 output is then interrogated over

$$[0, T_r].$$

This separation operationalizes the claim that later readout differences arise from retained internal history rather than merely from different present inputs.

Definition 11.3 (Matched present condition). Two test trajectories satisfy the matched-present condition at $t = 0$ when all externally declared control variables used in the readout model have the same values in both trajectories at the start of the readout interval.

The definition does not require the complete internal physical state to be equal; that inequality is exactly what the memory hypothesis seeks to detect.

11.3 Control families

Let

$$\mathcal{U}_{\text{prep}}$$

be a prospectively declared family of preparation histories. Useful members include:

- (i) a baseline or no-preparation history;
- (ii) two histories intended to prepare distinct hidden states;
- (iii) a repeated preparation to estimate within-condition variability;
- (iv) a polarity- or order-reversed history when the physical model predicts asymmetry; and
- (v) a reset or erasure history when such an operation is physically available.

The exact waveform is realization-specific and is not prescribed by the abstract theory.

11.4 Chemical and directional characterization

Before interpreting transport or retention, verify the realization scope associated with H_{chem} and H_{addr} : the declared linker/contact pattern must preserve the intended standard-locant chrysene node to the stated structural resolution and the C3–C9 and C12–C6 interfaces must be separately interrogable.

With the hidden state prepared by a common baseline condition, measure or compute the declared direction-resolved observables

$$\mathcal{T}_{xx}(\xi; \omega), \quad \mathcal{T}_{yy}(\xi; \omega), \quad \mathcal{T}_{xy}(\xi; \omega), \quad \mathcal{T}_{yx}(\xi; \omega)$$

over I_{ξ} and the relevant baseline subset of Ω_{op} . In a coherent scattering realization these may be the corresponding transmission probabilities. A failure to distinguish the intended C3–C9 and C12–C6 input–output pathways invalidates the proposed pathway assignment before any cognitive interpretation is considered.

11.5 Directional anisotropy test

The ideal core calculation of Appendix D motivates, but does not experimentally establish, a directional response inequality.

For a prospectively specified direction-resolved observable and operating family, test the physical hypothesis

$$H_{\text{aniso}} : \quad \mathcal{T}_{12,6}(\xi; \omega) > \mathcal{T}_{3,9}(\xi; \omega)$$

over the claim scope chosen before measurement. A statistically reproducible directional distinction supports the physical anisotropy claim. Failure to observe the declared distinction falsifies H_{aniso} without affecting the exact graph/Hückel statements of Appendix D.

11.6 Interference-selection test

A four-terminal realization should also test the complete terminal family

$$\mathcal{T}_{3,9}, \quad \mathcal{T}_{12,6}, \quad \mathcal{T}_{3,6}, \quad \mathcal{T}_{3,12}, \quad \mathcal{T}_{9,6}, \quad \mathcal{T}_{9,12}$$

over a prospectively declared near-symmetry window. The ideal bipartite model predicts suppression of the C3–C12 and C9–C6 pairings at the exact symmetry point, while C3–C6 and C9–C12 remain finite. In a realized junction exact zeros are not required; instead H_{QI} requires the declared selective suppression pattern to survive within a prospectively specified tolerance. If contacts, substitution, environment, or dephasing erase that pattern throughout the claimed window, the ideal bipartite model is not quantitatively adequate for that realization.

11.7 Speed-ordering test

A claim that

$$H_{\tau} : \quad \tau_{12,6} < \tau_{3,9}$$

requires an operational definition of each response time. Depending on the transport regime, τ may denote a transient junction response time, hopping mean-first-passage time, kinetic relaxation time, or coherent scattering delay. Static transmission magnitude is not a substitute for this dynamical observable. Failure of the prospectively measured ordering falsifies H_{τ} without affecting either the ideal core transmission hierarchy or the neuromorphic existence theorem.

11.8 Isolation test

The isolation test is conducted on the explicitly declared leakage-test domain

$$\Omega_{\text{leak}} = \{(\xi, \omega) \in I_{\xi} \times \Omega_{\text{op}} : \mathcal{T}_{xx}(\xi; \omega) + \mathcal{T}_{yy}(\xi; \omega) > 0\},$$

and evaluates

$$\eta_{\text{op}} = \sup_{(\xi, \omega) \in \Omega_{\text{leak}}} \eta(\xi; \omega).$$

The physical isolation hypothesis is supported only if $\Omega_{\text{leak}} \neq \emptyset$ and

$$\eta_{\text{op}} < \eta_{\text{max}}.$$

No exact-zero criterion is imposed. The state/environment dependence retained in Ω_{leak} prevents a baseline low-leakage state from masking excessive cross-talk in a programmed or retained state.

11.9 Modulation test

The modulation test varies the declared C3–C9 control input while measuring the C12–C6 readout. The central quantity is

$$G_{x \rightarrow y}(\xi; \omega).$$

A device fails the modulation hypothesis when

$$|G_{x \rightarrow y}(\xi; \omega)| \leq G_{\min}$$

throughout the declared operating family within the measurement resolution.

A useful control is a measurement in which the persistent-state preparation is held fixed while the contemporaneous control input is varied. This separates instantaneous modulation from retained-history effects.

11.10 Independent state-proxy test

When the manuscript claims the mechanistically resolved three-sector realization, the candidate retained state must be distinguishable through the independent proxy S_m . For two matched-present preparation histories define

$$\Delta S_m(t) = |\langle S_m \rangle_1(t) - \langle S_m \rangle_0(t)|.$$

Hypothesis H_{state} requires reproducible discrimination above $S_{\min}$ under the declared calibration procedure. When the two proxy-defined states are produced by prospectively specified C3–C9 preparation histories, the same comparison tests H_{prep} : the control pathway must causally prepare distinguishable hidden states rather than merely correlate with them.

The retained-state hypothesis H_{mem} requires that this independently observed state distinction persist over a nonzero portion of the declared retention window after present external conditions have been matched.

11.11 Hidden-state-to-readout test

For preparation histories that produce independently distinguished retained states, measure

$$M_{A_y}(t) = |\langle A_y \rangle_1(t) - \langle A_y \rangle_0(t)|.$$

Hypothesis H_{read} requires a reproducible readout difference above $M_{\min}$ associated with the independently distinguished retained states.

If no independent state proxy is available, the same matched-history protocol may support only the weaker claim H_{hist} : present-matched preparation history alters later readout. It does not independently establish the microscopic existence and readout coupling of a distinct $\mathcal{K}_m$ sector.

11.12 State dependence versus direct feedthrough

A retained-history effect should be distinguished from ordinary direct feedthrough from the control terminal. Compare:

- (a) different preparation histories with the same present control value; and
- (b) the same history with different present control values.

The first comparison estimates retained-state dependence; the second estimates contemporaneous control dependence. A physical implementation may exhibit both, but they should not be conflated.

11.13 Array-level test

A one-crosspoint realization establishes only the elementary node claim. A finite-array claim additionally requires H_{array} . For the declared array geometry, measure crosspoint selectivity and a prospectively defined worst-case unintended inter-node or sneak-path metric η_{array} . The array criterion is

$$\eta_{\text{array}} < \eta_{\text{array}, \max}.$$

Failure of this criterion rejects the claimed array-level realization without invalidating a successfully demonstrated single-crosspoint DCIT element.

11.14 Layer-localized physical falsification

Theorem 11.4 (Layer-localized physical falsification). *For a proposed physical realization:*

- (i) *failure of the declared linker/contact chemistry to preserve the required realization scope falsifies H_{chem} ;*
- (ii) *failure of separate directional addressability falsifies H_{addr} ;*
- (iii) *failure of the prospectively declared C12–C6 versus C3–C9 response distinction falsifies H_{aniso} ;*
- (iv) *loss of the declared selective cross-terminal suppression pattern falsifies H_{QI} for that realized junction;*
- (v) *failure of the prospectively defined ordering $\tau_{12,6} < \tau_{3,9}$ falsifies H_{τ} ;*
- (vi) *failure to realize the declared finite control–hidden-state–readout crosspoint falsifies H_{XB} ;*
- (vii) *failure of prospectively specified C3–C9 preparation histories to produce distinguishable proxy-defined hidden states falsifies H_{prep} for the mechanistically resolved three-sector claim;*
- (viii) *absence of measurable contemporaneous control-to-readout modulation throughout the operating family falsifies H_{ctrl} ;*

- (ix) *absence of an independently distinguishable retained-state proxy falsifies H_{state} for the mechanistically resolved three-sector claim;*
 - (x) *loss of that independently distinguished state under matched present conditions falsifies H_{mem} ;*
 - (xi) *absence of readout dependence on independently distinguished retained states falsifies H_{read} ;*
 - (xii) *leakage above the prospective threshold somewhere in the claimed operating family falsifies H_{iso} ; and*
 - (xiii) *excessive unintended inter-node or sneak-path transfer falsifies H_{array} for an array-level claim.*
- None of these outcomes, by itself, falsifies the protected twelve-axiom Chrysene foundation.*

Proof. Each hypothesis was defined by an independent operational condition or explicitly scoped physical premise. Failure of that condition removes the corresponding realization premise from the support of the physical claim. The protected foundation does not contain these realization hypotheses as axioms, so its validity is logically independent of their experimental failure. $\square$

11.15 Environmental robustness and optional Stark control

Temperature, field strength, contact chemistry, packing geometry, substrate, dielectric environment, and other external parameters may be swept to map the domain in which the realization criteria hold. The resulting operating region is part of the realization claim. A device that functions only under a restricted temperature or field window may still instantiate DCIT within that window; the theory does not require room-temperature operation as a logical prerequisite.

If Stark control is claimed, the experiment should additionally verify the optional H_{field} premise: the applied field window must remain within the independently established reversible operating regime of the realized device. The supplied CohBit source treats dielectric integrity and irreversible conformational change as unresolved device-level constraints rather than guaranteed consequences of the chrysene geometry [Sch26c].

11.16 Neuromorphic and cognitive falsification are separate

Let $\mathcal{T}_{\text{cog}}$ denote a declared family of external tests for a biological or behavioral target. The neuromorphic interpretation of Section 9 is not supported for a target that lacks the claimed structural combination of: rapid automatic or nonreportable processing; persistent or state-dependent latent variables; later influence on reportable output; and measurable but nonidentical

coupling between the processing levels. In particular, if no reproducible latent distinction changes later reportable behavior under matched present conditions, the accessible–hidden interpretation fails for that domain.

Failure of the biological analogy does not invalidate the molecular graph result, the DCIT mathematical existence theorem, or a separately successful crossbar realization.

Remark 11.5. Successful device realization would therefore establish a physical accessible–hidden information architecture to stated tolerances. It would not, by itself, establish that human conscious and nonconscious processing use the same mechanism. Behavioral priming or other nonreportable effects must not be numerically identified with a molecular transmission coefficient.

11.17 Prospective versus retrospective use

The thresholds and controls above are most informative when specified before the decisive experiment. Retrospective fitting may still be useful for model development, but it should be labeled as such and should not be presented as prospective falsification. This distinction mirrors the review architecture of Section 12: evidence is strongest when the admission criterion is fixed independently of the result it is used to certify.

12 Certification Frontier and Review Status

The development protocol follows the orthogonal-constraint review architecture described in the supplied review framework [Sch26a]. Mathematics, framework concordance, and chemistry/physical-domain admissibility are treated as non-substitutable questions, followed by targeted verification and a final meta-review. The present section adapts the certification-frontier language of the certified-growth framework to the three principal claim classes of this manuscript [Sch26b].

12.1 Claim classes

Let

$$\Theta_{\text{math}}$$

denote the central mathematical DCIT claim,

$$\Theta_{\text{core}}$$

the ideal four-terminal graph/Hückel claim,

$$\Theta_{\text{NM}}$$

the neuromorphic consistency claim,

$$\Theta_{\text{phys}}$$

Claim layer	Principal observable / evidence	Prospective failure condition	Consequence
Chemical realization	Structural/contact characterization	Declared linker/contact realization does not preserve required node/interface scope	Reject H_{chem}
Directional address-ability	Direction-resolved I/O map	C3–C9 and C12–C6 interfaces cannot be addressed separately	Reject H_{addr}
Directional anisotropy	$\mathcal{T}_{12,6}$ versus $\mathcal{T}_{3,9}$	Declared directional distinction is not reproduced	Reject H_{aniso}
Interference selection	Six-terminal-pair transfer pattern	C3–C12 / C9–C6 are not selectively suppressed in the declared window	Reject H_{OI}
Speed ordering	Prospectively defined $\tau_{12,6}, \tau_{3,9}$	$\tau_{12,6} < \tau_{3,9}$ is not reproduced	Reject H_{τ}
Direct isolation	$\eta(\xi; \omega)$	$\eta_{\text{op}} \geq \eta_{\text{max}}$	Reject H_{iso}
Control preparation	$\Delta S_m(t)$ under declared C3–C9 preparation histories	Control histories fail to prepare distinguishable proxy-defined states	Reject H_{prep}
Control modulation	$G_{x \rightarrow y}(\xi; \omega)$	Contemporaneous gain below G_{min} throughout the operating family	Reject H_{ctrl}
Independent hidden state	$\Delta S_m(t)$	No proxy distinction above S_{min}	Reject H_{state}
Retained state	$\Delta S_m(t)$ after matching present conditions	Proxy distinction does not persist over the declared interval	Reject H_{mem}
Hidden-to-readout coupling	$M_{A_y}(t)$ conditioned on proxy-defined state	Distinct retained states never alter readout beyond M_{min}	Reject H_{read}
Array realization	Crosspoint selectivity and η_{array}	Array leakage/sneak-path metric exceeds threshold	Reject H_{array}
Cognitive interpretation	External behavioral/biological observables	No reproducible latent distinction affects later reportability	Reject interpretation for that target domain

Table 1: Layer-localized falsification matrix. Failure of a downstream realization or interpretation claim does not automatically invalidate the protected mathematical foundation.

the candidate crossbar realization claim, and

$$\Theta_{\text{cog}}$$

the biological cognitive-adequacy claim. Their prerequisite families are deliberately different.

