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The Photoentrained Cycle

Existence and Stability of a Periodic Orbit in a Forced Hybrid Protocell

Charles D. Schaper, Ph.D.
Chief Editor, Annals of the Chrysene Formalism
San Francisco, California, USA
editor@chrysene.com

Abstract. The corpus supplies an enclosure, a strand anchored within it, a clock and a readable state. It does not say whether these compose. A cycle is not a set of steps but a closed orbit, and closure is a property of the composition. The question is posed here as the existence of a fixed point of a return map on a periodically forced hybrid system with an impulsive reset at division, and it receives two answers.

The flow is assembled from four parts. The clock is monotone at constant illumination, and the mechanochemical loop that couples it to the membrane is shown to have small gain and a sign that sharpens rather than destabilises, so the system does not oscillate autonomously; what the architecture has is entrainment, and since the cooperative coupling it needs for commitment makes the period exponentially sensitive to that coupling, a free-running clock would be unusable. Separation is a driven front in a quenched sequence landscape: processivity is topological rather than regulatory, the linking number is taken up by lateral sliding of the anchors at negligible cost and only in a fluid regime, and the origin is the low-force end of a compositional gradient rather than a marked site. Polymerisation is entropically catalysed by the scaffold itself, and the requirement that the encoding survive forces re-templating rather than polymerisation from free monomer. Division requires a reduced volume below $Q(\alpha)^{-3/2}$, of which symmetric fission is the most expensive case.

The first answer is unconditional and is the paper's principal result. Every process of the architecture leaves the number of impermeant internal particles unchanged or lowers it, so that number is conserved across a lineage. Each division restores the reduced volume exactly — shape is homeostatic with no feedback and no sensor, the restoration being a property of the reset — while area and particle number both fall. A founder therefore buds some forty times before reaching a size floor, and across a population total membrane grows only as the cube root of cell number. *The architecture reproduces and does not grow.*

The second answer is conditional. A steady lineage requires the cell to generate impermeant particles at the rate a bud removes them, about six per ten seconds at a bud fraction of a tenth. Under that condition the return map has a fixed point; under supply-limited growth it is a contraction of multiplier one half, so that size is stable and the contraction is supplied by the halving at division rather than by any measurement, and under surface-catalysed growth there is no stable size and the division interval is quantised to an integer multiple of the forcing period. The two growth laws are distinguishable by observation of a population.

Three caps on the length of the sequence carried — an information threshold, a geometric span and a diurnal clock — admit a common band of one hundred to four hundred base pairs, and the only free parameter among them is measurable. What the account does not supply is named rather than assumed: the missing particle source is the first requirement in this arc that is metabolic in character, and it is located not at the genome, the membrane or the clock but at the osmolyte.

Keywords: Chrysene Formalism; hybrid dynamical system; Poincaré map; photoentrainment; depinning; vesicle division; size homeostasis; branching process; hypothesis register

1 Introduction

The enclosure paper gives a membrane made from the architecture's own material and a strand anchored within it. The signalling paper gives a clock, a readable state, and a proof that no loop closes on that state. Between them they supply a set of steps, each of which is possible.

A cycle is not a set of steps. It is a closed orbit, and closure is a property of the composition rather than of the parts. Nothing in the corpus establishes that the steps return to their starting point, and the arithmetic of this paper shows that on the obvious reading they do not.

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1.1 What this paper addresses

Whether the composition closes; under what conditions; and what the closure implies for size, for population and for the length of the sequence carried. The question is made precise at Definition 5.4 and answered at Theorems 10.6 and 10.11, which give different answers: the architecture reproduces for a bounded number of generations unconditionally, and indefinitely only under a condition it does not meet.

1.2 What is established here, by kind

Recovered. The timescales, tether lengths, footprint, pitch, anchor ordering, pool ratio and line tension of the earlier papers, and from the patent the nucleotides, the tether linkages and the trisubstituted initiator. All at Section 4, which argues nothing.
Proved here. That the isomer subsystem admits no periodic orbit (Theorem 6.2), and that the mechanochemical loop coupling it to the membrane has small gain and the wrong sign (Theorem 6.4); that processivity is topological (Theorem 7.1); that the linking number is removable by sliding and only in a fluid regime (Theorem 7.2); that the origin is selected by a compositional gradient (Theorem 7.5); that the replication mode is forced by anchor conservation (Theorem 8.2); the division condition for general α (Theorem 9.1); that shape is restored at every division and size is not (Lemma 10.3, Theorem 10.6); the population law (Theorem 10.8); the stability dichotomy (Theorems 10.14, 10.15); and the three caps (Theorem 12.1).
Found. That the condition for a steady lineage and the condition for symmetric fission are the same condition (Theorem 10.5); that the architecture reproduces and does not grow, total membrane rising as the cube root of cell number (Theorem 10.8); that the contraction giving size stability under supply-limited growth is the halving at division and not any measurement (Scholium 10.4); and that entrainment is required rather than incidental, a free-running clock of the necessary sharpness being unusable (Corollary 6.8).
Computed here. Blocks V1 to V7 of Appendix B.
Supposed. The local regime, the coupling window, the growth law and the imported record, registered as H1 to H4 in Appendix A; and conditions C1′ to C4 of Section 10, of which C1′ names a process the architecture does not have.
Not claimed. Everything at Section 3, and the whole of column B of the ledger.

1.3 Organization

Section 2 fixes status and notation; Section 3 refuses seven readings before any construction; Section 4 is the materials ledger and establishes nothing. Section 5 defines the state, the flow and the reset, and poses the question as a fixed-point problem. Sections 6 to 9 supply the four components of the flow: the clock, the separation, the polymerisation and the division. Section 10 is the paper. Sections 11 and 12 are corollaries of it, and Sections 13 onward are the registers, the precedence entry, the targets and the conclusion.

2 Status, nomenclature, scope

Convention 2.1 (Status labels)**.** [A] As in the coordinating paper. [A] finite or axiom-free; [D] derived under named hypotheses, registered in Appendix A; [C] imported, with the source named; [E] molecular reading, entering no proof. Scholia carry no label and enter no proof.

Convention 2.2 (Feedforward)**.** [A] The signalling paper establishes that no loop closes on the orientational state. That corollary is imported and is not an obstacle here but a specification: no step below uses feedback, and Scholium 6.5 separates the three notions that are commonly run together, of which only synchronisation is used.

Convention 2.3 (Chemistry is imported)**.** [A] No statement of this paper derives a chemical transformation. Section 4 records what is imported, what is to be discovered, what is specified here and what is absent; every later section cites it rather than arguing. In particular no theorem depends on the azobenzene conversion, which is in column B and undischarged.

Convention 2.4 (Scope)**.** [A] The object is one enclosure and its lineage under a periodic illumination of fixed period and duty. Nothing is asserted about a population in a shared medium, about competition between lineages, or about any variation in the forcing.

Notation 2.5 (Imported quantities)**.** [C] Tether lengths of ten and thirteen backbone bonds; unit footprint 0.39 to 0.43 nm^2; helical pitch 3.57 nm; anchor ordering by the covalent backbone; pool ratio ν; line tension $\gamma = 0.59\,\varepsilon$ nm; the timescale table.

3 Non-implications

Proposition 3.1 (Non-implications)**.** [A] *None of the following is asserted anywhere below.*

(i) That a cell is exhibited. *A closed orbit of a reduced model is not an organism, and the reduction discards everything but four coordinates.*

(ii) That any chemistry is demonstrated. *Column B of the*

ledger is undischarged and no theorem depends on it.

(iii) That the architecture oscillates. *It is entrained; Theorems 6.2 and 6.4 exclude autonomous oscillation, the second by a computed loop gain rather than by a dimensional argument.*

(iv) That the cycle is regulated. *There is no feedback, no checkpoint, and no adjustment of any step to the state of another.*

(v) That the steady orbit is realised. *Its existence is conditional on C1′, which names a process the architecture does not have.*

(vi) That fidelity is addressed. *The error rate enters as an imported parameter of one cap and no proofreading is proposed.*

(vii) That anything reads the encoding. *The signalling paper's target stands untouched.*

Scholium 3.2 (Why (i) and (v) are stated together)**.** The two refusals guard one door from opposite sides. A model claiming a cell would be claiming a chemistry it imports; one claiming the steady orbit is realised would be claiming a condition it only states.
What lies between them is narrower and is the paper's content: that the steps compose; that the composition has a fixed point in a region whose boundaries are computed; and that without one named process the lineage is well formed at every step and nonetheless runs down.

4 The materials ledger

This section establishes nothing and is cited by every section below. Its purpose is that a reader should be able to see, before any argument, which materials and processes are taken from elsewhere, which are yet to be demonstrated, which this paper specifies, and which are absent from the account entirely.

A — Imported

From the patents. Nucleotides with the sugar ring and the phosphorus installed and unreacted; the trisubstituted initiator and its triphosphate, which begins the condensation in the $5' \to 3'$ sense; the tether linkages; the bridge topologies; the synthesis route by which a tetrasubstituted unit becomes a base on a disubstituted template.

From the residue account. The transannular centre and the transfer of configuration to it.

From the enclosure paper. Scission of the unrecruited dimer to a lamellar lipid; the two tether lengths; the unit footprint; the helical pitch; the ordering of anchors by the covalent backbone; the pool ratio ν.

From the signalling paper. The timescale hierarchy; the window and its fibre bound; the domain line tension $\gamma = 0.59\,\varepsilon$ nm; the corollary that no loop closes on the orientational state.

B — To be discovered

The conversion of a 6,12-disubstituted chrysene to a substituted azobenzene; the tethered trimer and its conversion in place; azo-bearing lipids by scission of converted units; the anaerobic oxidation route; whether three azo units in series isomerise independently.

No theorem below depends on any of these having been demonstrated, and every mechanism of Sections 6 and 9 depends on all of them.

C — Specified here

The forced hybrid system and its reset; the depinning formulation of separation; the division condition for general α; the return map; the dilution law; the viability condition; the three caps.

D — Absent

Phosphate activation beyond the initiator's triphosphate. Any source of impermeant internal particles, which condition C1′ requires and Target 16.3 names. Any export mechanism for a polar byproduct. Any checkpoint. Any reader of the encoding. Re-docking of daughters, which is specified in outline and derived nowhere.

Remark 4.1 (Why the fourth column)**.** [A] A section listing only what is imported, what is wanted and what is supplied is a specification. What makes it a ledger is the fourth column, and the fourth column is where the result of Section 10 was found: the absent source of impermeant particles was not identified as a gap until the existence proof was attempted, and it had been assumed away in a clause nobody would have thought to check.

5 The state space, the flow and the reset

Definition 5.1 (State)**.** [D] The state is $z = (\theta, x, A, N)$: the isomer fraction $\theta \in [0, 1]$ of the connector population, the fork position $x \in [0, L]$ along the duplex, the membrane area $A > 0$, and the number $N > 0$ of impermeant particles in the lumen. Two quantities are derived and not independent: the volume $V = N/c_{\mathrm{ext}}$, fixed by osmotic balance against a constant external concentration, and the reduced volume $v = V/V_{\mathrm{sph}}(A)$.

Definition 5.2 (Forcing)**.** [D] The illumination $I(t) \geq 0$ is periodic of period T with duty δ, and vanishes on the dark interval. It is the only time-dependent input, and nothing in the state acts upon it.

Definition 5.3 (Flow and reset)**.** [D] Let Φ_t be the flow of the vector field assembled in Sections 6 to 9, and let $\mathcal{D}_\alpha$ be the division map of Definition 10.2, applied at dusk when and only when the condition of Theorem 9.1 holds.

Definition 5.4 (The return map)**.** [D] Sampling at dawn,

$$\mathcal{P} \;=\; \mathcal{D}_\alpha^{\chi} \circ \Phi_T,$$

where $\chi \in \{0, 1\}$ indicates whether the division condition was met. A replication cycle is a fixed point of $\mathcal{P}$, and its stability is the spectral radius of $D\mathcal{P}$ there.

Remark 5.5 (Why this formulation and not a narrative)**.** [A] Three reasons, and the third is the one that decides the paper.
Closure becomes a property rather than a story. A list of steps each of which is possible does not establish that their composition returns to its starting point, and the corpus has so far supplied the list.
The absence of an internal oscillator becomes a theorem about the autonomous vector field (Theorem 6.2) rather than an apology.
And stability acquires a meaning. "The cycle works" admits no degree; the multiplier of a fixed point does, and it is what distinguishes the two growth laws of Theorems 10.14 and 10.15 — a distinction no narrative account could have drawn.

Convention 5.6 (What is not in the state)**.** [A] The orientational word is a parameter and not a coordinate: by the signalling paper it is not written on any timescale of this cycle, and by the feedforward convention nothing here reads it. It enters the vector field as the landscape $g(x)$ of Theorem 7.6 and nowhere else.

6 The clock

Definition 6.1 (Isomer dynamics)**.** [D] The connector population obeys

$$\dot{\theta} \;=\; \Phi_\sigma \, I(t) \, (1 - \theta) \;-\; k(\theta) \, \theta,$$

with quantum yield Φ_σ and thermal rate k decreasing in θ, the decrease expressing the stacking of the *trans* population.

Theorem 6.2 (The isomer subsystem admits no periodic orbit)**.** [D] *Hold A and N fixed. At constant I the right-hand side is then a continuous scalar function of θ on* $[0, 1]$*, positive at* 0 *and negative at* 1*; every trajectory is monotone between consecutive zeros and converges to one, and no periodic orbit exists.*
The hypothesis is not idle and is discharged at Theorem 6.4: A and N are not *fixed, and the coupling between them and θ closes a loop which this theorem does not address.*

Remark 6.3 (The loop the reduction omits)**.** [A] Three couplings asserted elsewhere in this paper close a cycle among the coordinates.
Proposition 9.6 has the isomer state alter the membrane area, the *cis* isomer occupying more of it. Theorem 9.3 has area and particle number fix the reduced volume and hence the tension. And an isomerisation that changes molecular area is shifted by tension, so the tension enters $k(\theta)$.
Hence $\theta \to A \to \sigma \to k \to \theta$, closed through mechanics and not through the orientational state that the signalling paper's corollary forbids. Across a division the loop acquires a delay, and delayed feedback with an impulsive reset is the standard construction of a relaxation oscillator. The question is therefore not whether the loop exists — it does — but what its gain is.

Theorem 6.4 (The loop gain is small, and its sign is wrong)**.** [A] *The tension enters the isomerisation free energy as $\sigma \, \delta a$ with $\delta a \simeq 0.15$ nm^2 the area change per molecule. At the lysis tension of a lipid bilayer, about* 10 mN m^{-1}*, this is* $0.37 \, k_B T$ *and shifts the equilibrium by a factor of* 1.44*; at the tensions the cycle visits it is a few per cent.*
The coupling is moreover asymmetric and saturates on the side that would matter. In the light phase θ rises and the area rises, and because the volume is pinned the resulting excess area drives σ to zero: the feedback term vanishes exactly where θ is largest. In the dark phase θ falls, the area falls, σ rises, and the shift favours trans *— the direction the transition is already taking. The feedback is positive there and sharpens the dusk transition.*
A feedback that vanishes on one side and is positive on the other is not the sign structure of a relaxation oscillator. The coupled system has no limit cycle, and the reason is

quantitative rather than dimensional.

Proof. Finite; Appendix B, block V8, computed at five tensions. □

Scholium 6.5 (Three things that are not the same)**.** Oscillation, regulation and synchronisation are distinct and only the third is used here.
Oscillation is an autonomous period that free-runs in constant conditions; it requires feedback, and Theorem 6.2 excludes it. *Regulation* is a response that adjusts to the system's own state; it requires feedback, and the signalling paper's no-loop corollary excludes it. *Synchronisation* is many units adopting one state at one time; it requires coupling and a common forcing, neither of which is feedback, and both are available.
A reader will supply "circadian" and "regulated" from habit. The architecture is entrained and coordinated, and is neither.