For the mathematical claim, relevant premises are the inherited full realization, the explicit channel definitions, and the stated bounded-operator hypotheses. The core-graph claim is conditional on the explicitly stated standard chrysene graph and ideal nearest-neighbor Hückel model. The neuromorphic consistency claim is a finite model-existence statement built from the certified mathematical DCIT structures together with explicitly declared response-scale and interpretation data. For the physical claim, additional premises include directional addressing, physical directional anisotropy, realized-junction interference selection, a persistent state degree of freedom, nonzero modulation, readable retention, bounded direct leakage, and any claimed speed ordering. For the biological cognitive claim, a formal interpretation and independent domain-evidence certificate are additionally required.

Let

$$\mathfrak{P}(\Theta)$$

denote the family of recorded finite proof or validation routes for Θ .

Definition 12.1 (Route support). For a target Θ and a recorded route $\mathbf{r}$, let

$$\text{Supp}(\mathbf{r}; \Theta)$$

denote the finite primitive premise set used by that route. The aggregate support is

$$\text{Supp}(\Theta) := \bigcup_{\mathbf{r} \in \mathfrak{P}(\Theta)} \text{Supp}(\mathbf{r}; \Theta).$$

Different routes may therefore have different support sets even though the aggregate support $\text{Supp}(\Theta)$ is uniquely defined once the recorded route family is fixed.

12.2 Recorded routes and deficits

For a route

$$\mathbf{r} \in \mathfrak{P}(\Theta),$$

let

$$\text{AdmReq}(\mathbf{r}; \Theta)$$

be the finite set of admission-requiring primitive premises used by that route.

For each such premise p , define a certification indicator

$$\text{PremCert}(p; \Theta) \in \{0, 1\}$$

according to the rule appropriate to its logical type. Mathematical premises require proof or explicit definition; physical premises require the declared physical evidence standard; cognitive-domain premises require external domain evidence.

Definition 12.2 (Route deficit). The certification deficit of a route r for Θ is

$$\Gamma(r; \Theta) := \{p \in \text{AdmReq}(r; \Theta) : \text{PremCert}(p; \Theta) = 0\}.$$

This definition mirrors the route-deficit construction of the certified-growth framework. A deficit records exactly the primitive support that remains uncertified for that route; premises retained explicitly as assumptions of a conditional theorem need not be misreported as established facts.

12.3 Certification frontier

Define the family of route deficits

$$\mathfrak{D}(\Theta) := \{\Gamma(r; \Theta) : r \in \mathfrak{P}(\Theta)\}.$$

For a family of sets $\mathcal{A}$, let

$$\text{Min}_{\subseteq} \mathcal{A}$$

denote its inclusion-minimal members.

Definition 12.3 (Certification frontier). If

$$\mathfrak{P}(\Theta) \neq \emptyset,$$

define

$$\text{Front}(\Theta) := \text{Min}_{\subseteq} \mathfrak{D}(\Theta).$$

The frontier is an antichain: no frontier member properly contains another. It need not have a unique member because several logically different routes may be equally close to complete certification.

Proposition 12.4 (Frontier non-emptiness). *If at least one finite route to Θ has been recorded, then*

$$\text{Front}(\Theta) \neq \emptyset.$$

Proof. Choose any finite route deficit. If it is not inclusion-minimal, choose a strictly smaller deficit from the same family and continue. Because the initial deficit is finite, the descending chain terminates at a minimal member. $\square$

Proposition 12.5 (Empty frontier criterion). *If*

$$\emptyset \in \text{Front}(\Theta),$$

then at least one recorded route to Θ contains no uncertified admission-requiring premise. Conversely, if some recorded route has empty deficit, then

$$\text{Front}(\Theta) = \{\emptyset\}.$$

Proof. The first statement is immediate from the definition. For the converse, the empty set is contained in every deficit and is therefore the unique inclusion-minimal member once it occurs. $\square$

12.4 Route-external readiness

A route deficit records missing primitive certification but does not encode every review condition. Following the certified-growth framework, write

$$\text{ExtReady}(\Theta) = 1$$

when all certification requirements applicable to Θ that are not represented by primitive route premises have passed. These include, as applicable, mathematical well-formedness, protected concordance, provenance completeness, validity of any formal interpretation map, and correctness of claim scope [Sch26b].

Definition 12.6 (Certificate-complete route). A recorded route r to Θ is certificate-complete when

$$\Gamma(r; \Theta) = \emptyset$$

and

$$\text{ExtReady}(\Theta) = 1.$$

Proposition 12.7 (Frontier criterion with route-external readiness). *Assume $\mathfrak{P}(\Theta) \neq \emptyset$. Under*

$$\text{ExtReady}(\Theta) = 1,$$

a certificate-complete recorded route exists if and only if

$$\emptyset \in \text{Front}(\Theta).$$

Proof. The empty-frontier criterion identifies an empty recorded route deficit. Route-external readiness supplies the remaining certification requirements by definition. The converse is immediate. $\square$

12.5 Mathematical frontier

For the manuscript-relative mathematical claim, one convenient route is

$$\begin{aligned} &\mathfrak{R} + \text{explicit channel definitions} \\ &\quad + \text{bounded realization hypotheses} \\ &\implies \Theta_{\text{math}}. \end{aligned}$$

The relevant mathematical premises are either inherited from Article 39, explicitly defined, or stated as hypotheses of the conditional theorems. The explicit finite witness in Appendix A additionally establishes non-vacuity of the nontrivial memory construction.

The targeted post-review verification of the repaired controlling manuscript checked the observation-space/algebra separation, the nonzero-coupling hypotheses, lattice-coordinate typing, memory-kernel wording, norm-weighted interaction strengths, crossbar indexing, and the proof-preserving cognitive-interpretation contract. No new mathematical inconsistency was introduced by those repairs. Accordingly, for the manuscript-relative mathematical claim under its explicitly stated hypotheses, at least one recorded route has empty mathematical deficit:

$$\emptyset \in \text{Front}(\Theta_{\text{math}}).$$

By the empty-frontier criterion,

$$\text{Front}(\Theta_{\text{math}}) = \{\emptyset\}.$$

Remark 12.8 (Scope of mathematical certification). The empty mathematical frontier is manuscript-relative and hypothesis-relative. It establishes a premise-complete recorded mathematical route. Following the targeted mathematics and concordance verifications, the route-external conditions applicable to Θ_{math} —well-formedness, protected concordance, provenance, interpretation typing where used, and scope—have also been checked. Accordingly,

$$\text{ExtReady}(\Theta_{\text{math}}) = 1,$$

so the recorded mathematical route is certificate-complete in the manuscript-relative sense. This does not certify the physical realization hypotheses or the cognitive domain-evidence claim, whose frontiers remain nonempty below.

12.6 Core-graph and neuromorphic-consistency frontiers

Appendix D contains two distinct levels of result. Conditional on the explicitly stated 18-vertex standard chrysene graph and nearest-neighbor Hückel Hamiltonian, the bipartition, graph distances, inverse-adjacency matrix, terminal amplitude hierarchy, and same-sublattice interference zeros are mathematical consequences of the declared model. Those conditional mathematical

results therefore have a recorded premise-complete route:

$$\text{Front}(\Theta_{\text{core}}) = \{\emptyset\}$$

relative to the stated ideal model.

This status does not certify the realized-junction claims

$$H_{\text{aniso}}, \quad H_{\text{QI}}, \quad H_{\tau}.$$

Directional anisotropy in a substituted/contacted junction, persistence of the selective interference pattern after symmetry breaking and environmental coupling, and the response-time ordering remain empirical realization premises.

Section 9 proves a separate finite model-existence result. The corresponding mathematical consistency claim Θ_{NM} is certified as a conditional existence theorem: there is a finite realization in which common substrate, distinguishable pathways, unequal selectable response scales, persistent hidden state, bounded nonzero cross-talk, state-mediated influence, and later accessibility coexist. The neuromorphic interpretation map Φ_{NM} is therefore mathematically admissible on its declared fragment. Its biological adequacy is not certified by that existence proof.

12.7 Physical frontier

The physical claim requires additional realization premises. For a mechanistically resolved single-crosspoint realization write

$$\begin{aligned} \mathcal{H}_{\text{phys}} := \{ &H_{\partial\pi}, H_{\text{chem}}, H_{\text{addr}}, H_{\text{prop}}, H_{\text{aniso}}, H_{\text{QI}}, H_{\text{XB}}, \\ &H_{\text{prep}}, H_{\text{ctrl}}, H_{\text{state}}, H_{\text{mem}}, H_{\text{read}}, H_{\text{iso}} \}. \end{aligned}$$

A finite-array claim additionally requires

$$H_{\text{array}},$$

a speed-ordered realization additionally requires H_{τ} , and a Stark-specific implementation may additionally require the optional field-window premise H_{field} .

These are proposed experimental or realization hypotheses rather than completed experimental certificates. Therefore a natural current deficit for the mechanistically resolved physical claim satisfies

$$\Gamma(\mathfrak{r}_{\text{phys}}; \Theta_{\text{phys}}) \supseteq \mathcal{H}_{\text{phys}}.$$

If no independent state proxy is available, the manuscript permits the weaker operational history route based on H_{hist} , but that route does not independently certify the separate H_{mem} and H_{read} premises.

A future experiment may shrink the deficit one premise at a time. For example, chemical characterization may certify H_{chem} without certifying directional addressability; direction-resolved addressing may certify H_{addr} without certifying physical anisotropy, interference selection, or retention; a static transfer hierarchy may support H_{aniso} while leaving H_{τ} open; an independent state proxy may certify H_{state} without yet establishing

state-dependent readout; and a single-node realization may succeed while H_{array} remains open. This premise-local structure is one reason to use a frontier rather than a single scalar confidence score.

Remark 12.9 (Domain-review status versus empirical frontier). The chemistry/physical-domain review gate asks whether the realization claims are chemically and physically admissible as stated and correctly conditionalized. It does not experimentally certify the realization premises themselves. For the pre-neuromorphic physical overlay examined in the completed domain-review cycle, the manuscript recorded

$$H = 1$$

for domain admissibility while keeping the empirical physical frontier open. The R43 enlargement adds the distinct realization claims H_{aniso} , H_{QI} , and H_{τ} . They are explicitly conditionalized here and remain empirically open; extending the formal domain-review predicate $H = 1$ to those new claims requires the corresponding renewed chemistry/physical-domain review rather than inheritance by notation.

12.8 Cognitive frontier

Let

$$H_{\text{cog}}$$

denote the Cognitive Dual-Channel Interpretation Hypothesis and

$$E_{\text{cog}}$$

a domain-evidence certificate for a specified target system. Then the cognitive adequacy claim requires at least

$$\{H_{\text{cog}}, E_{\text{cog}}\}.$$

At present,

$$\Gamma(\mathbf{r}_{\text{cog}}; \Theta_{\text{cog}}) \supseteq \{H_{\text{cog}}, E_{\text{cog}}\}.$$

The crossbar experiment cannot remove E_{cog} from this deficit because device evidence and cognitive-domain evidence have different evidentiary types. The map Φ_{NM} of Section 9 is mathematically admissible as an interpretation of a speed-ordered DCIT realization, but its existence does not certify E_{cog} .

12.9 Non-substitutability of review dimensions

Let

$$M, \quad C, \quad H$$

denote the mathematical, concordance, and chemistry/physical-domain review predicates. The admissible foundational region is

$$F_{\text{adm}} = M \cap C \cap H.$$

Success in two dimensions cannot compensate for failure in the third. In particular:

$$M = 1, \quad C = 1, \quad H = 0$$

describes mathematically coherent and framework-concordant mathematics whose physical interpretation is not admissible as stated; it is not a “two-thirds pass.” Likewise, successful chemistry cannot repair an invalid proof.

12.10 Review provenance

The review process applied during drafting identified and repaired issues involving new-axiom inflation, symbol collisions, insufficient typing of open-system generators, conflation of channel path metrics with the inherited admissibility metric, hard-coded informational roles, physical overinterpretation of tensor products, temporal language in the reachability order, overstatement of concordance, exact-zero channel assumptions, and the initial placement of memory directly on the C3–C9 pathway. Appendix F records those actions and their targeted verification status.

The revision history is deliberately not erased from the manuscript-development record. A criticism may be resolved in the final text while remaining historically relevant as evidence of what was checked and changed.

12.11 Conditionalization as a certification operation

Assume throughout this subsection a classical proof calculus satisfying the deduction theorem. An uncertified realization premise need not force deletion of every theorem that depends on it. Instead, the claim can be conditionalized.

Proposition 12.10 (Conditionalization of an open premise). *Suppose a formally valid proof derives Q from certified premises Γ together with an uncertified premise p . Then the manuscript may retain the conditional theorem*

$$p \implies Q$$

without certifying p .

Proof. The proof establishes the implication from the combined support. Reclassifying p as an explicit hypothesis preserves the mathematical derivation while removing the false assertion that p has already been independently established. $\square$

This operation is used repeatedly in the present manuscript. The physical crossbar statements are conditional on the realization family $\mathcal{H}_{\text{phys}}$ defined above; H_{array} and H_{field} enter only for the corresponding stronger claims. The cognitive statements remain conditional on H_{cog} and external domain evidence.

12.12 Cross-track targeted verification

After the separate mathematics, concordance, and chemistry/physical-domain review gates passed, the assembled controlling manuscript was checked for inconsistencies introduced by combining their repairs. Three cross-track issues were identified and repaired: the generalized direction-resolved transfer observable was required to be a nonnegative common-scale magnitude before entering a leakage ratio; the control-to-hidden-state preparation claim was separated from contemporaneous control-to-readout modulation; and the physical frontier/protocol were updated to carry that distinction consistently.

The resulting repairs do not alter the proved mathematical theorems, the Article-41 protected-concordance certificate, or the conditional status of the physical and cognitive claims. Accordingly, the final targeted verification predicate for the review sequence is recorded as

$$V_{\text{cross}} = 1.$$

This statement concerns internal consistency of the assembled manuscript after the recorded repairs. It does not close the empirical physical frontier or the cognitive domain-evidence frontier.

12.13 Review gate after neuromorphic integration

A predecessor version of the manuscript completed the sequence

$$M \longrightarrow C \longrightarrow H \longrightarrow V_{\text{cross}} \longrightarrow \text{meta-review},$$

and subsequently closed its publication-stage minor revisions. The present controlling version is substantively enlarged by the corrected four-terminal core-graph/Hückel analysis of Appendix D and by the neuromorphic consistency construction of Section 9. The predecessor post-meta closure therefore is not automatically inherited by the new claims.

The present integration has undergone a targeted internal consistency check of the added material. The 18-vertex graph, bipartition, terminal parities, inverse-adjacency restriction, ideal amplitude hierarchy, and selective interference zeros were recomputed; the neuromorphic model-existence proof was checked for type consistency with the DCIT finite witness; static transmission magnitude was kept separate from the optional response-time hypothesis H_τ ; and all cognitive mappings were kept at interpretation level. Denote this targeted integration predicate by

$$V_{\text{NM}} = 1.$$

This targeted status certifies internal integration of the new conditional mathematics as stated. It does not certify H_{aniso} , H_{QI} , H_τ , or biological domain adequacy. Those remain members of the appropriate empirical frontiers. If the journal's

full orthogonal-constraint workflow is applied to the present enlarged version, the final meta-review should treat this version—including Section 9, Appendix D, and revision R43—as the controlling manuscript.

13 Discussion

13.1 What is mathematically new in this extension

The DCIT construction does not claim novelty for crossed products, trace-class projection methods, observability algebras, Alexandrov spaces, or Heyting algebras individually. Its proposed contribution is their organized use over a fixed Chrysene realization to distinguish direct channel transport, persistent hidden state, accessible readout, and contextual information semantics.

The key architectural distinction is

$$\text{direct leakage} \neq \text{hidden-state modulation}.$$

A small off-diagonal transport amplitude does not force a small causal influence of the hidden state on the readout. The three-sector model makes this difference explicit by separating the C3–C9 control pathway, a persistent state factor, and the C12–C6 readout pathway.

13.2 Relation to the revised Chrysene foundation

The extension preserves the controlling separation between canonical molecular specificity and downstream model plurality. Article 39 expressly allows the same canonical chrysene framework to support different physical Hamiltonians, contact operators, environmental states, and response models, and warns that geometric directional inequivalence does not itself establish quantum-channel isolation or transfer anisotropy [Sch26d]. DCIT therefore treats every electronic, dynamical, and cognitive specialization beyond the inherited physical motif spaces as explicit overlay data.

Likewise, the new four-state channel lattice

$$\mathbb{B}_{\text{ch}} \simeq \mathcal{P}(\{x, y\})$$

is independent of the inherited four-state dual-cipher/registry alphabet. The construction does not solve the deferred $GF(4)$ problem; it creates a different quaternary object with a fully specified Boolean operation law.

13.3 Role of intuitionistic semantics

The intuitionistic layer is deliberately modest. It does not state that human reasoning is intuitionistic or that consciousness is a topos. It records a mathematically natural fact: persistent propositions on an oriented reachability poset form a Heyting

rather than necessarily Boolean algebra. This supplies a language in which locally or contextually valid information need not be globally decidable. Under the cognitive interpretation, reportability may be represented by an additional nucleus, but that mapping remains interpretive.

13.4 Relation to the CohBit program

The CohBit paper supplies a useful candidate device narrative: C12–C6 as a longitudinal data path, C3–C9 as a control path, electrostatic modulation, a vertically coupled retained gate state, and crossbar scaling [Sch26c]. The present work narrows several of its stronger assumptions. Exact physical channel orthogonality is replaced by a measured leakage ratio over the declared operational-state family; ambient-temperature ballistic coherence is not required by the abstract information topology; and persistent memory is placed in a distinct hidden-state factor rather than identified directly with a directional wire. In the CohBit-inspired specialization the primary hidden-state candidate is retained charge or electrostatic occupation. Configurational, redox, or other memory mechanisms remain separate possible DCIT realizations rather than asserted properties of substituted chrysene.

These changes make the present theory easier to falsify locally. A device can fail the isolation threshold while the mathematical topology remains intact; a retention mechanism can fail while directional transport remains meaningful; and a successful device can exist without validating any cognitive interpretation.

13.5 Dual processing does not require dual matter

The bipartite/Hückel analysis of Appendix D changes the physical picture in an important way. The two nominal channels are not disjoint molecular wires. They are terminal-defined response functions of one delocalized chrysene π network. Distinguishability can therefore arise from terminal connectivity, interference structure, transfer amplitude, state-dependent perturbation of the common substrate, and temporal response rather than from spatial separation of the underlying electrons.

This observation gives the neuromorphic analogy its central structural content:

dual processing does not require dual matter.

A single physical substrate may support rapid feedforward response, slower state-dependent integration, selective direct leakage, and indirect state-mediated influence at the same time.

The comparison with biological information processing remains structural rather than homologous. Masked-priming studies show that information unavailable to immediate report can nevertheless influence later behavior [VVR09], and contemporary reviews document substantial sensory processing without awareness while emphasizing limits on strong conclusions about

unconscious representation [MD22]. Within the Global Neuronal Workspace framework, conscious access is associated with recurrent amplification and sustained global availability rather than with an initial feedforward sweep alone [Mas+20]. GNW is one theoretical framework rather than a settled account of consciousness, but these literatures motivate the abstract distinction among rapid automatic processing, persistent state-dependent integration, and later reportable access.

The analogy should therefore not be written as a numerical identification between a behavioral priming effect and a molecular transmission coefficient. The defensible correspondence is qualitative:

small direct feedthrough
+
nonzero indirect state-mediated influence

The quantities χ_{dir} and $\chi_{\text{state}}(T)$ introduced in Section 9 are engineering/model observables, not measured biological “cross-talk coefficients.”

The optional stacked extension of Section 9.9 and Appendix E enlarges this realization class without changing the single-layer theory. A second cofacial node can supply additional state and field-control degrees of freedom, including a candidate retained interlayer electrostatic state, while coherent interlayer hybridization remains an optional specialization rather than a prerequisite. Thus stacking enlarges capability but is not required by the core DCIT or neuromorphic constructions.

13.6 Limits of the cognitive interpretation

The formal cognitive mapping uses reportability and delayed causal influence rather than attempting to define phenomenal experience. It therefore addresses an operational distinction among rapid automatic/feedforward processing, latent or state-dependent integration, and later accessibility. The terms *conscious* and *nonconscious* are secondary labels within the formal interpretation, not conclusions about neural anatomy, subjective experience, or molecular mechanisms in the brain. In particular, conscious access belongs to the integrated process culminating in reportability, not to C3–C9, C12–C6, or any other molecular channel in isolation.

The strongest empirical claim that could eventually emerge would require a separately specified biological target, measurements demonstrating an accessible/latent decomposition with the predicted causal structure, and evidence that the DCIT interpretation outperforms relevant alternative models. None of those empirical steps is supplied by the mathematical construction alone.

13.7 Future mathematical directions

Natural extensions include unbounded Lindblad generators, process-tensor formulations of history dependence, higher-rank

graph or product-system algebras for multi-channel transport, spectral invariants of the channel quotient spaces, and a fully internal topos-theoretic treatment of reportability. These directions should remain downstream of the present typed core rather than being imported as additional foundational assumptions.

14 Conclusion

This paper has constructed a dual-channel information topology as a model-relative extension of the twelve-axiom Chrysene Formalism. The construction begins with no assumption that either distinguished molecular direction carries a cognitive role. Instead, the C3–C9 and C12–C6 structures generate mathematical channel objects; physical information roles and cognitive interpretations are added only through explicitly typed realization and interpretation hypotheses.

The mathematical result begins with channel subgroups

$$L_x = \mathbb{Z}e_x, \quad L_y = \mathbb{Z}e_y,$$

whose inclusion structure yields the four-element Boolean lattice

$$\mathbb{B}_{\text{ch}} \simeq \mathcal{P}(\{x, y\}).$$

This object is independent of the inherited structural alphabet $\Sigma_C = \mathbb{Z}_2^{dc} \times \mathbb{Z}_2^z$ and carries no field structure by cardinality alone.

At the transport level, the direct-sum Hamiltonian

$$H_{\varepsilon}^{\text{tr}} = \begin{pmatrix} H_x & \varepsilon K \\ \varepsilon K^* & H_y \end{pmatrix}$$

produces the finite-time estimate

$$\|\Pi_y e^{-itH_{\varepsilon}^{\text{tr}}/\hbar} \Pi_x\| \leq \frac{|\varepsilon| |t| \|K\|}{\hbar}.$$

Thus channel isolation is represented quantitatively rather than postulated absolutely.

The complete information architecture introduces a persistent internal-state sector and takes

$$\mathcal{H}_{\text{XB}} = \mathcal{H}_x \otimes \mathcal{H}_m \otimes \mathcal{H}_y = \mathcal{H}_{\text{hid}} \otimes \mathcal{H}_{\text{acc}}.$$

The accessible projection

$$\mathbf{P}_{\text{acc}} \rho = \sigma_{\text{hid}} \otimes \text{Tr}_{\text{hid}} \rho$$

yields the exact reduced equation

$$\frac{dp}{dt} = \mathbf{P}_{\text{acc}} \mathcal{L} \mathbf{P}_{\text{acc}} p(t) + \int_0^t \mathcal{K}(t-s) p(s) ds + I(t).$$

Hence complete Markovian or Hamiltonian dynamics can induce history dependence in the accessible readout after elimination of the hidden block.

The information distinction is operational. Two complete states may satisfy

$$D_{\text{acc}}(\rho, \sigma) = 0$$

while

$$D_{\text{full}}(\rho, \sigma) > 0.$$

The finite-time observation space determines whether such an initially hidden difference is distinguishable from the declared readout history, while the generated dynamical algebra records multiplicative algebra growth. In finite dimension, full state separation occurs exactly when

$$\mathfrak{B}_T = B(\mathcal{H}_{\text{XB}})_{\text{sa}}.$$

The positive channel-translation semigroups also define oriented reachability orders whose Alexandrov topologies form complete Heyting algebras. The explicit failure of excluded middle shows that this intuitionistic layer is generated by the reachability structure rather than imposed as a philosophical premise.

The proposed physical realization assigns C3–C9 to control access, $\mathcal{H}_m$ to persistent internal state, and C12–C6 to readout. Appendix D shows that, in the ideal nearest-neighbor bipartite/Hückel core model, the two intended terminal pairs are both transmission-permitted but electronically inequivalent, while selected cross-terminal amplitudes are interference-suppressed and others remain finite. The resulting microscopic picture is therefore not two isolated wires but

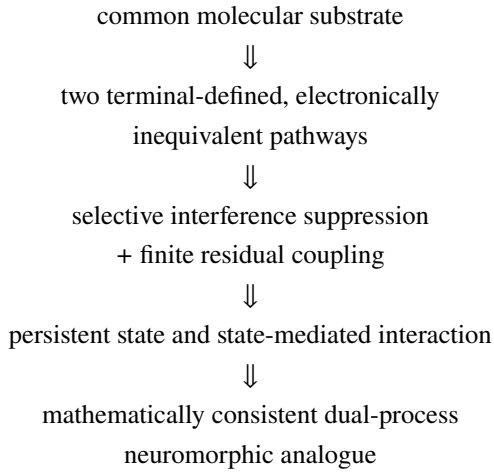
one delocalized chrysene π substrate $+$ two terminal-defined inequivalent pathways $+$ selective interference filtering
--

The paper further proves that a finite model can support unequal response scales, persistent hidden state, bounded nonzero direct cross-talk, nonzero state-mediated later influence, and delayed emergence into accessibility on that common substrate. Under the optional empirical speed-ordering hypothesis

$$H_{\tau} : \quad \tau_{12,6} < \tau_{3,9},$$

the neuromorphic interpretation may associate C12–C6 with a rapid automatic/feedforward analogue and the C3–C9 plus persistent-state architecture with a slower state-dependent integrative analogue. Conscious or reportable access is not assigned to either molecular channel in isolation; it is the composite dynamical event in which previously latent information becomes accessible.

The final conceptual hierarchy is therefore



At every stage the inferential boundaries remain explicit:

mathematical consistency $\nRightarrow$ physical realization,

and

physical realization $\nRightarrow$ biological cognitive identity.

Static transmission magnitude likewise does not imply a transport-time ordering.

The single-layer architecture remains sufficient for these results. An optional stacked two-node extension, developed in Appendix E, consistently enlarges the available state and control space through candidate interlayer memory and field gating, with coherent interlayer effects remaining optional. This extension adds capability without becoming a premise of the core theory.

The principal result is therefore not that a molecular axis has been proved to be “conscious” or “subconscious.” It is the construction and consistency proof of a class of systems in which distinct fast and slow information processes, latent state, later accessibility, and bounded cross-influence can coexist on a single physical substrate. The cognitive question is thereby posed in a framework that does not identify topology, transport, memory, cognition, and empirical mechanism by definition.

A An Explicit Finite DCIT Model

This appendix constructs a finite operator realization of the DCIT extension. Its purpose is mathematical non-vacuity rather than first-principles device prediction.

A.1 Inherited finite directional motifs

Use the explicit Article-39 Axiom-A₁₀ witness, for which each left, bridge, and right terminal space is $\mathbb{C}$. Thus for $\alpha \in \{x, y\}$,

$$\mathcal{K}_\alpha \simeq \mathbb{C}^3,$$

with bounded self-adjoint Hamiltonian

$$H_\star = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}.$$

The inherited witness transfer is $T_\alpha^{(1)}(z) = z^{-1}$ for $z \neq 0$ [Sch26d].

Set

$$\mathcal{K}_x = \mathcal{K}_y = \mathbb{C}^3, \quad H_x = H_y = H_\star.$$

A.2 Direct leakage witness

Let

$$K = \kappa I_3$$

with $\kappa \geq 0$. On $\mathcal{K}_{\text{tr}} = \mathbb{C}^3 \oplus \mathbb{C}^3$, define

$$H_\varepsilon^{\text{tr}} = \begin{pmatrix} H_\star & \varepsilon \kappa I_3 \\ \varepsilon \kappa I_3 & H_\star \end{pmatrix}.$$

This is a finite-dimensional witness for both the ideal $\varepsilon = 0$ and residual-coupling regimes.