Theorem 6.6 (Cooperative threshold)**.** [D] *Treating the stacked connectors as an Ising array of coupling J and coordination number z: in two dimensions an ordering transition occurs for $J/k_BT > 0.44$, and in one dimension the correlation length is $\xi \sim e^{2J/k_BT}$, so blocks of that size switch together. Stacking contacts are several k_BT, so a genuine threshold is easily met.*

Theorem 6.7 (The reset constraint, and its sensitivity)**.** [D] *A unit switching back must switch against its neighbours, so the dark half-life is set by $E_a + Jz$ and*

$$\frac{\partial \ln \tau}{\partial (Jz)} = \frac{1}{k_BT}.$$

One k_BT of coupling changes the period by a factor of e. A bare barrier giving a twelve-hour half-life gives thirty-three hours at $Jz = k_BT$ and eighty-nine at $2k_BT$; the intrinsic barrier must therefore be lowered to pay for the coupling.

Proof. Finite; Appendix B, block V1, computed. □

Corollary 6.8 (Entrainment is required, not incidental)**.** [D] *A free-running clock with the sensitivity of Theorem 6.7 decoheres across a population within a few cycles: a spread of one k_BT in coupling gives a spread of e in period. Under forcing the error does not accumulate, dawn restoring every cell to the same phase whatever its coupling.*
Coupling supplies sharpness within a cell; forcing supplies phase across cells. Neither substitutes for the other, and given the coupling the architecture needs for commitment, an internal oscillator would be worse than useless.

7 Separation

Theorem 7.1 (Processivity is topological)**.** [D] *A duplex of L base pairs carries a linking number $L/10.5$. If separation nucleated at several interior sites, each intervening segment would be held at both ends by separated strands anchored in two membranes and could not unwind; the strands would be topologically locked and would not part at any force. Separation must therefore begin at one end and propagate.*

Theorem 7.2 (Unwinding requires a fluid regime)**.** [A] *The linking number is taken up by lateral sliding of the anchors, which trace helical paths in the membrane. At Saffman–Delbrück drag the cost is $0.02\,k_BT$ per 10^3 base pairs at a fork speed of $1\ \mathrm{nm\,s^{-1}}$: negligible at any plausible speed, and available only if the anchors can move.*

Proof. Finite; Appendix B, block V3, computed at four fork speeds. □

Hypothesis 7.3 (Local regime)**.** [D] The regime is local: dense stacked patches where force is transmitted, dispersal elsewhere.

Remark 7.4 (The regime is load-bearing three times)**.** [A] The enclosure paper requires dispersal, for a leaflet map defined on independently placed anchors. The signalling paper requires aggregation, which fixes relative positions and sharpens the window. Separation requires dispersal, so that anchors may slide.
Two of the three want one branch and one wants the other, and no global choice satisfies all three. Hypothesis 7.3 is the only reconciliation available, and the coordination ledger should record the three-way form rather than the two-way form previously entered.

Theorem 7.5 (Origin selection without a marker)**.** [D] *The critical unzipping force is composition-dependent: of order 10–11 pN for AT-rich tracts and 15–20 pN for GC-rich. A membrane tension delivering an intermediate force opens the first and not the second, so the fork nucleates at the low-force end of the compositional gradient and propagates up it.*
The origin is therefore not a recognised site. It is the extremum of a gradient which the sequence already carries, and which by the signalling paper's anchor ordering cannot

diffuse away.
The two force figures are single-molecule values obtained by pulling a free end, and the geometry here differs in lever arm and loading rate. The result uses only their ordering, *which is robust to both; the magnitudes are indicative and are used nowhere else.*

Theorem 7.6 (Depinning)**.** [D] *Writing $g(x)$ for the local composition and $F_c(g)$ for the corresponding critical force,*

$$\dot{x} \;=\; v_0\, H\big[F(\theta) - F_c\big(g(x)\big)\big]$$

is a driven front in a quenched disordered landscape. There is a critical drive I_c below which the front is pinned and above which it propagates, and I_c is a functional of the extremes *of the composition distribution rather than of its mean: the fork is held by the strongest barrier it meets.*

Corollary 7.7 (A selection pressure on composition)**.** [D] *Sequences with fewer strong barriers replicate at lower drive. This is a pressure on base composition arising from the mechanics and not from any coding function, and it acts on the distribution's tail rather than on its mean.*

8 Polymerisation

Theorem 8.1 (The scaffold is an entropic catalyst)**.** [D] *The nucleotides arrive with sugar and phosphorus installed and unreacted, held in slots interleaved with the anchor units. The scaffold therefore fixes their relative position and orientation, which is what an enzyme supplies and it supplies it by entropy rather than by chemistry. Juxtaposition on a rigid scaffold gives effective molarities of 10^2 to 10^8 M; against free monomer at millimolar that is 10^5 to 10^{11} in rate. The upper end of that range requires rigid juxtaposition with little residual freedom, which a chrysene held in a membrane by a flexible connector is not, so the conservative half — 10^2 to 10^5 M — is taken here, and condition C3 inherits it through t_{add}.*
The anchor unit is not a label on the reaction. It is the reason the reaction proceeds without an enzyme, and it is the same structure that carries the encoding.

Theorem 8.2 (Anchor conservation forces the mode)**.** [A] *A daughter strand polymerised from free monomer carries no template units and therefore no tethers; the anchored fraction halves every generation and the encoding is lost within a handful of cycles. A daughter re-templated on fresh complexes acquires, with each base, the unit that templated it, and the anchored fraction is conserved exactly.*
The replication mode is therefore forced by the requirement that the encoding survive, and is not a choice among mechanisms.

Proof. Finite; Appendix B, block V4, computed. □

Proposition 8.3 (The initiator supplies the registered origin)**.** [C] *The trisubstituted initiator is chemically distinct from both the disubstituted anchor and the tetrasubstituted base precursor, and carries the triphosphate that begins the condensation in the $5' \to 3'$ sense. Being distinct it is addressable, and it supplies the registered starting point that the enclosure paper assumed as a premise.*
A premise of that paper is thereby discharged, which is the first such discharge in this arc.

Remark 8.4 (Two origins, and a decision the paper must make)**.** [A] Separation nucleates at the low-force end of the compositional gradient; polymerisation begins at the initiator. Nothing requires these to coincide.
If they coincide, synthesis follows the fork and the two run together. If they lie at opposite ends, synthesis waits upon completion of separation and the period must accommodate both in series, tightening condition C3 by roughly a factor of two. Both close; they are different cycles, and this paper adopts the coincident arrangement and records the alternative here.

9 Division

The section is written for a general volume fraction α from the outset. Symmetric fission is the case $\alpha = \frac{1}{2}$ and is, as Theorem 9.1 shows, the most expensive one; the cycle of Section 10 does not use it.

Theorem 9.1 (The geometric condition)**.** [A] *Let a vesicle of volume V and area A divide into two spheres of volume fractions α and $1-\alpha$. Writing $Q(\alpha) = \alpha^{2/3} + (1-\alpha)^{2/3}$, the event requires*

$$A \;\geq\; Q(\alpha)\, A_{\mathrm{sph}}(V), \qquad \textit{equivalently} \qquad v \;\leq\; Q(\alpha)^{-3/2},$$

where $v = V/V_{\mathrm{sph}}(A)$. The function $Q(\alpha)$ is maximal at $\alpha = \frac{1}{2}$, so symmetric fission demands the largest area gain and the smallest reduced volume of any division.

Proof. The two daughters have areas $\alpha^{2/3}A_{\mathrm{sph}}(V)$ and $(1-\alpha)^{2/3}A_{\mathrm{sph}}(V)$; their sum is $Q(\alpha)\,A_{\mathrm{sph}}(V)$. Since $V_{\mathrm{sph}}(A) = V\,Q(\alpha)^{3/2}$ when $A = Q(\alpha)A_{\mathrm{sph}}(V)$, the reduced-volume form follows. Concavity of $t \mapsto t^{2/3}$ gives the maximum at $\alpha = \frac{1}{2}$. Appendix B, block V5, computed at seven values of α. □

Corollary 9.2 (The cost of symmetry). [A] *Symmetric fission requires a* 26.0% *area gain and* $v \le 0.7071$. *Budding at* $\alpha = 0.10$ *requires* 14.8% *and* $v \le 0.8134$*; at* $\alpha = 0.02$, 6.0% *and* $v \le 0.9147$. A cycle unable to reach the symmetric threshold may still bud.

Theorem 9.3 (Volume is pinned, not tracked). [A] *With water permeability* $P_f \simeq 20\ \mu\mathrm{m\,s^{-1}}$ *and* $R = 100$ *nm the osmotic relaxation time is* $\tau_{\mathrm{osm}} \simeq 0.9$ *s, against a forcing period* $T \simeq 4\times10^{4}$ *s: a ratio of* 5×10^{4}. *Volume therefore follows area essentially instantaneously and cannot lag it. Excess area arises only because the internal particle number* N *is impermeant, whence* $V = N/c_{\mathrm{ext}}$ *and*

$$v \;=\; 6\sqrt{\pi}\,\frac{N}{c_{\mathrm{ext}}\,A^{3/2}} \;\propto\; N\,A^{-3/2}.$$

Proof. Finite; Appendix B, block V5, computed. □

Remark 9.4 (A correction, and what it cost). [A] An earlier form of this section stated that a 26% area gain at constant N reaches the two-sphere threshold, and inferred that the area required for division and the area producing the required reduced volume are the same quantity.
The first clause is true of a *single* division from a taut sphere. The inference is false of a *cycle*: daughters then begin at $2^{-2/3} = 0.63$ of the parent's area and the lineage shrinks. The correct statement is Theorem 10.5 below, and the error is recorded at Remark D.1.

Proposition 9.5 (Drive). [C] *The division is driven by the line tension at the boundary of the anchored domain,* $\gamma = 0.59\,\varepsilon$ *nm, imported from the signalling paper, where* ε *is the free energy of lateral contact per unit area of stacking face.*

Proposition 9.6 (Gate). [D] *The azo connectors are some four per cent of the membrane and modulate its area by under one per cent; they do not gate the geometry and nothing here asks them to. What the isomer coordinate does at dusk is release the tether constraint that holds the two territories together.*
The area comes from growth, the drive is the line tension, and the isomer coordinate sets the moment and supplies no force.

Theorem 9.7 (An azo-bearing lipid would unpin the volume). [A] *A population of azo-bearing lipids large enough to modulate the membrane area was considered and is excluded. Pinning over a period with a factor of ten in hand requires* $P_s < R/30T = 7.8\times10^{-14}\ \mathrm{m\,s^{-1}}$. *Modelling the switched permeability as* $P_s = P_0\big(1 + f\theta(E-1)\big)$ *with* f *the azo-lipid mole fraction and* E *the enhancement on switching, a bare bilayer at* $P_0 = 10^{-14}$ *and* $E = 100$ *permits* $f\theta \le 0.068$.
The enhancement E *is not taken from a measurement. It is inferred from the packing change that Theorem 6.4 quantifies: the* cis *isomer occupies more area and packs less well, and defect-mediated leakage rises with disorder. The exclusion below is therefore conditional on* $E \gtrsim 10$*, and is stated as such at Remark 9.8.*
An area-gating role requires $f \simeq 0.5$. *No mole fraction satisfies both, and the conflict is phase-locked: the light phase is when the* cis *fraction is highest, hence when excess area accumulates and pinning is most needed, and it is then that the leak is worst.*

Proof. Finite; Appendix B, block V9, computed at four combinations. □

Remark 9.8 (On what the exclusion rests). [A] The exclusion of Theorem 9.7 turns on the enhancement E, which is inferred rather than measured. At $E = 10$ and a bare bilayer at $10^{-14}\ \mathrm{m\,s^{-1}}$, a mole fraction of three quarters is compatible with pinning and the conflict does not arise; at $E = 100$ the limit is seven per cent and it does. At a bare bilayer of 10^{-13} no enhancement at all is tolerated.
A measurement of E for an azo-bearing lipid of this class would therefore settle whether the population is excluded or merely constrained. It is entered as Target 16.4. The paper withdraws the population in either case, because it does not need it.

Remark 9.9 (What the exclusion costs). [A] Nothing that this paper uses. The area-gating role was never the primary mechanism — Proposition 9.6 states that the area comes from growth and the drive from the line tension — and removing the population removes a mechanism the account did not need while preserving a premise it does. The withdrawal is recorded at Remark D.2.

Corollary 9.10 (Waste is division-neutral)**.** [A] *Every byproduct of the architecture — the excised alkane fragments of the scission and the four carbons per azo conversion — is apolar and partitions into the bilayer rather than the lumen, so none enters N. A polar byproduct would raise N, raise v and arrest the cycle, and the architecture has no export mechanism for one.*

10 The periodic orbit

10.1 The reduced map

Lemma 10.1 (Reduction to two coordinates)**.** [D] *Sampled at dawn, the isomer coordinate has relaxed ($\theta = 0$, by Theorem 6.7 under condition C4) and the fork coordinate has been reset ($x = 0$). The return map therefore acts on (A, N) alone.*
The fork resets in all three cases and not only on completion. If the fork reaches the end, the strand is copied and x returns to zero at the reset. If it does not, the tethers extend at dusk, the separated strands are released from the wall, and the duplex re-anneals behind the stalled fork; x returns to zero and the cycle is lost, which is the failure event of Theorem 11.1. Re-annealing is available precisely because separation is ratcheted by the tethers and not by any covalent change.

Definition 10.2 (The budding reset)**.** [D] At a division of fraction α occurring exactly at threshold,

$$\mathcal{D}_\alpha : (A, N) \longmapsto \big((1-\alpha)^{2/3} A_{\mathrm{sph}}(V),\ (1-\alpha)N\big)$$

for the mother, and $(\alpha^{2/3} A_{\mathrm{sph}}(V), \alpha N)$ for the bud.

Lemma 10.3 (Both products are taut)**.** [A] *After the reset, mother and bud each satisfy $v = 1$ exactly: their areas are by construction the sphere areas of their volumes. The cycle therefore begins each period in the same shape state, whatever α.*

Scholium 10.4 (Shape homeostasis without regulation)**.** Lemma 10.3 is the sense in which this architecture is homeostatic, and it is worth separating from the sense in which it is not.
The reduced volume is restored exactly at every division, by no sensor and no feedback: it is a consequence of the geometry of the reset. A feedforward architecture forbidden a control loop nonetheless returns to a definite shape every cycle.
It does not return to a definite *size*, and Theorem 10.6 shows it cannot. Shape homeostasis and size homeostasis come apart here, and the map exhibits the reason: v is a ratio and is restored, while A and N are magnitudes and are not.

10.2 The dilution-limited orbit

Theorem 10.5 (Steady lineage requires balanced doubling)**.** [A] *A lineage in which each daughter begins where its parent began requires A and N each to be restored across a full cycle. For symmetric fission this means both must double over the period, and the resulting reduced volume at dusk is*

$$v = \frac{2}{2^{3/2}} = 2^{-1/2} = 0.7071,$$

which is exactly the threshold of Theorem 9.1 at $\alpha = \frac{1}{2}$. The condition for a steady lineage and the condition for symmetric fission are the same condition.

Theorem 10.6 (Without a solute source the lineage runs down)**.** [A] *Every process of the architecture leaves N unchanged or lowers it: conversion of an internalised unit to membrane removes it from the lumen; polymerisation replaces m monomers by one chain; and the byproducts are apolar and never enter N (Corollary 9.10). Hence N is non-increasing, and at bud fraction α the mother satisfies*

$$A_{n+1} = (1-\alpha)^{2/3} A_n, \qquad N_{n+1} = (1-\alpha) N_n,$$

with $v_n \equiv 1$. The mother shrinks geometrically and no coordinate of the map signals it.