A.3 Persistent-state factor

Let

$$\mathcal{K}_m = \mathbb{C}^2$$

with Pauli matrices

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Then

$$\mathcal{K}_{\text{XB}} = \mathcal{K}_x \otimes \mathcal{K}_m \otimes \mathcal{K}_y \simeq \mathbb{C}^{18}, \quad \mathcal{K}_{\text{hid}} \simeq \mathbb{C}^6.$$

Let $P_{x,2} = |e_2\rangle\langle e_2|$ and define

$$V_{xm} := P_{x,2} \otimes \sigma_x.$$

Choose $g_{xm} \in \mathbb{R}$ and put

$$H_{\text{hid}} = H_\star \otimes I_2 + g_{xm} V_{xm}.$$

Let

$$\sigma_{\text{hid}} = I_6/6.$$

Then $[H_{\text{hid}}, \sigma_{\text{hid}}] = 0$, so the hidden reference state is stationary under the unitary hidden generator.

A.4 Explicit control-to-memory preparation

The control-path interaction also acts nontrivially on the persistent state. For

$$U_{xm}(\theta) := e^{-i\theta V_{xm}},$$

one has, on the control basis state $|e_2\rangle$,

$$U_{xm}(\theta)(|e_2\rangle \otimes |0\rangle) = |e_2\rangle \otimes (\cos \theta |0\rangle - i \sin \theta |1\rangle).$$

In particular,

$$U_{xm}(\pi/2)(|e_2\rangle \otimes |0\rangle) = -i|e_2\rangle \otimes |1\rangle.$$

Thus the finite witness contains an explicit control operation that changes the persistent-state factor before the hidden-to-readout interaction is applied.

A.5 Hidden-to-readout coupling

Let

$$B_y = |f_1\rangle\langle f_2| + |f_2\rangle\langle f_1|, \quad A_{\text{hid}} = I_3 \otimes \sigma_z,$$

and define

$$V_{\text{hy}} := A_{\text{hid}} \otimes B_y.$$

Set

$$\mathcal{V}(\rho) = -\frac{i}{\hbar} [V_{\text{hy}}, \rho],$$

$$\mathcal{L}_0(\rho) = -\frac{i}{\hbar} [H_{\text{hid}} \otimes I_y + I_{\text{hid}} \otimes H_\star, \rho],$$

and

$$\mathcal{L}_\lambda = \mathcal{L}_0 + \lambda \mathcal{V}.$$

A.6 Nonzero memory kernel

Let $P_{y,1} = |f_1\rangle\langle f_1|$, $P_{y,2} = |f_2\rangle\langle f_2|$. One checks

$$[B_y, [B_y, P_{y,1}]] = 2(P_{y,1} - P_{y,2}).$$

Because

$$\sigma_{\text{hid}} = I_6/6$$

and

$$\text{Tr } A_{\text{hid}} = 0,$$

one has

$$P_{\text{acc}} \mathcal{V} P_{\text{acc}} = 0.$$

At $t = 0$,

$$\mathcal{K}_\lambda(0) = \lambda^2 P_{\text{acc}} \mathcal{V} Q_{\text{hid}} \mathcal{V} P_{\text{acc}}.$$

Evaluating on $\sigma_{\text{hid}} \otimes P_{y,1}$ yields

$$\mathcal{K}_\lambda(0)(\sigma_{\text{hid}} \otimes P_{y,1}) = -\frac{2\lambda^2}{\hbar^2} \sigma_{\text{hid}} \otimes (P_{y,1} - P_{y,2}),$$

so $\mathcal{K}_\lambda(0) \neq 0$ whenever $\lambda \neq 0$.

A.7 Explicit hidden-state modulation

Define

$$\rho_{\text{hid}}^{(0)} = |e_1\rangle\langle e_1| \otimes |0\rangle\langle 0|,$$

$$\rho_{\text{hid}}^{(1)} = |e_1\rangle\langle e_1| \otimes |1\rangle\langle 1|.$$

Let

$$|\psi_y\rangle = (|f_1\rangle + i|f_2\rangle)/\sqrt{2}, \quad \rho_y = |\psi_y\rangle\langle\psi_y|,$$

and choose $A_y = P_{y,1}$. Then

$$\langle\psi_y|[B_y, A_y]|\psi_y\rangle = -i$$

and

$$\text{Tr}[(\rho_{\text{hid}}^{(0)} - \rho_{\text{hid}}^{(1)})A_{\text{hid}}] = 2.$$

Thus the hidden-state modulation condition is nonzero, and the two complete states generate distinct readout trajectories for sufficiently small nonzero times.

Theorem A.1 (Explicit finite DCIT witness). *There exists a finite-dimensional DCIT extension over the explicit Article-39 physical motif witness for which the direct leakage model is well defined, the three-sector crosspoint is nonzero, the accessible projection exists, the hidden-state memory kernel is nonzero, and distinct hidden states can influence readout while the separate direct-sum leakage model is evaluated at its ideal reference value $\varepsilon = 0$.*

Proof. All assertions were established by the preceding explicit matrices. $\square$

Remark A.2. The witness is an existence model, not a claim that these finite matrices are chemically realistic parameters of a synthesized device.

B Analytic Proof Details

B.1 Duhamel estimate

Let $H_\varepsilon^{\text{tr}} = H_0^{\text{tr}} + \varepsilon V_K$. Duhamel's identity gives

$$U_\varepsilon(t) - U_0(t) = -\frac{i\varepsilon}{\hbar} \int_0^t U_\varepsilon(t-s) V_K U_0(s) ds.$$

Because $\Pi_y U_0(t) \Pi_x = 0$,

$$\Pi_y U_\varepsilon(t) \Pi_x = -\frac{i\varepsilon}{\hbar} \int_0^t \Pi_y U_\varepsilon(t-s) V_K U_0(s) \Pi_x ds.$$

Moreover,

$$V_K^2 = \begin{pmatrix} KK^* & 0 \\ 0 & K^*K \end{pmatrix},$$

so $\|V_K\| = \|K\|$. Unitarity then yields

$$\|\Pi_y U_\varepsilon(t) \Pi_x\| \leq \frac{|\varepsilon| |t| \|K\|}{\hbar}.$$

B.2 Resolvent estimate

Let $R_\varepsilon(z) = (zI - H_\varepsilon^{\text{tr}})^{-1}$ and $R_0(z) = (zI - H_0^{\text{tr}})^{-1}$. For $z \in \rho(H_0^{\text{tr}})$, self-adjointness gives

$$\|R_0(z)\| = \delta(z)^{-1}.$$

If $|\varepsilon|\|K\| < \delta(z)$, then $\|\varepsilon V_K R_0(z)\| < 1$, so the Neumann criterion implies $z \in \rho(H_\varepsilon^{\text{tr}})$ and

$$\|R_\varepsilon(z)\| \leq [\delta(z) - |\varepsilon|\|K\|]^{-1}.$$

Using

$$R_\varepsilon - R_0 = \varepsilon R_\varepsilon V_K R_0$$

and $\Pi_y R_0 \Pi_x = 0$ gives the displayed cross-channel estimate in the main text.

B.3 Accessible projection and decoupled reduction

For

$$P_{\text{acc}}\rho = \sigma_{\text{hid}} \otimes \text{Tr}_{\text{hid}} \rho,$$

idempotence follows from $\text{Tr} \sigma_{\text{hid}} = 1$. The partial trace is trace-norm contractive, hence $\|P_{\text{acc}}\| \leq 1$, while equality holds because P_{acc} is the identity on its nonzero range.

If

$$\mathcal{L}_0 = \mathcal{L}_{\text{hid}} \otimes \text{id} + \text{id} \otimes \mathcal{L}_y$$

with $\mathcal{L}_{\text{hid}}(\sigma_{\text{hid}}) = 0$, then

$$\mathcal{L}_0(\sigma_{\text{hid}} \otimes \eta) = \sigma_{\text{hid}} \otimes \mathcal{L}_y \eta,$$

so $\text{Ran } P_{\text{acc}}$ is invariant and $Q_{\text{hid}} \mathcal{L}_0 P_{\text{acc}} = 0$. If $q \in \text{Ran } Q_{\text{hid}}$, then $\text{Tr}_{\text{hid}} q = 0$. Trace preservation gives $\text{Tr}_{\text{hid}}(\mathcal{L}_0 q) = 0$, hence $P_{\text{acc}} \mathcal{L}_0 Q_{\text{hid}} = 0$.

B.4 Exact memory equation

Writing $p = P_{\text{acc}}\rho$ and $q = Q_{\text{hid}}\rho$,

$$\dot{p} = P_{\text{acc}} \mathcal{L} P_{\text{acc}} p + P_{\text{acc}} \mathcal{L} Q_{\text{hid}} q,$$

$$\dot{q} = Q_{\text{hid}} \mathcal{L} P_{\text{acc}} p + Q_{\text{hid}} \mathcal{L} Q_{\text{hid}} q.$$

Variation of constants yields

$$\begin{aligned} q(t) &= e^{t Q_{\text{hid}} \mathcal{L} Q_{\text{hid}}} q(0) \\ &+ \int_0^t e^{(t-s) Q_{\text{hid}} \mathcal{L} Q_{\text{hid}}} Q_{\text{hid}} \mathcal{L} P_{\text{acc}} p(s) ds. \end{aligned}$$

Substitution gives the memory equation, kernel, and initial hidden contribution in Section 5.

If $\mathcal{L}_\lambda = \mathcal{L}_0 + \lambda \mathcal{V}$, the decoupled identities imply

$$P_{\text{acc}} \mathcal{L}_\lambda Q_{\text{hid}} = \lambda P_{\text{acc}} \mathcal{V} Q_{\text{hid}},$$

$$Q_{\text{hid}} \mathcal{L}_\lambda P_{\text{acc}} = \lambda Q_{\text{hid}} \mathcal{V} P_{\text{acc}}.$$

which proves the λ^2 kernel formula.

B.5 Finite-time observation and generated algebra

Let

$$\mathfrak{A}_{\text{acc}} = I_{\text{hid}} \bar{\otimes} B(\mathcal{H}_y)$$

and let

$$\tau_t(A) = U(t)^* A U(t).$$

Define the real observation space

$$\mathfrak{B}_T = \text{span}_{\mathbb{R}} \{\tau_t(A) : A = A^* \in \mathfrak{A}_{\text{acc}}, 0 \leq t \leq T\}$$

and the generated von Neumann algebra

$$\mathfrak{D}_T = W^* \{\tau_t(A) : A \in \mathfrak{A}_{\text{acc}}, 0 \leq t \leq T\}.$$

The first object controls state separation from expectation values of the declared readout family; the second records multiplicative algebraic closure.

Assume

$$d := \dim \mathcal{H}_{\text{XB}} < \infty.$$

If

$$\mathfrak{B}_T = B(\mathcal{H}_{\text{XB}})_{\text{sa}},$$

then equality of expectation values on $\mathfrak{B}_T$ implies equality of density operators.

Conversely, suppose

$$\mathfrak{B}_T \subsetneq B(\mathcal{H}_{\text{XB}})_{\text{sa}}.$$

Using the real Hilbert–Schmidt pairing on self-adjoint operators, choose nonzero

$$X = X^* \in \mathfrak{B}_T^\perp.$$

Since

$$I \in \mathfrak{B}_T,$$

one has

$$\text{Tr } X = 0.$$

Choose

$$0 < \epsilon < \frac{1}{d\|X\|}.$$

Then

$$\rho_\pm = \frac{I}{d} \pm \epsilon X$$

are positive density matrices, and for every $A \in \mathfrak{B}_T$,

$$\text{Tr}[(\rho_+ - \rho_-)A] = 2\epsilon \text{Tr}(XA) = 0.$$

Thus distinct states remain observationally equivalent whenever the finite-time observation space is proper.

The generated algebra may be strictly larger than $\mathfrak{B}_T$. Expectation values on the original generating family do not automatically determine expectation values of arbitrary products in $\mathfrak{D}_T$, so the two objects are kept separate throughout the manuscript.

B.6 Heyting implication

For $U, V \in O_S^+$, define

$$U \Rightarrow_S V = \{k : (\uparrow_S k) \cap U \subseteq V\}.$$

If $k \preceq_S \ell$, then $\uparrow_S \ell \subseteq \uparrow_S k$, so this set is upper. An upper set W satisfies $W \cap U \subseteq V$ iff $W \subseteq U \Rightarrow_S V$. Thus the displayed formula is the Heyting implication. Setting $V = \emptyset$ gives the pseudocomplement.

C Crossbar Experimental Protocol

C.1 Purpose and scope

This appendix specifies an operational protocol for testing a candidate physical realization of the DCIT architecture. It does not prescribe a unique chemical synthesis, linker, electrode material, substrate, temperature, or fabrication route. Those choices belong to the realization being tested.

The protocol distinguishes two levels of claim:

- (i) an *operational DCIT realization*, established by limited direct leakage, control-to-readout modulation, and matched-condition history dependence; and
- (ii) a *mechanistically resolved three-sector realization*, which additionally requires an independently calibrated hidden-state proxy and evidence that the proxy-defined retained state causally alters the readout.

A finite-array claim is a further level and requires an array-specific addressability and sneak-path test.

C.2 Prospective declaration

Before data collection specify:

$$I_\xi, \quad I_t, \quad \Omega_{\text{op}},$$

the operating-parameter, retention-time, and operational-state/environment windows, together with thresholds

$$\eta_{\text{max}} > 0, \quad G_{\text{min}} > 0, \quad S_{\text{min}} > 0, \quad M_{\text{min}} > 0.$$

For an array-level claim also specify

$$\eta_{\text{array,max}} > 0.$$

Measurement resolution, uncertainty handling, replication rules, and statistical decision criteria should be fixed prospectively.

C.3 Stage I: chemical and structural scope

Document the chemical realization supporting H_{chem} . The declared node must preserve the standard-locant substituted-chrysene framework and the contact/linker assignments used to identify C3–C9 and C12–C6. The characterization method is

realization-specific; the manuscript does not prescribe a unique structural assay.

If the realized chemistry no longer supports the claimed conjugated framework or the designated directional interfaces, the device is outside the scope of the proposed physical realization regardless of later electrical behavior.

C.4 Stage II: directional addressability

Verify H_{addr} by showing that the C3–C9 and C12–C6 interfaces can be interrogated separately to the prospectively declared resolution. The path assignment is

$$x : \text{C3} \leftrightarrow \text{C9}, \quad y : \text{C12} \leftrightarrow \text{C6}.$$

This assignment is a realization hypothesis and not an automatic consequence of molecular directional inequivalence.

C.5 Stage III: direction-resolved transfer

Measure or compute

$$\mathcal{T}_{xx}(\xi; \omega), \quad \mathcal{T}_{yy}(\xi; \omega), \quad \mathcal{T}_{xy}(\xi; \omega), \quad \mathcal{T}_{yx}(\xi; \omega)$$

over the declared operating family. In a coherent scattering realization these quantities may be transmission probabilities $T_{\alpha\beta}(E; \omega)$. In hopping, kinetic, or mixed regimes they may instead be direction-resolved conductance magnitudes, rates, current-response magnitudes, or other prospectively normalized nonnegative transfer measures. The four entries used in the leakage ratio must share a common normalization or common physical units. Signed primitive measurements must be converted by a prospectively declared magnitude, norm, or other nonnegative leakage functional before entering the ratio.

Define the leakage-test domain

$$\Omega_{\text{leak}} = \{(\xi, \omega) \in I_\xi \times \Omega_{\text{op}} : \mathcal{T}_{xx}(\xi; \omega) + \mathcal{T}_{yy}(\xi; \omega) > 0\}.$$

Require

$$\Omega_{\text{leak}} \neq \emptyset.$$

For $(\xi, \omega) \in \Omega_{\text{leak}}$, define

$$\eta(\xi; \omega) = \frac{\mathcal{T}_{xy}(\xi; \omega) + \mathcal{T}_{yx}(\xi; \omega)}{\mathcal{T}_{xx}(\xi; \omega) + \mathcal{T}_{yy}(\xi; \omega)},$$

and

$$\eta_{\text{op}} = \sup_{(\xi, \omega) \in \Omega_{\text{leak}}} \eta(\xi; \omega).$$

Operating points outside Ω_{leak} have no defined leakage ratio and cannot support H_{iso} .

The isolation criterion is

$$\eta_{\text{op}} < \eta_{\text{max}}.$$

C.6 Stage IV: directional anisotropy and interference selection

Using the complete terminal family, compare

$$\mathcal{T}_{3,9}, \quad \mathcal{T}_{12,6}, \quad \mathcal{T}_{3,6}, \quad \mathcal{T}_{3,12}, \quad \mathcal{T}_{9,6}, \quad \mathcal{T}_{9,12}$$

over a prospectively declared near-symmetry window. The physical anisotropy hypothesis H_{aniso} requires the declared C12–C6 versus C3–C9 distinction to be reproduced. The interference-selection hypothesis H_{QI} requires selective suppression of C3–C12 and C9–C6 relative to the intended responses within a prospectively specified tolerance. These realized-junction tests are distinct from the exact conditional graph/Hückel results of Appendix D.

C.7 Stage V: response-time ordering

If the speed-ordered realization is claimed, define the response-time observable before data collection. Depending on the regime, it may be a transient junction response, hopping mean-first-passage time, kinetic relaxation time, or coherent scattering delay. Then test

$$H_{\tau} : \quad \tau_{12,6} < \tau_{3,9}.$$

The static transmission proxy of Appendix D is not accepted as a substitute for this dynamical measurement.

C.8 Stage VI: control-to-readout modulation

Let u_x be the declared C3–C9 control input. Evaluate the readout response through

$$G_{x \rightarrow y}(\xi; \omega) = \frac{\partial \mathcal{T}_{yy}(\xi; u_x, \omega)}{\partial u_x}$$

when differentiable, or by a prospectively specified finite-difference estimator.

The modulation criterion is

$$\sup_{\substack{\xi \in I_{\xi} \\ \omega \in \Omega_{\text{op}}}} |G_{x \rightarrow y}(\xi; \omega)| > G_{\text{min}}.$$

C.9 Stage VII: preparation–readout protocol

Choose two preparation histories

$$u_x^{(0)}(t), \quad u_x^{(1)}(t), \quad -T_p \leq t \leq 0,$$

intended to prepare distinct candidate internal states. At $t = 0$, return the externally applied control variables to the same declared readout condition for both histories. Thus any later difference is not attributed to a difference in the contemporaneous applied control value.

C.10 Stage VIII: independent retained-state proxy

For a mechanistically resolved three-sector claim, use an independently calibrated hidden-state proxy S_m that is not identical to the principal C12–C6 readout. Define

$$\Delta S_m(t) = |\langle S_m \rangle_1(t) - \langle S_m \rangle_0(t)|.$$

Hypothesis H_{state} requires reproducible state discrimination above

$$S_{\text{min}}.$$

The retained-state hypothesis H_{mem} requires that this state distinction remain above threshold for a nonzero part of the declared retention interval after present external conditions have been matched. Define

$$\tau_{\text{ret}} = \sup(\{t \in I_t : \Delta S_m(t) \geq S_{\text{min}}\} \cup \{0\}).$$

C.11 Stage IX: state-dependent readout

For a declared C12–C6 readout observable A_y , define

$$M_{A_y}(t) = |\langle A_y \rangle_1(t) - \langle A_y \rangle_0(t)|.$$

Hypothesis H_{read} requires independently distinguished retained states to alter the readout beyond

$$M_{\text{min}}$$

under the declared comparison protocol.

If no independent state proxy can be calibrated, the protocol may support only the weaker operational claim H_{hist} : different preparation histories produce different later readouts under matched present conditions. In that case the separate physical existence and readout coupling of a distinct hidden-state sector are not independently certified.

C.12 Stage X: matched-condition controls

At minimum compare:

- (i) different preparation histories followed by identical present control conditions;
- (ii) repeated trials of the same preparation history;
- (iii) a no-preparation or sham-preparation condition;
- (iv) a reset condition, where physically available;
- (v) polarity- or order-reversed histories when the device model predicts distinct state preparation; and
- (vi) where possible, a contemporaneous-control sweep with retained-state preparation held fixed, to separate direct feedthrough from retained-state effects.

C.13 Stage XI: finite-array realization

A single crosspoint does not certify a crossbar. For a finite-array claim, verify H_{array} by demonstrating preservation of row/column addressability, crosspoint-selective interrogation, and bounded unintended inter-node or sneak-path transfer. Define a prospective array leakage metric η_{array} and require

$$\eta_{\text{array}} < \eta_{\text{array},\text{max}}.$$

A failed array test does not invalidate a successful single-node realization.

C.14 Stage XII: optional environmental and field sweeps

Temperature, field strength, electrode configuration, packing geometry, substrate, dielectric environment, and other variables may be swept to map the realized operating region. The abstract theory does not require a particular temperature or ballistic regime.

For a Stark-specific implementation, also verify the optional premise H_{field} : the applied field window remains within the independently established reversible operating range of the realized device and below declared irreversible-change or dielectric-failure limits.

C.15 Layer-localized failure

Failure of chemical scope rejects H_{chem} . Failure of separate directional addressing rejects H_{addr} . Failure of the declared directional response distinction rejects H_{aniso} . Loss of the declared selective cross-terminal suppression pattern rejects H_{QI} for that junction. Failure of the prospectively defined ordering $\tau_{12,6} < \tau_{3,9}$ rejects H_{τ} . Excessive operational leakage rejects H_{iso} . Failure of declared C3–C9 preparation histories to prepare distinguishable proxy-defined states rejects H_{prep} for the mechanistically resolved claim. Absence of contemporaneous modulation rejects H_{ctrl} . Absence of independent hidden-state discrimination rejects H_{state} for the mechanistically resolved claim. Failure of that proxy distinction to persist rejects H_{mem} . Failure of proxy-defined state differences to alter readout rejects H_{read} . Excessive sneak-path or inter-node transfer rejects H_{array} for an array claim. These failures do not, by themselves, invalidate the abstract topology or protected Chrysene foundation.

C.16 Interpretive boundary

Successful completion of the physical protocol establishes only a device-level accessible–hidden architecture to stated tolerances and claim scope. It does not establish the Cognitive Dual-Channel Interpretation Hypothesis.

D Bipartite Core-Graph Analysis of the Four-Terminal Chrysene Node

D.1 Scope and logical status

This appendix examines whether the carbon-connectivity graph of the standard chrysene core supplies an intrinsic topological distinction among the four substituted terminal positions

$$\{C3, C6, C9, C12\}.$$

The analysis is intentionally restricted to the ideal nearest-neighbor π -electron graph. Connectivity-dependent quantum interference and Green-function descriptions of single-molecule transport provide the conventional molecular-electronics context for this idealization [Lam15; LL18]. It does not assert that the resulting graph amplitudes are identical to measured conductances of a substituted molecular junction.

Accordingly, three levels are distinguished throughout:

1. exact graph-theoretic statements concerning the chrysene carbon skeleton;
2. exact consequences of an ideal nearest-neighbor Hückel Hamiltonian on that graph; and
3. realization-level hypotheses concerning actual molecular conductance, propagation time, electrode coupling, and device cross-talk.

In particular, the present calculation does not replace the realization hypotheses H_{chem} , H_{addr} , H_{prop} , or H_{iso} of the main text.

D.2 The 18-vertex chrysene graph

Let

$$G_{\text{chr}} = (V_{\text{chr}}, E_{\text{chr}})$$

denote the carbon-connectivity graph of the standard chrysene framework, with vertex set

$$V_{\text{chr}} = \{1, 2, 3, 4, 4a, 4b, 5, 6, 6a, 7, 8, 9, 10, 10a, 10b, 11, 12, 12a\}.$$

The perimeter cycle is

$$\begin{aligned} &C12 - C12a - C1 - C2 - C3 - C4 - C4a - C4b \\ &\quad - C5 - C6 - C6a - C7 - C8 - C9 - C10 \\ &\quad - C10a - C10b - C11 - C12, \end{aligned}$$

together with the fused-ring edges

$$C12a - C4a, \quad C4b - C10b, \quad C6a - C10a.$$

The graph is bipartite. One admissible bipartition is

$$V_{\text{chr}} = A \sqcup B,$$

where

$$A = \{C1, C3, C4a, C5, C6a, C8, C10, C10b, C12\},$$

and

$$B = \{C2, C4, C4b, C6, C7, C9, C10a, C11, C12a\}.$$

The exchange $A \leftrightarrow B$ gives the equivalent alternative coloring. Consequently, the four substituted positions satisfy

$$C3, C12 \in A, \quad C6, C9 \in B.$$

D.3 Directional graph distances

The shortest $C3$ – $C9$ path is

$$C3 - C4 - C4a - C4b - C10b - C10a - C10 - C9,$$

and therefore

$$d_{\text{chr}}(C3, C9) = 7.$$

For the $C12$ – $C6$ pair,

$$d_{\text{chr}}(C12, C6) = 5.$$

There are three distinct shortest paths of this length:

$$C12 - C11 - C10b - C4b - C5 - C6,$$

$$C12 - C12a - C4a - C4b - C5 - C6,$$

and

$$C12 - C11 - C10b - C10a - C6a - C6.$$

Thus both intended directional pairs have odd graph distance:

$$d_{\text{chr}}(C3, C9) = 7, \quad d_{\text{chr}}(C12, C6) = 5.$$

Because every edge of a bipartite graph connects opposite sublattices, an odd path joins opposite sublattices. Hence

$$C3 \in A, \quad C9 \in B,$$

and

$$C12 \in A, \quad C6 \in B.$$

The two intended pathways are therefore both opposite-sublattice connectivities:

$$C3 \leftrightarrow C9 : A \leftrightarrow B,$$

$$C12 \leftrightarrow C6 : A \leftrightarrow B.$$

This observation is graph-theoretic. It does not by itself establish a conductance magnitude or a propagation time.

D.4 Cross-terminal parity structure

The remaining pairwise graph distances among the four substituted positions are

$$d_{\text{chr}}(C3, C12) = 4, \quad d_{\text{chr}}(C9, C6) = 4,$$

and

$$d_{\text{chr}}(C3, C6) = 5, \quad d_{\text{chr}}(C9, C12) = 5.$$

Accordingly,

$$C3 \leftrightarrow C12 \quad \text{and} \quad C9 \leftrightarrow C6$$

are same-sublattice connections, whereas

$$C3 \leftrightarrow C6 \quad \text{and} \quad C9 \leftrightarrow C12$$

are opposite-sublattice connections.

The resulting parity table is

terminal pair	sublattice relation	graph distance
$C3 - C9$	$A - B$	7
$C12 - C6$	$A - B$	5
$C3 - C12$	$A - A$	4
$C9 - C6$	$B - B$	4
$C3 - C6$	$A - B$	5
$C9 - C12$	$B - A$	5

Thus bipartiteness does not separate the two nominal channels into two disconnected molecular wires. Instead, it predicts a terminal-selective interference structure.

D.5 Ideal nearest-neighbor Hückel model

Let $\mathbf{A}_{\text{chr}}$ denote the adjacency matrix of G_{chr} and consider the ideal nearest-neighbor Hückel Hamiltonian

$$H_{\pi} = -t \mathbf{A}_{\text{chr}}, \quad t > 0.$$

All carbon on-site energies are set equal and only nearest-neighbor π coupling is retained.

With the bipartite ordering

$$V_{\text{chr}} = A \sqcup B,$$

the Hamiltonian has the block form

$$H_{\pi} = -t \begin{pmatrix} 0 & C \\ C^{\dagger} & 0 \end{pmatrix}.$$

For the present chrysene graph,

$$\det \mathbf{A}_{\text{chr}} \neq 0,$$

so the gap-center Green operator is well defined:

$$G_{\pi}(0) = (0 - H_{\pi})^{-1} = \frac{1}{t} \mathbf{A}_{\text{chr}}^{-1}.$$

Because the inverse of an invertible balanced bipartite matrix of this form is also block off-diagonal,

$$G_{\pi,AA}(0) = 0, \quad G_{\pi,BB}(0) = 0.$$

Hence same-sublattice terminal pairs possess exact gap-center zeros in this ideal model, while opposite-sublattice matrix elements need not vanish.

It follows immediately that

$$G_{3,12}(0) = 0, \quad G_{9,6}(0) = 0.$$

By contrast,

$$G_{3,9}(0), \quad G_{12,6}(0), \quad G_{3,6}(0), \quad G_{9,12}(0)$$

are not forced to vanish by bipartite symmetry.