Proof. Finite; Appendix B, block V6d, computed at six values of α. □

Corollary 10.7 (How far a founder gets)**.** [A] *From $R = 100$ nm to a floor at $R = 25$ nm, a founder buds $\log(A_{\min}/A_0)/\log(1-\alpha)^{2/3}$ times: six at $\alpha = \frac{1}{2}$, nineteen at $\alpha = 0.2$, forty at $\alpha = 0.1$ and eighty-one at $\alpha = 0.05$.* Forty daughters from one founder on a fixed solute budget, with no metabolism.

Theorem 10.8 (The population law)**.** [A] *The total impermeant solute N_{tot} is conserved. Each cell sits at $v = 1$, so $A_i \propto N_i^{2/3}$, and with N_{tot} divided among k cells*

$$A_{\mathrm{tot}} \ \propto\ k\Big(\frac{N_{\mathrm{tot}}}{k}\Big)^{2/3} = N_{\mathrm{tot}}^{2/3}\, k^{1/3}.$$

Total membrane grows as the cube root of cell number and mean radius falls as $k^{-1/3}$.

Proof. Finite; Appendix B, block V6e, computed. □

Scholium 10.9 (The architecture reproduces and does not grow)**.** Theorem 10.8 is the paper's least welcome result and should be stated where a reader will not miss it.
A thousand cells carry ten times the founder's membrane, not a thousand times. The architecture converts a fixed solute budget into an increasing number of diminishing cells: it reproduces, and it does not grow. Nothing in the preceding sections is wrong on that account — the clock runs, the strand separates, the polymer forms, the bud departs — and the lineage nonetheless runs down, because the one quantity none of those processes makes is the one that sets how large a vesicle may be.
This locates, quantitatively and for the first time in this corpus, where the absence of a metabolism becomes binding. It is not at the genome, not at the membrane and not at the clock. It is at the osmolyte.

10.3 The steady orbit, and what it would cost

Definition 10.10 (The conditions)**.** [D] Four conditions are used below and are collected here, none of them being a hypothesis about nature in the sense of Appendix A.

(C1) The internal particle number is impermeant over a period: $P_s < R/30T$, which at $R = 100$ nm and $T = 4.3 \times 10^4$ s is 7.8×10^{-14} m s^{-1} (Theorems 9.3, 9.7).

(C2) The membrane area gains at least the factor $Q(\alpha)$ within one period, so that the division condition of Theorem 9.1 is reached.

(C3) Replication completes within the active phase: $L\, t_{\mathrm{add}} < \delta T$ (Theorem 11.1).

(C4) The coupling lies in the window of Section 6: $Jz \geq 0.44\, k_B T$ for the threshold of Theorem 6.6, and

$$E_a + Jz \;\leq\; k_B T \,\ln\!\left(\frac{A_0\,(1-\delta)T}{\ln 2}\right)$$

so that the dark relaxation of Theorem 6.7 completes within the dark interval.

Condition C1′ of Theorem 10.11 is of a different kind and is discussed at Remark 10.13.

Theorem 10.11 (Existence of a steady orbit)**.** [D] *Suppose in addition to C2 to C4 that*

(C1′) the cell generates impermeant internal particles at a fractional rate $\alpha/(1-\alpha)$ per period,

so that the mother recovers exactly what the bud removes. Then $\mathcal{P} = \mathcal{D}_\alpha \circ \Phi_T$ maps the compact set $\{(A, N) : v = 1,\ A \in [A_{\min}, A_{\max}]\}$ into itself and, being continuous, has a fixed point by Brouwer's theorem. At that point the lineage is steady and the cycle closes.

Proof. By Lemma 10.3 the reset returns both products to $v = 1$, so the invariant set may be taken within the one-parameter family $K = \{(A, N) : v = 1,\ A \in [A_{\min}, A_{\max}]\}$, which is compact and, being the image of a closed interval under a continuous injection, convex in the coordinate A.
Invariance at the endpoints: at $A_{\min}$ condition C2 supplies a gain of at least $Q(\alpha)$ over the period and C1′ restores N, so $\mathcal{P}(A_{\min}) > A_{\min}$ provided $A_{\min}$ is chosen below the fixed area of Theorem 10.14; at $A_{\max}$ the reset removes the factor $(1-\alpha)^{2/3}$ and the gain is bounded above by $Q(\alpha)$, so $\mathcal{P}(A_{\max}) < A_{\max}$ for $A_{\max}$ chosen above it. Hence $\mathcal{P}(K) \subseteq K$.
Continuity: Φ_T is smooth in (A, N) away from the division threshold, the vector field of Sections 6 to 9 being smooth there; and at the threshold $\mathcal{D}_\alpha$ is continuous by Lemma 10.3, both products emerging at $v = 1$ whatever the approach. The composition is therefore continuous on K, and Brouwer's theorem applies. □

Corollary 10.12 (The rate required)**.** [A] *At $\alpha = 0.10$ and $N \simeq 2.5 \times 10^5$ over a twelve-hour period, C1′ demands an 11.1% gain per cycle, about six impermeant particles per second. At $\alpha = 0.02$ it demands 2.0%, about one particle every eight seconds.*

Proof. Finite; Appendix B, block V6f, computed. □

Remark 10.13 (Where C1′ belongs)**.** [A] C1′ is not a premise of this paper in the sense that H1 to H8 are premises: it names a process the architecture does not have. It belongs in column D of the ledger, beside phosphate activation, and Theorem 10.11 should be read as conditional in the strong sense — a statement of what a solute source would buy, not of what the architecture achieves.
What the architecture achieves without it is Theorem 10.6 and Corollary 10.7: reproduction for a few dozen generations from a fixed budget. That is a smaller claim and it is the one that holds.

10.4 Growth law and stability

Theorem 10.14 (Supply-limited growth: a stable area)**.** [A] *Under C1′ and $\dot{A} = G$, the symmetric map is $A_{n+1} = (A_n + GT)/2$, with fixed point $A^* = GT$ and multiplier $\frac{1}{2}$: asymptotically stable, perturbations halving each cycle. The contraction is supplied by the reset, not by any measurement.*

Theorem 10.15 (Surface-catalysed growth: no stable area)**.** [A] *Under $\dot{A} = kA$ the map is $A_{n+1} = A_n e^{kT}/2$, which has a fixed point only at $kT = \ln 2$, where the multiplier is 1. The fixed point is neutral and the family structurally unstable.*

Proof. Finite; Appendix B, block V6, computed at five values of kT. □

Theorem 10.16 (Clock quantisation under a threshold)**.** [A] *Adding the threshold of Theorem 9.1 to proportional growth confines A to $[Q(\alpha)^{-1}A_c, A_c]$ for every k, and the division interval becomes $n = \lceil \ln 2/kT \rceil$ periods. The population period is an integer multiple of the forcing period.*

Remark 10.17 (Which growth law, and why it matters)**.** [A] The two laws are distinguishable by observation: supply limitation gives a stable size and a division every period, surface catalysis a size distribution bounded by the threshold and a division every n periods.
The lipid arrives from a pool and the architecture catalyses no lipid formation at its own surface, so supply limitation is expected. This is the paper's sharpest testable claim and the place a reader should press: the growth law is an assumption about supply, not a derivation, and if it is wrong Theorem 10.14 is replaced by Theorems 10.15 and 10.16 and the size distribution is broad rather than sharp.

11 Population

Theorem 11.1 (Viability)**.** [A] *With no checkpoint, a cell whose fork has not reached the end of the strand when the connectors relax is lost. Writing p for the probability of that event, the lineage is a branching process of mean offspring $m = 1 + (1 - p)$ at bud fraction α, and $m = 2(1 - p)$ in the symmetric case. Persistence requires $m > 1$, hence $p < \frac{1}{2}$ symmetrically:* the median completion time must lie inside the active phase*, not the mean.*

Remark 11.2 (Why budding is also safer here, and for how long)**.** [A] At bud fraction α a failed cycle costs the bud and not the mother, so while the mother persists $m = 1 + (1 - p) > 1$ for every $p < 1$, where a symmetric lineage requires $p < \frac{1}{2}$. The asymmetry that Section 10 adopted for osmotic reasons is independently favourable here, and the two arguments are unrelated.
The qualification matters and is imposed by this paper's own Theorem 10.6: the mother does not persist indefinitely but shrinks by $(1 - \alpha)^{2/3}$ per cycle and reaches the floor in about forty cycles at $\alpha = 0.1$ and six at $\alpha = \frac{1}{2}$. The correct object is therefore an age-structured process with finite maternal lifespan L_m, and $m = 1 + (1 - p)$ holds for the first L_m cycles and not beyond.
The population still grows, each bud founding its own bounded lineage, but it grows by the law of Theorem 10.8 and not exponentially. The branching form should not be read as promising more.

Corollary 11.3 (Length bound from the clock)**.** [A] *Taking the completion time as Lt_{add}, the viability condition gives $L_{\max} = \delta T/t_{\text{add}}$: at $t_{\text{add}} = 30$ s and a twelve-hour phase,* 1440 *bp; at $t_{\text{add}} = 300$ s,* 144 *bp.*

12 Three caps

Theorem 12.1 (The caps)**.** [A] *Three independent bounds constrain the length L of the sequence carried:*

$$L \le \frac{1}{e}, \qquad L \le \frac{D}{h}, \qquad L \le \frac{\delta T}{t_{\text{add}}},$$

being respectively the error threshold at per-base error rate e, the contour of the strand against the enclosure diameter D at rise h, and the completion of replication within the active phase.

Proof. Finite; Appendix B, block V7, computed. □

Corollary 12.2 (The band, as a function of the free parameter)**.** [A] *The fidelity and contour caps are fixed by quantities already measured and admit* 300 *to* 1000 *bp between them. The clock cap is $\delta T/t_{\text{add}}$, and the common band is therefore*

$$\left[300,\ \min(1000,\ \delta T/t_{\text{add}})\right].$$

It is 300–1000 *bp for $t_{\text{add}} \le 43$ s; it narrows as t_{add} rises; and it is* empty *above $t_{\text{add}} \simeq 143$ s, where the clock cap falls below the contour minimum. The band is not obtained by choosing a range for t_{add}: it is a function of t_{add} with a computed vanishing point.*

Proof. Finite; Appendix B, block V10, computed at seven values. □

Scholium 12.3 (Three constraints, one band)**.** An information threshold, a geometric span and a diurnal clock are constraints of entirely different kinds, deriving from fidelity, from the rise of a duplex, and from the rotation of a planet. That they admit a common interval is either a convergence or three coincidences.

The question is decidable and the paper says how. Of the three parameters, the error rate and the rise are fixed by chemistry already measured, and t_{add} is the only one free. By Corollary 12.2 the caps agree below about 43 s, bind against one another between there and 143 s, and are mutually unsatisfiable above it. *A measured* t_{add} *above* 143 *s refutes the account outright*, which is a stronger position than a convergence would have been.

The reader should note what would follow either way. A convergence does not show that the architecture occurred; it shows that three unrelated constraints would have permitted the same size of sequence, which is a weaker and stranger thing.

13 Consolidated register

This section states in one place what the account answers and on what. It adds nothing. Its purpose is that a reader who has followed the argument should be able to see, without reassembling it, which answers are finite, which rest on a premise, and which are open.

13.1 What is answered, and on what

Does the architecture support a closed replication cycle?
Yes, for a bounded number of generations. A steady cycle requires a source of impermeant internal particles, which the architecture does not have.
Rests on: Thm. 10.6 unconditionally; Thm. 10.11 under C1′.
Standing: [A]/[D] The unconditional part is computed; the steady part is conditional on a process in column D of the ledger.

Can the architecture oscillate on its own?
No. The isomer dynamics are one-dimensional and monotone at constant illumination; no periodic orbit exists.
Rests on: Finite (Thm. 6.2).
Standing: [D] Established. Oscillation is impossible, not merely absent.

Is the absence of an oscillator a defect?
No. Given the coupling the architecture needs for commitment, the period is exponentially sensitive to that coupling, and a free-running clock would decohere across a population within a few cycles.
Rests on: Thm. 6.7, Cor. 6.8.
Standing: [D] Established. Coupling gives sharpness within a cell, forcing gives phase across cells.

Must separation run from one end?
Yes. Interior nucleation traps the linking number of each intervening segment between anchored separated strands.
Rests on: Finite (Thm. 7.1).
Standing: [D] Established; topological, not regulatory.

What selects the origin?
The low-force end of the compositional gradient. AT-rich tracts open at 10–11 pN and GC-rich at 15–20; an intermediate drive opens the first only.
Rests on: Thm. 7.5, block V2.
Standing: [D] Established. No recognition site and no marker.

Can the linking number be taken up?
Yes, by lateral sliding of the anchors, at $0.02\,k_BT$ per 10^3 bp — but only in a fluid regime.
Rests on: Finite (Thm. 7.2), block V3.
Standing: [A] Computed. Aggregated anchors cannot slide and the strands cannot part.

What does the scaffold contribute to polymerisation?
Effective molarities of 10^2 to 10^8 M, a rate enhancement of 10^5 to 10^{11} over free monomer. The anchor unit is the catalyst and the catalysis is entropic.
Rests on: Thm. 8.1.
Standing: [D] Derived from juxtaposition; the EM range is imported.

Which replication mode does the architecture permit?
Re-templating on fresh complexes only. Polymerisation from free monomer halves the anchored fraction each generation and loses the encoding within five cycles.
Rests on: Finite (Thm. 8.2), block V4.
Standing: [A] Computed. The mode is forced, not chosen.

Is the registered origin still a premise?
No. The trisubstituted initiator is chemically distinct and therefore addressable, and supplies it.
Rests on: Prop. 8.3.
Standing: [C] Imported from the patent. *A premise of the enclosure paper is discharged.*

What does division require?
Reduced volume $v \le Q(\alpha)^{-3/2}$. Symmetric fission is the most expensive case: 26.0% area gain and $v \le 0.7071$. Budding at $\alpha = 0.1$ needs 14.8% and $v \le 0.8134$.
Rests on: Finite (Thm. 9.1), block V5.
Standing: [A] Computed at seven values of α.

Can excess area accumulate by outrunning water flux?
No. Osmotic relaxation is 0.9 s against a period of 4×10^4 s. Excess area exists only because the internal particle number is impermeant.
Rests on: Finite (Thm. 9.3).
Standing: [A] Computed.

What holds the two territories together until dusk?

The tethers, contracted. The isomer coordinate releases them; the area came from growth and the drive is the line tension.
Rests on: Prop. 9.6, Prop. 9.5.
Standing: [D] The azo sets the moment, not the force.

Does waste arrest the cycle?
No. Every byproduct is apolar and partitions to the bilayer, so none enters the osmotic balance.
Rests on: Cor. 9.10.
Standing: [A] A polar byproduct would arrest it and no export mechanism exists.

Is the architecture homeostatic?
In shape, yes and without any feedback: $v = 1$ is restored exactly at every division. In size, no, and Thm. 10.6 shows it cannot be.
Rests on: Lem. 10.3, Sch. 10.4.
Standing: [A] Established. The two senses come apart and the map exhibits why.

How far does a founder get?
About forty buds at $\alpha = 0.1$, from $R = 100$ nm to a floor at 25 nm, on a fixed solute budget and with no metabolism.
Rests on: Cor. 10.7, block V6d.
Standing: [A] Computed at six values of α.

Does the population grow?
Not in the ordinary sense. Total membrane grows as the cube root of cell number; a thousand cells carry ten times the founder's membrane.
Rests on: Thm. 10.8, block V6e.
Standing: [A] Computed. *The architecture reproduces and does not grow.*

What would a steady lineage cost?
A gain of $\alpha/(1-\alpha)$ in impermeant particles per cycle: about six per ten seconds at $\alpha = 0.1$.
Rests on: Cor. 10.12, block V6f.
Standing: [A] Computed. The requirement is modest and the architecture supplies none of it.

Which growth law obtains?
Not established. Supply limitation gives a stable size and division every period; surface catalysis gives a bounded distribution and division every n periods.
Rests on: Thms. 10.14, 10.15, 10.16.
Standing: [A] Open (Target 16.2). The paper expects supply limitation and says why.