D.6 Explicit four-terminal core matrix

Restrict the inverse adjacency matrix to the terminal ordering

$$(C3, C9, C12, C6).$$

Direct inversion gives

$$A_{\text{chr}}^{-1}|_{\{3,9,12,6\}} = \frac{1}{8} \begin{pmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 4 \\ 2 & 0 & 4 & 0 \end{pmatrix}.$$

Define the dimensionless gap-center core matrix

$$M := 8t \quad G_{\pi}(0)|_{\{3,9,12,6\}} = \begin{pmatrix} 0 & 1 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 4 \\ 2 & 0 & 4 & 0 \end{pmatrix}.$$

The relevant off-diagonal magnitudes are therefore

$$|M_{3,9}| = 1, \quad |M_{12,6}| = 4,$$

$$|M_{3,6}| = 2, \quad |M_{9,12}| = 2,$$

and

$$M_{3,12} = M_{9,6} = 0.$$

Thus the ideal graph predicts simultaneously:

1. nonzero gap-center propagation amplitude for both intended directional pairs;
2. a larger ideal core amplitude for $C12$ – $C6$ than for $C3$ – $C9$;
3. exact same-sublattice interference zeros for two of the four cross-terminal connections; and
4. finite amplitudes for the remaining two cross-terminal connections.

D.7 Ideal weak-contact transmission hierarchy

In a weak-contact, off-resonant realization in which the terminal coupling factors are comparable, the pairwise core transmission proxy follows the same connectivity/midgap logic used in molecular Green-function and magic-ratio approaches [LL18] and scales as

$$\mathcal{T}_{ij}^{(\text{core})} \propto |G_{\pi,ij}(0)|^2 \propto |M_{ij}|^2.$$

Under this restricted approximation,

$$\mathcal{T}_{3,9}^{(\text{core})} : \mathcal{T}_{3,6}^{(\text{core})} : \mathcal{T}_{9,12}^{(\text{core})} : \mathcal{T}_{12,6}^{(\text{core})} = 1 : 4 : 4 : 16,$$

while

$$\mathcal{T}_{3,12}^{(\text{core})} = \mathcal{T}_{9,6}^{(\text{core})} = 0$$

at the exact bipartite symmetry point.

In particular,

$$\frac{\mathcal{T}_{12,6}^{(\text{core})}}{\mathcal{T}_{3,9}^{(\text{core})}} = 16$$

under the equal-contact idealization.

The factor 16 is not introduced as an experimentally established conductance ratio. It is the square of the ideal gap-center Green-function amplitude ratio

$$\frac{|M_{12,6}|}{|M_{3,9}|} = 4.$$

Electrode self-energies, linker chemistry, Fermi-level alignment, substitution-induced site-energy shifts, next-nearest-neighbor couplings, dielectric polarization, vibronic coupling, dephasing, and contact asymmetry may alter this ratio substantially.

D.8 Interpretation of directional anisotropy

The graph calculation supplies a stronger statement than simple path-length comparison.

Both intended pathways are opposite-sublattice and are therefore compatible with finite gap-center transmission in the ideal model:

$$C3 \leftrightarrow C9, \quad C12 \leftrightarrow C6.$$

Nevertheless, they are not equivalent. The $C3$ – $C9$ pair has a single shortest path of graph length 7, whereas the $C12$ – $C6$ pair has graph length 5 and three distinct shortest paths. The exact inverse-adjacency calculation correspondingly gives

$$|M_{12,6}| > |M_{3,9}|.$$

This motivates the realization-level anisotropy hypothesis

$$H_{\text{aniso}} : \quad T_{12,6} > T_{3,9}$$

over a prospectively declared energy, bias, structural, and environmental operating window.

The graph calculation supports this inequality only in the ideal core model. The physical hypothesis must be evaluated using the direction-resolved observables of the main text.

D.9 Selective suppression of cross-talk

The bipartite graph does not imply complete physical isolation between the $C3$ – $C9$ and $C12$ – $C6$ directions.

Instead, it predicts selective interference filtering:

$$M_{3,12} = 0, \quad M_{9,6} = 0,$$

but

$$M_{3,6} = 2, \quad M_{9,12} = 2.$$

Thus two primitive cross-terminal amplitudes are exactly suppressed at the ideal symmetry point, whereas the other two remain finite.

The corresponding qualitative pattern is

connection	ideal gap-center status
$C3 \leftrightarrow C9$	allowed
$C12 \leftrightarrow C6$	allowed
$C3 \leftrightarrow C12$	interference-suppressed
$C9 \leftrightarrow C6$	interference-suppressed
$C3 \leftrightarrow C6$	allowed
$C9 \leftrightarrow C12$	allowed

Consequently,

bipartite topology $\nRightarrow$ complete channel isolation.

Rather,

bipartite topology
 $\Downarrow$
 terminal-selective interference structure

within the ideal Hückel realization.

This distinction is important for the direct-leakage hypothesis H_{iso} . A physical four-terminal device requires a full junction calculation or measurement including electrode self-energies and simultaneous terminal boundary conditions. Pairwise isolated-core amplitudes cannot by themselves certify the array-level leakage ratio η_{op} .

D.10 Shared internal carbon framework

The two nominal directional channels are not disjoint carbon wires.

The $C3$ – $C9$ shortest route contains the central edge

$$C4b - C10b,$$

and one of the $C12$ – $C6$ shortest routes also contains

$$C4b - C10b.$$

The appropriate microscopic interpretation is therefore

one delocalized chrysene π network
 supporting terminal-defined directional responses

rather than two electronically independent carbon pathways.

The graph-theoretic channel distinction arises from different terminal-to-terminal Green-function matrix elements and interference conditions. It does not require spatial separation of the underlying π electrons.

This shared internal network also leaves open the possibility that a perturbation applied through one terminal pair can modify the Green function associated with another terminal pair without requiring large direct carrier transfer between the corresponding external interfaces.

D.11 Implication for control and readout modulation

Let a realization-specific control or retained state perturb the core Hamiltonian according to

$$H_{\pi} \mapsto H_{\pi} + \Delta H_x.$$

The readout Green-function element becomes

$$G_{12,6}(E; \Delta H_x) = [E - H_{\pi} - \Delta H_x - \Sigma(E)]_{12,6}^{-1},$$

where $\Sigma(E)$ denotes the realization-specific environmental and electrode self-energy.

Because molecular interference features can be highly sensitive to site energies, electrostatic potentials, and coupling strengths, a small perturbation of the common π network may produce a measurable change in

$$T_{12,6}$$

even when a particular direct cross-terminal amplitude remains small.

The graph therefore motivates, but does not by itself prove, the physical regime

small selected direct cross-transmission
 $+$
 appreciable state-dependent modulation

This is compatible with the distinction made in the main text between direct leakage and hidden-state-mediated modulation.

D.12 Transmission magnitude versus propagation time

The present calculation concerns static Green-function amplitudes and corresponding transmission proxies. It does not establish a transport-time ordering.

In particular,

$$T_{12,6} > T_{3,9}$$

does not logically imply

$$\tau_{12,6} < \tau_{3,9}.$$

A propagation-time statement requires an independently defined dynamical observable. Depending on the physical regime, an appropriate quantity may include a transient current-response time, a hopping mean-first-passage time, a kinetic relaxation time, or a coherent scattering delay.

The directional-speed hypothesis should therefore remain separate:

$$H_\tau : \quad \tau_{12,6} < \tau_{3,9}.$$

The shorter graph distance

$$d_{\text{chr}}(C12, C6) = 5 < 7 = d_{\text{chr}}(C3, C9)$$

and the larger ideal core transmission amplitude make H_τ physically motivated, but do not prove it.

D.13 Experimental and computational test

The graph-level prediction can be tested prospectively by computing or measuring the complete direction-resolved transfer family

$$T_{\alpha\beta}(E; \omega), \quad \alpha, \beta \in \{3, 6, 9, 12\},$$

over a declared operating window.

A particularly informative first-principles test is a four-terminal DFT-NEGF calculation comparing

$$\begin{aligned} &T_{3,9}(E), \quad T_{12,6}(E), \quad T_{3,6}(E), \\ &T_{3,12}(E), \quad T_{9,6}(E), \quad T_{9,12}(E). \end{aligned}$$

The ideal bipartite calculation predicts, near the corresponding midgap symmetry region,

$$T_{3,12}, T_{9,6} \ll T_{3,9}, T_{12,6},$$

together with a tendency toward

$$T_{12,6} > T_{3,9}.$$

The prediction is falsified as a description of the realized junction if substitution, contacts, environment, or symmetry breaking remove these distinctions over the claimed operating window.

D.14 Conclusion

The standard 18-carbon chrysene graph supplies more structure than mere geometric inequivalence of the C3–C9 and C12–C6 directions.

Within the ideal nearest-neighbor bipartite model:

$$\begin{aligned} &C3 - C9 \text{ and } C12 - C6 \\ &\text{are both opposite-sublattice} \\ &\text{transmission-permitted connectivities} \end{aligned}$$

$$|M_{3,9}| : |M_{12,6}| = 1 : 4,$$

and therefore, under the equal weak-contact midgap approximation,

$$\mathcal{T}_{3,9}^{(\text{core})} : \mathcal{T}_{12,6}^{(\text{core})} = 1 : 16.$$

At the same time,

$$M_{3,12} = M_{9,6} = 0,$$

while

$$M_{3,6} = M_{9,12} = 2.$$

Thus the bare chrysene topology predicts neither complete isolation nor indiscriminate cross-talk. It predicts a structured four-terminal network in which the intended directions are both transport-capable, the two intended directions are intrinsically inequivalent in the ideal core model, and selected cross-terminal processes are suppressed by bipartite quantum interference.

These results provide a graph-theoretic and ideal molecular-electronic motivation for directional anisotropy and selective cross-talk suppression. Their extension to an actual substituted-chrysene junction remains conditional on explicit linker chemistry, electrode coupling, energy alignment, environmental state, and experimental or first-principles verification.

E Stacked Dual-Channel Dimer Architectures

E.1 Purpose and scope

The single-core DCIT construction is sufficient to establish the existence of two distinguishable terminal-defined pathways, a hidden state, bounded cross-talk, and later state-dependent accessibility. The purpose of the present appendix is different. We examine the larger realization class obtained by stacking two such molecular nodes and determine which additional computational functions become available without altering the logical structure of the single-core theory.

The stacked construction is therefore an extension and not a prerequisite:

$$\text{DCIT}_1 \subset \text{DCIT}_2.$$

Here DCIT_1 denotes the single-orientation realization and DCIT_2 the class of finite two-layer realizations defined below.

No statement in this appendix asserts that interlayer charge retention, coherent tunneling, Coulomb blockade, or room-temperature phase coherence has already been experimentally demonstrated for a substituted-chrysene dimer. Such mechanisms enter only through explicit realization hypotheses.

E.2 Single-layer reference architecture

Let a single substituted-chrysene node carry the three-sector state space

$$\mathcal{K}^{(1)} = \mathcal{K}_x \otimes \mathcal{K}_m \otimes \mathcal{K}_y,$$

where

$$x = C3 - C9, \quad y = C12 - C6.$$

The factors have the operational interpretation

$$\mathcal{K}_x = \text{control/integrative pathway},$$

$$\mathcal{K}_m = \text{persistent internal state},$$

and

$$\mathcal{K}_y = \text{accessible propagation/readout pathway}.$$

A schematic finite effective Hamiltonian is

$$H^{(1)} = H_x + H_m + H_y + \lambda_{xm} V_{xm} + \lambda_{my} V_{my} + \varepsilon V_{xy}.$$

The direct cross-channel scale is

$$g_{xy} = |\varepsilon| \|V_{xy}\|,$$

whereas the state-mediated coupling scales are

$$g_{xm} = |\lambda_{xm}| \|V_{xm}\|, \quad g_{my} = |\lambda_{my}| \|V_{my}\|.$$

The neuromorphically relevant regime is not exact physical decoupling but rather

$$g_{xy} \ll g_{xm}, g_{my},$$

whenever such an inequality is realized experimentally.

E.3 Two-layer state space

Let

$$A \quad \text{and} \quad B$$

denote two cofacially arranged substituted-chrysene nodes. For the embedding used below, layer A retains the full single-layer DCIT state space,

$$\mathcal{K}_A = \mathcal{K}_{x,A} \otimes \mathcal{K}_{m,A} \otimes \mathcal{K}_{y,A},$$

while the added layer contributes at least its terminal-defined channel space

$$\mathcal{K}_B = \mathcal{K}_{x,B} \otimes \mathcal{K}_{y,B}.$$

A realization may adjoin further internal state factors to layer B without changing the embedding argument.

The elementary dimer state space is

$$\mathcal{K}_D = \mathcal{K}_A \otimes \mathcal{K}_B.$$

The eight substituted terminal sites give an eight-terminal interface, but terminal number and transport-channel number are not identified. The latter is determined by the rank and spectral structure of the realized transmission operator.

Let

$$H_A, \quad H_B$$

be the intralayer Hamiltonians and let

$$V_{AB} = V_{AB}^*$$

describe the finite interlayer interaction. The dimer Hamiltonian is

$$H_D = H_A \otimes I_B + I_A \otimes H_B + \lambda_{AB} V_{AB}.$$

An externally applied field may additionally contribute

$$H_F(\mathbf{F}) = -\boldsymbol{\mu} \cdot \mathbf{F} - \frac{1}{2} \mathbf{F}^T \boldsymbol{\alpha} \mathbf{F}$$

or a more general realization-specific field operator.

Thus

$$H_D(\mathbf{F}) = H_D + H_F(\mathbf{F}).$$

E.4 Single-layer embedding

Proposition E.1 (Single-layer embedding). *Every finite single-layer DCIT realization admits an embedding into a finite two-layer realization.*

Proof. Let

$$\mathfrak{M}_1$$

be any finite single-layer realization on $\mathcal{K}_A$. Choose any nonzero finite-dimensional space $\mathcal{K}_B$ and fix a normalized reference state

$$\sigma_B \in \mathcal{S}(\mathcal{K}_B).$$

Set

$$\lambda_{AB} = 0.$$

Then

$$H_D = H_A \otimes I_B + I_A \otimes H_B$$

has factorized evolution. The map

$$\rho_A \mapsto \rho_A \otimes \sigma_B$$

is injective and preserves every observable acting on the A layer. Consequently the dimer realization contains an isomorphic copy of the original single-layer dynamics. Hence the stacked extension does not remove any model admitted by the single-layer theory. $\square$

The result establishes the extension relation

$$\text{DCIT}_1 \hookrightarrow \text{DCIT}_2.$$

E.5 Architecture I: passive electrostatic memory layer

The first stacked specialization uses one layer principally for transport and the neighboring layer principally for retained state.