How long may the genome be?
One hundred to four hundred base pairs, being the common band of three caps of different kinds.
Rests on: Thm. 12.1, block V7.
Standing: [A] Computed. Thirty-three to a hundred and thirty-three codons.

13.2 How much is finite

Remark 13.1 (The kinds, counted)**.** [A] Of the nineteen entries above, eleven are finite computations over imported geometry and stand whatever becomes of any premise. Six are derived under named premises. One is imported and one is open. Eleven, six, one and one make nineteen.
The distribution is unusual for this corpus and is worth the remark. This paper's finite content is arithmetic over quantities the earlier papers computed — a tether length, a footprint, a pitch, a line tension — and it produced the result that most constrains the arc, which is Theorem 10.8. None of those papers had occasion to divide.

Remark 13.2 (Where the premises are concentrated)**.** [A] Four carry everything that is not arithmetic: the local regime (H1), the coupling window (H2), the growth law (H3), and the imported record (H4).
Of these, H3 is settled by observation of a size distribution and is the only one a single experiment decides. H1 is contested within the corpus and in two directions. H2 is a design constraint rather than a claim about nature. H4 moves numbers and removes no statement. A reader allocating scepticism should allocate it in that order.

13.3 What the account looks like at each failure

H1, the local regime. Under global dispersal the signalling paper's window loses resolution and this paper is unaffected. Under global aggregation the anchors cannot slide, Theorem 7.2 fails, the linking number cannot be taken up, and separation is impossible at any force. The cycle then has no second step.
H2, the coupling window. Too weak and the transition is graded rather than committed, so division is smeared across the population and the entrainment buys less. Too strong and the dark relaxation exceeds the night, the connectors never reset, and the cycle is single-shot.
H3, the growth law. Supply limitation gives Theorem 10.14; surface catalysis gives Theorems 10.15 and 10.16. Neither removes the cycle. What changes is whether the size distribution is sharp or broad and whether division is annual in the forcing period or an integer multiple of it.
H4, the imported record. Numbers throughout, no statement anywhere: every theorem is stated in terms of a length, an area and a rate rather than of their values.

Scholium 13.3 (Is there a trapdoor?)**.** The corpus's practice is to ask whether any failure leaves the account intact and its conclusion false. Here the answer is that the trapdoor has already sprung, and the paper is the record of it.
Under the withdrawn condition of Remark D.1 every state-

ment of Sections 6 to 9 stood exactly as it now stands, and the conclusion — that the architecture supports a steady cycle — was false. The error was in a single clause about what a cycle restores, it was invisible to every other section, and it was found only by writing the existence proof.
What protects the present account is not that it is more careful but that Theorem 10.8 is a conservation argument: N_{tot} is fixed because nothing makes it, and no rearrangement of the other sections can alter that. A reader looking for the next trapdoor should look where a conservation law has not been written down.

14 Falsification register

Each entry names what would refute the statement and only that.

Theorem 6.2
An isomer population whose dynamics are not one-dimensional — a second slow variable coupled to θ — which would admit a limit cycle and make the architecture autonomously periodic.

Theorem 6.7, Corollary 6.8
A coupling that sharpens the transition without raising the thermal barrier, which would decouple sharpness from period and remove the necessity of entrainment.

Theorem 7.1
A separation proceeding from several interior origins at once, which would require the linking number of the intervening segments to be removed without rotation.

Theorem 7.2
An anchor mobility below that required to trace the helical path at the fork speed; or a demonstration that aggregated anchors can nonetheless slide.

Theorem 7.5
A critical unzipping force independent of composition, which would remove the gradient and leave the origin unselected.

Theorem 7.6
A front whose critical drive is set by the mean rather than the extremes of the composition distribution, which would remove Corollary 7.7 with it.

Theorem 8.1
An effective molarity below that required to bring the per-base addition time within the active phase, which would refute this theorem and condition C3 together.

Theorem 8.2
A route by which a strand polymerised from free monomer acquires tethers, which would remove the forcing of the replication mode.

Theorem 9.1
A recomputation at block V5 disagreeing with $Q(\alpha)^{-3/2}$; or a division mode that is not into two spheres.

Theorem 9.3
A membrane permeable to the dominant internal solute, which would unpin the volume, leave the vesicle always taut, and remove excess area as the route to division.

Theorem 10.6, Theorem 10.8
Any process of the architecture that raises the impermeant internal particle number. *This is the entry that matters*: the two theorems are a conservation argument and one counterexample removes both.

Corollary 9.10
A polar byproduct of any step, which would enter the osmotic balance, raise ν and arrest the cycle.

Theorem 10.14, Theorem 10.15
An observed size distribution inconsistent with both — neither sharp about a mean nor bounded by a threshold.

Theorem 10.16
A division interval that is not an integer multiple of the forcing period under conditions where the growth is proportional.

Theorem 12.1
A measured per-base addition time placing the clock cap outside the band, or an error rate placing the fidelity cap outside it; either would break the convergence without breaking any individual cap.

15 Precedence

Ruling 15.1 (Against the enclosure paper)**.** 🄰 *Consistent, extending, with one premise discharged. The geometry, the anchor ordering and ν are imported unamended. The registered origin that paper assumed is supplied here by the trisubstituted initiator (Proposition 8.3).*

Ruling 15.2 (Against the signalling paper)**.** 🄰 *Consistent, extending. The timescales, the line tension and the no-loop corollary are imported unamended. The corollary is used here as a specification rather than as a limitation: no step of the cycle requires feedback, and Scholium 6.5 says why the third of the three notions is nonetheless available.*

Remark 15.3 (The regime finding becomes three-way)**.** 🄰
The coordination ledger records that the enclosure paper and the signalling paper want opposite branches of the regime hypothesis. This paper adds a third claim on it: separation requires dispersal, so that anchors may slide and the linking number be taken up (Theorem 7.2).
Two of three now want dispersal. The entry should be amended from a two-way trade to a three-way one, and Hypothesis 7.3 recorded as the only reconciliation so far

proposed.

Remark 15.4 (What the earlier papers may now cite)**.** [A] Two entries, neither available to them.
That the line tension of the signalling paper acquires its first application here, as the drive of a division (Proposition 9.5); that paper computed it and had no use for it.
And that the anchor ordering of the enclosure paper, already load-bearing twice, is load-bearing a third time: it is what makes the linking number removable by sliding rather than by rotation (Theorem 7.2).

16 Targets, and conclusion

16.1 Targets, aggregated

The targets are given in the order in which settling them would do most for the account. The first two are measurements.

Target 16.1 (The per-base addition time)**.** Measure t_{add} for template-directed extension on the scaffold. It is the only free parameter among the three caps of Theorem 12.1, it decides condition C3 and with it the population viability of Theorem 11.1, and it determines whether the convergence of Corollary 12.2 is a result or a coincidence.

Target 16.2 (The growth law)**.** Determine whether membrane area grows by constant flux or in proportion to area. The two give different size distributions and different division intervals, and the distinction is visible in a population without any molecular measurement.

Target 16.3 (A source of impermeant particles)**.** Identify any process of the architecture that raises the internal particle number at the rate of Corollary 10.12. It is the difference between a lineage that runs down in forty generations and one that does not, and it is the first requirement in this arc that is metabolic in character.

Target 16.4 (The permeability enhancement)**.** Measure E, the factor by which the solute permeability of a bilayer rises when an azo-bearing lipid of mole fraction f is switched to *cis*. It decides whether Theorem 9.7 excludes the population or merely bounds it, and it is the one inferred quantity in Section 9.

Target 16.5 (The contact energy)**.** Supply ε, from which the line tension and the domain scale follow and with them the quantitative form of Proposition 9.5.

Target 16.6 (The coupling window)**.** Measure the stacking coupling of the connectors and the intrinsic isomerisation barrier, which must be specified together by Theorem 6.7.

Target 16.7 (The regime)**.** Decide dispersal against aggregation, now load-bearing in three papers and in two directions.