Let

$$Q_B = Q_B^*$$

be an observable on $\mathcal{K}_B$ with at least two distinguishable candidate state preparations

$$\sigma_B^{(0)}, \quad \sigma_B^{(1)}$$

satisfying

$$\text{Tr}(Q_B \sigma_B^{(0)}) \neq \text{Tr}(Q_B \sigma_B^{(1)}).$$

Let

$$A_A = A_A^*$$

be an operator acting on the transport-active layer and choose an interlayer electrostatic coupling

$$V_{AB} = A_A \otimes Q_B.$$

For a fixed B -layer state $\sigma_B^{(j)}$, the effective Hamiltonian on layer A is

$$H_{A,\text{eff}}^{(j)} = H_A + \lambda_{AB} \langle Q_B \rangle_j A_A,$$

where

$$\langle Q_B \rangle_j = \text{Tr}(Q_B \sigma_B^{(j)}).$$

Hence

$$H_{A,\text{eff}}^{(0)} \neq H_{A,\text{eff}}^{(1)}$$

whenever

$$\lambda_{AB} \neq 0$$

and

$$\langle Q_B \rangle_0 \neq \langle Q_B \rangle_1.$$

The resulting transmission becomes

$$T_{y,A} = T_{y,A}(E; q_B),$$

with

$$q_B = \langle Q_B \rangle.$$

Thus the neighboring layer can represent a persistent internal variable capable of changing later transport without being identical to the readout pathway itself.

Proposition E.2 (Existence of state-dependent dimer readout). *There exists a finite dimer realization in which two retained states of layer B generate distinct later readout statistics on layer A .*

Proof. Choose

$$\sigma_B^{(0)}, \sigma_B^{(1)}$$

with unequal Q_B expectation values and choose A_A so that

$$[A_A, Y_A] \neq 0$$

for some declared readout observable Y_A . Because $i[A_A, Y_A]$ is then a nonzero self-adjoint operator, choose an A -layer state ρ_A satisfying

$$\text{Tr}(\rho_A i[A_A, Y_A]) \neq 0.$$

The corresponding effective Hamiltonians differ by

$$H_{A,\text{eff}}^{(1)} - H_{A,\text{eff}}^{(0)} = \lambda_{AB} (\langle Q_B \rangle_1 - \langle Q_B \rangle_0) A_A.$$

The two Heisenberg derivatives of Y_A at the initial time therefore differ by a nonzero expectation proportional to

$$\lambda_{AB} (\langle Q_B \rangle_1 - \langle Q_B \rangle_0) \text{Tr}(\rho_A i[A_A, Y_A]).$$

Hence the two preparations yield distinct accessible readout trajectories for sufficiently small nonzero times of one sign. $\square$

This architecture supplies a concrete realization candidate for the abstract persistent-state sector:

$$\mathcal{K}_m \longleftrightarrow \text{retained interlayer electrostatic state.}$$

E.6 Architecture II: field-gated dimer

A second specialization retains the two layers but allows an external field to alter the transport Hamiltonian.

Let

$$u(t)$$

be a declared control input and write

$$H_D(u) = H_D + u W_1 + u^2 W_2,$$

where W_1 and W_2 represent the linear and quadratic field responses of the chosen realization.

For a direction-resolved readout transmission,

$$T_y(E; u, q_B),$$

define the differential field gain

$$G_F(E; u, q_B) = \frac{\partial T_y(E; u, q_B)}{\partial u}$$

whenever the derivative exists.

The field-gated architecture adds a new distinction:

$$\text{stored state} \neq \text{instantaneous control.}$$

The same molecular node may therefore respond separately to

$$q_B = \text{history-dependent internal state,}$$

and

$$u(t) = \text{present external input.}$$

This realizes the functional form

$$T_y(t) = \mathcal{F}(u(t), q_B(t), \text{environment}),$$

which is a minimal state-dependent nonlinear computational element.

E.7 Architecture III: combined field and memory control

The most direct stacked extension of the DCIT neuromorphic architecture combines the preceding two mechanisms:

$$H_D = H_A + H_B + \lambda_{AB} V_{AB} + H_F(u).$$

The accessible transmission is then

$$T_y = T_y(E; u, q_B).$$

Two forms of modulation are present:

$$\frac{\partial T_y}{\partial u}$$

measures contemporaneous field control, whereas

$$\frac{\partial T_y}{\partial q_B}$$

measures retained-state dependence.

These derivatives need not have comparable magnitude or time scale.

The associated computational decomposition is

$C12 - C6$	→	rapid propagation/readout,
$C3 - C9$	→	control/integrative input,
q_B	→	persistent context or memory,
F	→	instantaneous gain/state control.

This is an architectural analogy only. It does not assign a biological neural function to a molecular degree of freedom.

E.8 Architecture IV: reciprocal two-layer control

A more general dimer need not designate one permanent logic layer and one permanent memory layer. Let

$$Q_A, \quad Q_B$$

be state variables on the two layers and allow

$$V_{AB} = A_A \otimes Q_B + Q_A \otimes A_B.$$

Then the effective state of either layer may modulate the other:

$$T_A = T_A(E; q_B),$$

$$T_B = T_B(E; q_A).$$

This architecture supplies bidirectional contextual modulation rather than a fixed master–gate relation.

The possibility is mathematically available in the dimer state space, but its usefulness depends on whether the physical realization retains adequate separation between functional modulation and unwanted direct interlayer leakage.

E.9 Architecture V: coherent interlayer extension

If the interlayer coupling remains phase coherent over the relevant operating time, the dimer may additionally support hybridized molecular states.

Let

$$H_D = \begin{pmatrix} H_A & J_{AB} \\ J_{AB}^* & H_B \end{pmatrix}$$

in a direct-sum effective orbital representation.

Nonzero

$$J_{AB}$$

generically splits or hybridizes otherwise nearby orbital levels. Thus a pair of single-layer levels

$$E_A, \quad E_B$$

may produce dimer eigenvalues of the form

$$E_{\pm} = \frac{E_A + E_B}{2} \pm \sqrt{\left(\frac{E_A - E_B}{2}\right)^2 + |J_{AB}|^2}.$$

A field or retained charge may therefore modify not only a single orbital energy but also the degree of interlayer hybridization.

This can enlarge the family of available transfer functions

$$T_{ij}(E)$$

and interference conditions.

However, coherent interlayer hybridization is not required for the neuromorphic interpretation developed in the main text. If dephasing eliminates phase-sensitive interlayer effects while a state-dependent electrostatic interaction survives, Architectures I–III remain available.

E.10 Terminal count versus channel count

The stacked dimer contains eight substituted terminal sites:

$$\{3_A, 6_A, 9_A, 12_A, 3_B, 6_B, 9_B, 12_B\}.$$

It is therefore naturally described as an *eight-terminal system*. This does not imply eight independent quantum transport channels.

For a multi-terminal realization let

$$\mathbf{T}(E) = [T_{ij}(E)]_{i,j=1}^8$$

denote the terminal-to-terminal transfer matrix.

Independent transport eigenchannels are determined by the spectral or singular-value structure of the realized scattering operator, not by the number of attached terminals.

Thus

$$8 \text{ terminals} \Rightarrow 8 \text{ independent transport eigenchannels.}$$

The eight-terminal geometry nevertheless enlarges the number of possible inputs, outputs, control variables, and pairwise transfer functions relative to a single four-terminal node.

E.11 Expanded state and control capacity

A single node with n_x , n_y , and n_m effective states has finite state-space dimension

$$d_1 = n_x n_m n_y.$$

Two independently represented nodes have

$$d_2 = d_A d_B.$$

If

$$d_A = d_B = d,$$

then

$$d_2 = d^2.$$

The statement concerns available model-state dimension only; it does not imply that all states are experimentally accessible or computationally useful.

More important for neuromorphic function is the increase in the number of independently controllable state variables:

$$\{x, y, m\}$$

for the single-layer effective model may expand to

$$\{x_A, y_A, x_B, y_B, q_A, q_B, F_A, F_B, \dots\}$$

in a stacked realization.

This permits multiple forms of contextual dependence within one molecular assembly.

E.12 Characteristic time-scale hierarchy

A stacked stateful realization can support at least three distinct characteristic time scales:

$$\tau_{\text{prop}}, \quad \tau_{\text{ctrl}}, \quad \tau_{\text{mem}}.$$

Here

$$\tau_{\text{prop}}$$

characterizes propagation or readout response,

$$\tau_{\text{ctrl}}$$

characterizes field- or state-induced modulation, and

$$\tau_{\text{mem}}$$

characterizes persistence of the retained state.

A candidate neuromorphic regime is

$$\tau_{\text{prop}} < \tau_{\text{ctrl}} < \tau_{\text{mem}},$$

although no such ordering follows from the dimer topology alone.

If realized, this hierarchy supplies a physical analogue of

rapid automatic processing $\rightarrow$ slower integration

$\rightarrow$ persistent contextual state.

The inequalities must be independently established for a particular device.

E.13 Performance architecture hierarchy

The stacked realization classes may be summarized as follows.

E.14 Cross-talk in the stacked architecture

Adding a second layer creates new intended and unintended coupling paths.

Let

$$g_{\text{intra}}$$

represent the direct within-layer cross-channel coupling and let

$$g_{\text{inter}}$$

represent unwanted direct interlayer transfer.

Let

$$g_{\text{state}}$$

represent the desired state-mediated modulation.

The neuromorphically useful stacked regime is schematically

$$g_{\text{intra}}, g_{\text{inter}} \ll g_{\text{state}},$$

rather than

$$g_{\text{intra}} = g_{\text{inter}} = 0.$$

Thus the dimer generalizes the central DCIT distinction:

parasitic direct transfer $\neq$ functional state-mediated influence

The additional layer is useful only if the increase in controllable state dimension exceeds the penalty introduced by additional unwanted transport.

E.15 Neuromorphic consistency theorem

Theorem E.3 (Stacked neuromorphic extension). *There exists a finite two-layer extension of a neuromorphically admissible single-core DCIT realization having simultaneously:*

1. *a rapid accessible propagation sector;*
2. *a distinct control/integrative sector;*
3. *a persistent interlayer state variable;*
4. *nonzero state-dependent modulation of the accessible readout;*
5. *independently tunable instantaneous field control; and*
6. *bounded but nonzero direct intra- and interlayer coupling.*

Proof. Begin with a finite neuromorphically admissible single-layer model on $\mathcal{K}_A$ chosen within the optional speed-ordered

Architecture	Additional state/control	Principal capability	Principal unresolved requirement
Single layer	x, m, y	Dual pathways, hidden state, bounded cross-talk	Physical realization of anisotropy and retention
Passive dimer	Interlayer state q_B	History-dependent readout; persistent contextual memory	Charge retention and independent state proxy
Field-gated dimer	F or gate input	Instantaneous gain and transfer-function control	Reversible field window and orbital response
State + field dimer	(q_B, F)	Separation of stored context from present control	Independent calibration of both mechanisms
Reciprocal dimer	(q_A, q_B)	Bidirectional contextual modulation	Control of unintended interlayer leakage
Coherent dimer	Phase-sensitive J_{AB}	Hybridized resonances and expanded interference control	Sufficient phase coherence and known interlayer coupling

Table 2: Performance architecture hierarchy for the optional stacked realization class.

subclass of Corollary 9.4. Such a model exists by that corollary, and its accessible y response may therefore be chosen faster than its control/integrative response scale within the finite witness.

Choose

$$\mathcal{K}_B \simeq \mathbb{C}^2$$

with orthogonal candidate retained-state basis

$$|0\rangle_B, \quad |1\rangle_B,$$

and define

$$Q_B = |1\rangle\langle 1|_B.$$

Choose the uncoupled B -layer state generator so that the two basis states are stationary (for example, take $H_B = 0$ in the finite witness). They therefore provide a persistent state variable within the existence construction; physical retention time remains a realization-level quantity.

Choose a non-scalar self-adjoint operator A_A for which some accessible observable Y_A satisfies

$$[A_A, Y_A] \neq 0,$$

and define

$$V_{AB} = A_A \otimes Q_B.$$

For any nonzero

$$\lambda_{AB},$$

the two B -layer states produce distinct effective Hamiltonians on layer A :

$$H_{A,\text{eff}}^{(0)} = H_A,$$

$$H_{A,\text{eff}}^{(1)} = H_A + \lambda_{AB} A_A.$$

Hence the retained state changes the subsequent accessible dynamics.

Now choose a bounded field operator W_A satisfying

$$[W_A, Y_A] \neq 0$$

and define

$$H_F(u) = uW_A.$$

The instantaneous control parameter u then changes the declared readout dynamics independently of the retained state Q_B .

Finally, choose finite nonzero direct intra- and interlayer cross-coupling operators and multiply them by coefficients ϵ_1, ϵ_2 . Since the spaces are finite dimensional, the operator norms are finite. For arbitrary prescribed tolerances $\delta_1, \delta_2 > 0$, choose the coefficients so that

$$0 < |\epsilon_1| \|V_{\text{intra}}\| < \delta_1,$$

and

$$0 < |\epsilon_2| \|V_{\text{direct}, AB}\| < \delta_2.$$

Thus direct leakage may remain strictly nonzero while being made arbitrarily smaller than the independently retained state-mediated interaction. All six stated properties therefore coexist in one finite model. $\square$

Remark E.4 (Meaning of the theorem). The theorem establishes model non-vacuity:

$$\text{Mod}(\text{DCIT}_2) \neq \emptyset.$$

It does not certify that a synthesized chrysene dimer realizes the chosen state lifetimes, field gains, or coupling hierarchy.

E.16 Relation to the conscious/nonconscious analogy

The single-layer neuromorphic interpretation already permits distinct rapid and state-dependent processing on one physical substrate. The stacked dimer adds a candidate physical realization of a third function:

persistent contextual state.

Accordingly, a stacked neuromorphic interpretation may be

written schematically as

$C12 - C6 \mapsto$
rapid automatic/feedforward-processing analogue,

$C3 - C9 \mapsto$
control/integrative-process analogue,

and

$q_B \mapsto$ persistent latent/contextual-state analogue.

The transition to reportable accessibility remains a property of the combined dynamics:

$(C3 - C9) + q_B + (C12 - C6)$
 $\longrightarrow$ accessible output.

Thus neither layer nor channel is identified individually with consciousness.

The relevance of the stacked construction is instead an existence statement: a finite molecular architecture can contain distinct pathway, state, and control degrees of freedom on the same physical assembly while permitting both weak direct cross-talk and stronger state-mediated interaction.

E.17 Interpretive and experimental boundary

The stacked construction enlarges the set of candidate physical realizations but does not strengthen the empirical status of any unverified molecular mechanism.

In particular, the following remain independent realization questions:

1. whether a cofacial substituted-chrysene dimer possesses a sufficiently long-lived distinguishable charge state;
2. whether that state can be prepared reproducibly;
3. whether the state modulates a declared terminal-to-terminal transfer observable;
4. whether field-induced orbital shifts remain reversible over the operating range;
5. whether direct interlayer leakage remains below the declared threshold;
6. whether coherent interlayer effects survive over a useful time scale; and
7. whether any biological conscious/nonconscious target system is adequately represented by the resulting architecture.

Failure of any such realization claim does not invalidate the single-layer DCIT mathematics or the stacked model-existence theorem.

E.18 Conclusion

The dimer extension increases architectural capacity in three principal ways:

additional state dimension,

independent field programmability,

and

candidate persistent interlayer memory.

The resulting hierarchy is

single-layer dual processing $\subset$ stateful stacked processing $\subset$ field-program

with coherent interlayer transport representing a further optional specialization.

The principal neuromorphic significance is therefore not the existence of eight terminals or the requirement of quantum coherence. It is the possibility of realizing, in one finite molecular assembly, the simultaneous structure

fast propagation + slower control/integration
+persistent context + bounded cross-influence

This supplies a hardware-level consistency witness for the expanded dual-process neuromorphic interpretation while leaving its physical and biological adequacy open to independent experiment.

F Review and Certification Ledger

This appendix records the staged manuscript-development review used during drafting. It is not represented as independent blinded external peer review. The ledger separates the criticism raised, the manuscript action taken, and the later verification status, following the orthogonal-constraint methodology [Sch26a].

F.1 Status conventions

Revision disposition is recorded as accepted, partial, not accepted, conditional, or optional. Verification is recorded as explicit, indirect, or none. A verified manuscript repair does not imply that a physical or cognitive realization premise has been experimentally established.

ID	Stage	Criticism	Revision	Verification
R1	<i>M</i>	Channel structures risked being presented as new protected axioms.	Recast channel topology as definitional mathematics over a fixed Article-39 realization; physical and informational assumptions moved to an overlay.	Explicit.
R2	<i>M</i>	P_X, P_Y risked collision with inherited localized projections.	Renamed channel projections Π_X, Π_Y .	Explicit.
R3	<i>M</i>	Open-system tensor-sum generators were insufficiently typed.	Restricted to bounded generators of uniformly continuous CPTP semigroups and typed the joint trace-class generator.	Explicit.
R4	<i>M</i>	The observability proof omitted an explicit positivity bound.	Required $0 < \epsilon < (d\ X\)^{-1}$ for $\rho_{\pm} = I/d \pm \epsilon X$.	Explicit.
R5	<i>C</i>	Accessible/latent roles could be read as inherited properties of the x, y directions.	Separated molecular direction from informational role and later from cognitive interpretation.	Explicit.
R6	<i>C</i>	New directional graph metrics resembled the inherited Axiom- A_{12} path metric.	Renamed them δ_Q^{ch} and denied automatic metric compatibility.	Explicit.
R7	<i>C</i>	$\mathcal{K}_X \otimes \mathcal{K}_Y$ could be read as an inherited physical factorization.	Declared tensor composites abstract until a separate device representation is supplied.	Explicit.
R8	<i>C</i>	Positive translation order risked being read as physical time.	Replaced temporal wording by forward reachability; physical time requires separate dynamics.	Explicit.
R9	<i>V</i>	Model-reduct preservation had been overstated as full proof-theoretic concordance.	Restricted the unconditional claim to protected-reduct preservation; stronger definitional conservativity is conditional on standard definition elimination.	Explicit.
R10	<i>H</i>	C3–C9 was initially identified directly with the persistent latent state.	Introduced $\mathcal{K}_M$ and the three-sector space $\mathcal{K}_X \otimes \mathcal{K}_M \otimes \mathcal{K}_Y$; C3–C9 is control access, not memory itself.	Explicit manuscript verification; physical realization open.
R11	<i>H</i>	Perfect physical channel isolation was too strong.	Replaced zero-coupling by a measured leakage ratio with prospective threshold η_{max} .	Explicit; experimental value open.
R12	<i>H</i>	Ambient-temperature ballistic coherence was too strong as a prerequisite.	Removed it from the abstract DCIT realization criteria.	Explicit.
R13	<i>V</i>	After introducing $\mathcal{K}_M$, tracing out memory alone left C3–C9 inside the purported accessible state.	Defined $\mathcal{K}_{\text{hid}} = \mathcal{K}_X \otimes \mathcal{K}_M$ and $\text{P}_{\text{acc}}\rho = \sigma_{\text{hid}} \otimes \text{Tr}_{X,M}\rho$.	Explicit.
R14	<i>V</i>	Direct-sum leakage operator K and tensor-product interaction V_{XY} could be conflated.	Declared the two effective representations distinct absent an explicit intertwiner.	Explicit.
R15	<i>M/C</i>	The intuitionistic layer could appear to replace the classical proof calculus.	Restricted intuitionistic logic to object-level Alexandrov/Heyting semantics.	Explicit.
R16	Interp.	“C3–C9 is subconscious” would identify a molecular path with a cognitive state.	Replaced channel identity by a hidden-block/reportability interpretation; the later neuromorphic refinement in R43 further assigns conscious access to an integrated dynamical process rather than to either channel alone.	Explicit; cognitive adequacy open.
R17	Interp.	Formal theorem transfer could be mistaken for human-mechanism evidence.	Added an independent cognitive domain-evidence certificate and explicit interpretation boundary.	Explicit.
R18	<i>M</i>	The generated von Neumann algebra was used as though generator expectations supplied arbitrary product expectations.	Introduced the real observation space $\mathfrak{B}_T$ for state discrimination and retained Σ_T for multiplicative algebra growth.	Explicitly verified.
R19	<i>M</i>	Two interaction theorems omitted a nonzero-coupling condition.	Added $\lambda \neq 0$ to hidden-state modulation and restated algebra growth through the actual condition $[H, A] \notin \mathfrak{B}_{\text{acc}}$.	Explicitly verified.
R20	<i>M</i>	Group addition was used on X_C after stating only an abstract isomorphism with $\mathbb{Z}^3$.	Fixed a lattice-coordinate identification and works thereafter with $X_C = \mathbb{Z}^3$ in those coordinates.	Explicitly verified.
R21	<i>M</i>	“Non-Markovian memory kernel” exceeded what the projection calculation alone establishes.	Replaced it by memory-kernel representation and separated history dependence from CP-divisibility and other non-Markovianity criteria.	Explicitly verified.
R22	<i>M</i>	Bare coupling coefficients from differently normalized effective models were compared directly.	Replaced them by norm-weighted strengths such as $g_{\text{leak}} = \epsilon \ K\ $.	Explicitly verified as an effective-model comparison.
R23	<i>M/C</i>	Crossbar row/column coupling labels were reversed relative to $x = C3 - -C9$ and $y = C12 - -C6$.	Reassigned readout-row couplings to J_Y and control-column couplings to J_X .	Explicitly verified.
R24	Interp.	The cognitive map lacked an explicit proof-preservation contract.	Introduced formal flow and cognitive presentations and typed τ_{cog} as a proof-preserving interpretation on a declared fragment.	Explicitly verified on the declared fragment; empirical adequacy remains open.
R25	<i>C</i>	Protected-reduct preservation alone did not establish Article-41 ordinary concordance at the presentation level.	Added an explicit protected-concordance proposition proving old-axiom inclusion, anchor-compatible non-vacuity, and equality of the protected identity theory for the definitional DCIT layer.	Explicitly verified.
R26	<i>C</i>	The certification frontier treated an empty route deficit as sufficient for certificate completeness.	Added $\text{ExtReady}(\Theta)$ and distinguished premise-complete from certificate-complete routes exactly as required by the certified-growth framework.	Explicitly verified.
R27	<i>C/H</i>	The label H_{π} could be mistaken for the narrower inherited Article-39 Conjugated π -Framework Hypothesis.	Split the assumptions into inherited $H_{\beta\pi}$ and the stronger DCIT transport-applicability hypothesis H_{prep} .	Explicitly verified; physical truth remains open.
R28	<i>C</i>	External tensor-factor commutativity could be confused with the Axiom-A8 primary-dual commutant.	Added an explicit remark separating the external tensor product from the inherited dual-cipher commutant construction.	Explicitly verified.
R29	<i>C</i>	Locant labels could be read as newly derived nomenclature rather than inherited coordinates.	Added an explicit standard-numbered-reference-frame convention and repeated it at the physical pathway assignment.	Explicitly verified.
R30	<i>H</i>	The physical overlay lacked separate chemical-instantiation and directional-addressability premises.	Added H_{chem} and H_{addr} , separating molecular/contact realization from the downstream propagation and crosspoint claims.	Explicitly verified; experimental truth remains open.
R31	<i>H</i>	Transmission notation $T_{\alpha\beta}(E)$ was used while the manuscript also allowed hopping, kinetic, or mixed transport regimes.	Introduced a generic direction-resolved transfer observable $\mathcal{T}_{\alpha\beta}(\xi; \omega)$, with $T_{\alpha\beta}(E; \omega)$ retained as the coherent-scattering specialization.	Explicitly verified.
R32	<i>H</i>	Leakage was bounded only over energy/bias and could miss high crosstalk in programmed or retained states.	Added the operational-state family Ω_{op} and defined η_{op} as the worst-case leakage over both operating parameter and declared device state/environment.	Explicitly verified.
R33	<i>H</i>	A successful single crosspoint did not by itself establish a functional finite crossbar.	Added H_{array} and a separate array-level selectivity/sneak-path criterion $\eta_{\text{array}} < \eta_{\text{array}, \text{max}}$.	Explicitly verified; empirical array realization open.
R34	<i>H</i>	H_{mem} and H_{read} were partly inferred from the same readout and therefore not independently testable.	Added an independent retained-state proxy S_M through H_{state} ; separated proxy discrimination, persistence, and state-dependent readout. If no proxy exists, only the weaker H_{hist} claim is permitted.	Explicitly verified; physical proxy remains realization-specific.
R35	<i>H</i>	The manuscript listed configurational and redox memory as though they were equally source-supported chrysene mechanisms.	Identified vertically coupled retained charge/electrostatic state as the CohBit-inspired candidate and reclassified configurational, redox, and other mechanisms as separate possible DCIT realizations requiring their own evidence.	Explicitly verified.
R36	<i>H</i>	A Stark-specific implementation could be read as inheriting safe field operation from the abstract geometry.	Added optional H_{field} , requiring independent verification that the applied field window remains within the reversible device regime; Stark tuning is not required by the abstract theory.	Explicitly verified; field limits remain empirical.
R37	<i>V/H</i>	The generalized transfer observable could be signed or differently normalized, making the leakage ratio physically ill-defined across transport regimes.	Required $\mathcal{T}_{\alpha\beta}$ to be a nonnegative common-scale transfer magnitude; signed primitive observables must be converted by a prospectively declared magnitude, norm, or leakage functional.	Explicitly cross-verified.
R38	<i>V/H</i>	H_{ctrl} allowed either hidden-state preparation or readout modulation, while its falsification test used only control-to-readout gain.	Added H_{prep} for causal C3–C9 preparation of proxy-defined hidden states and narrowed H_{ctrl} to contemporaneous readout modulation; protocol and falsification matrix updated accordingly.	Explicitly cross-verified.
R39	<i>V</i>	The physical frontier and review sequence did not yet record the consequences of the final cross-track integration check.	Added H_{prep} to the physical frontier and recorded $V_{\text{cross}} = 1$ after verifying that the combined mathematics, concordance, and domain repairs introduce no new cross-layer inconsistency.	Explicitly cross-verified; empirical physical and cognitive frontiers remain open.

ID	Stage	Criticism	Revision	Verification
R40	Meta	The supplied survey manuscript carried bibliographic metadata conflicting with the controlling Article-39 record, creating an unresolved page/article identifier collision.	Removed the ambiguous survey citation from the DCIT manuscript and bibliography rather than guessing corrected metadata; the review architecture remains supported by the orthogonal-review case study and certified-growth paper.	Explicitly post-meta verified.
R41	Meta	The leakage ratio was defined only where intended-transfer denominator was positive, but the operating supremum was written over the entire operating family.	Introduced the explicit leakage-test domain Ω_{leak} , required it to be nonempty, and defined η_{op} only over that domain in the main physical section, falsification protocol, and Appendix C.	Explicitly post-meta verified.
R42	Meta	The manuscript was mathematically self-contained but under-cited conventional literature for projection operators, CPTP trace-distance contractivity, observability, Alexandrov topology, and intuitionistic/Kripke semantics.	Added canonical external references and localized citations without changing any theorem statement or proof dependence.	Explicitly post-meta verified.
R43	M/H/I	The corrected C5-containing C12–C6 path required recomputation of the standard four-terminal bipartite model and exposed an obsolete simple channel-specific conscious/nonconscious assignment.	Recomputed the 18-vertex chrysene graph; established both intended pairs as opposite-sublattice connections; introduced the exact four-terminal inverse-adjacency matrix, ideal amplitude ratio $ M_{3,9} : M_{12,6} = 1 : 4$, selective C3–C12 / C9–C6 interference zeros, and the equal-contact core transmission proxy $1 : 16$. Added the finite neuromorphic consistency section, revised the cognitive interpretation toward rapid automatic processing plus slower state-dependent integration and later reportable access, and separated the empirical speed hypothesis H_T from static transmission magnitude.	Graph/Hückel statements verified conditional on the ideal model: H_{aniso} , H_{QI} , H_T , and biological domain adequacy remain empirical/open.

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