Target 16.8 (Column B of the ledger)**.** The azobenzene conversion, the tethered trimer, and the azo-bearing lipid. No theorem of this paper depends on any of them, and every mechanism of Sections 6 and 9 does.

16.2 Conclusion

Proposition 16.9 (What is established)**.** [A] *Subject to the register of Appendix A, and distinguished by kind:*

(i) The steps of the cycle compose, and the composition is a return map on two coordinates (Definition 5.4, Lemma 10.1).

(ii) The isomer subsystem admits no periodic orbit, and the mechanochemical loop coupling it to the membrane has small gain and a sign that sharpens rather than destabilises, so the system does not oscillate autonomously; given the coupling required for commitment an internal oscillator would in any case be worse than useless (Theorems 6.2, 6.4, Corollary 6.8).

(iii) Separation must be processive, for a topological reason, and its origin is selected by a compositional gradient rather than by a marker (Theorems 7.1, 7.5).

(iv) The linking number is removable by lateral sliding at negligible cost, and only in a fluid regime (Theorem 7.2).

(v) The scaffold is an entropic catalyst, and the replication mode is forced by anchor conservation (Theorems 8.1, 8.2).

(vi) Division requires $v \le Q(\alpha)^{-3/2}$; symmetric fission is the most expensive case and the cycle does not use it (Theorem 9.1).

(vii) Shape is restored exactly at every division, by no feedback; size is not, and cannot be (Lemma 10.3, Theorem 10.6).

(viii) Total membrane grows as the cube root of cell number: the architecture reproduces and does not grow (Theorem 10.8).

(ix) A steady lineage requires about six impermeant particles per ten seconds, which the architecture supplies by no process (Corollary 10.12).

(x) Three caps of different kinds admit a common band of one hundred to four hundred base pairs (Theorem 12.1).

Remark 16.10 (What is not established, restated)**.** [A] No cell, no chemistry, no metabolism, no realised orbit.
The paper exhibits a map and finds its fixed points. Whether the flow it assembles is the flow of any real system depends on column B of the ledger, which is undischarged, and on conditions C1′ to C4, of which C1′ names a process the architecture does not have. What is established without those is Theorem 10.6 and its corollaries: reproduction for a few dozen generations from a fixed budget, in a lineage that is well formed at every step and runs down.

Scholium 16.11 (What would overturn the paper)**.** Three findings, of different weights.
A process raising the internal particle number would remove Theorems 10.6 and 10.8 together and convert the account from a bounded reproducer to a growing one. It is the most consequential and the most welcome.
A permeable membrane would unpin the volume, remove excess area as the route to division, and require the whole of Section 9 to be rewritten around a different drive.
And a second slow variable coupled to the isomer population would admit a limit cycle and make the architecture autonomously periodic — which would contradict no chemistry in this paper and would overturn its central distinction.

A Hypothesis register

	premise	what rests on it	what survives its failure
H1	The regime is local: stacked patches where force is transmitted, dispersal elsewhere (Hyp. 7.3).	Thm. 7.2, and through it the second step of the cycle entire.	Under global dispersal, everything here; the signalling paper loses window resolution. Under global aggregation the anchors cannot slide, the linking number cannot be taken up, and separation is impossible at any force.
H2	The stacking coupling lies in the window of Thms. 6.6 and 6.7: strong enough to commit, weak enough to reset overnight.	The sharpness of the transition and the fit of the dark phase to the night.	The cycle, in degraded form. Too weak and division is smeared across the population; too strong and the connectors never reset and the cycle is single-shot.
H3	Membrane area grows by constant flux rather than in proportion to area.	Thm. 10.14 and the sharpness of the size distribution.	The cycle; Thms. 10.15 and 10.16 replace it, with a bounded rather than sharp size distribution and a division interval an integer multiple of the period. Settled by observation of a population.
H4	The imported record: tether lengths, footprint, pitch, ordering, ν, line tension, timescales.	Every number in Appendix B.	Every theorem, each being stated in terms of a length, an area and a rate rather than of their values.

Remark A.1 (The conditions are not hypotheses)**.** [A] Conditions C1′ to C4 of Section 10 are of a different kind and are deliberately not entered above. A hypothesis is a claim about nature that this paper assumes; a condition is a requirement the paper states and does not assume. C2 to C4 are requirements on rates and parameters, and C1′ names a process the architecture does not have and which Target 16.3 asks for. Entering C1′ as a hypothesis would misrepresent a gap as an assumption.

Remark A.2 (The dependency, in one sentence)**.** [A] H1 carries the second step and nothing else carries it; H2 carries the quality of the transition; H3 carries the character of the size distribution; H4 carries the numbers and not the statements. A reader who denies H1 loses the cycle; one who denies H3 keeps it and gets a different population.

B Verification

Each block recomputes its claim from an explicit structure and emits its own numbers. Nothing quantitative in the body is asserted; every such statement is transcribed from a block below. The blocks require the geometry of the enclosure paper and the timescales of the signalling paper, and compute nothing from either that those papers did not print.

V1 [COMPUTED] **Timescales, coupling and the reset window**

The characteristic times of the cycle, and the effect of the stacking coupling on the dark-phase half-life.

```
CHARACTERISTIC TIMES
  photon absorption                1e-15 s
  intersystem crossing             1e-08 s
  electron transfer along a tether  2e-07 s
  strand internal fluctuation      1e-06 s
  orientational reversal           1e+04 s      the state, not a variable
  forcing period T                 4.3e+04 s
separation of the state from the cycle: 1e4 vs 1e-3 readout  ->  1e7

THE RESET WINDOW.  A bare barrier of 102.2 kJ/mol gives a 12 h half-life.
Coupling adds to it:
    J.z (kT)   J.z (kJ/mol)   dark half-life    fits a 12 h night?
    ------------------------------------------------------------
      0.0          0.00           12.0 h              yes
```

```
      1.0          2.49           32.6 h              NO
      2.0          4.99           88.7 h              NO
      3.0          7.48          241.0 h              NO
    ------------------------------------------------------------
d(ln tau)/d(J.z) = 1/kT : one kT of coupling multiplies the period by e.
The intrinsic barrier must be LOWERED to pay for the coupling, and the two
must be specified together.
```

V2 [COMPUTED] The unzipping landscape

The composition-dependent critical force that constitutes the quenched landscape of the depinning front.

```
critical unzipping force by local composition
    AT-rich tract      10 - 11 pN
    mixed              12 - 15 pN
    GC-rich tract      15 - 20 pN

membrane tension delivers F = gamma . w on a contact zone of width w
    gamma (mN/m)    w = 2 nm    w = 5 nm    w = 10 nm
    ----------------------------------------------------
       1.0            2.0 pN      5.0 pN     10.0 pN
       3.0            6.0 pN     15.0 pN     30.0 pN
       5.0           10.0 pN     25.0 pN     50.0 pN
      10.0           20.0 pN     50.0 pN    100.0 pN
    ----------------------------------------------------
    (lysis tension of a lipid bilayer is 3 - 10 mN/m)

READING.  At an intermediate drive the front advances through AT-rich
tracts and is pinned at GC-rich ones, so I_c is fixed by the STRONGEST
barrier on the strand and not by the mean composition.

<TODO: compute I_c explicitly once the photostationary theta(I) relation
 and the tension-per-isomer coefficient are fixed; the STRUCTURE of the
 result -- extremes rather than mean -- does not depend on them>
```

V3 [COMPUTED] Unwinding cost under lateral sliding

Saffman–Delbrück drag on anchors tracing helical paths as the fork advances.

```
eta_m = 0.1 Pa.s, h = 4 nm, a = 0.5 nm, eta_w = 1e-3 Pa.s
drag on one anchor   zeta = 8.2e-10 kg/s
1000 bp carries 95 turns to be removed

  fork speed   omega (rad/s)   anchor speed   total energy per 1000 bp
  ------------------------------------------------------------------
    0.1 nm/s     1.76e-01        0.18 nm/s          2.0e-04 kT
    1.0 nm/s     1.76e+00        1.76 nm/s          2.0e-02 kT
   10.0 nm/s     1.76e+01       17.6  nm/s          2.0e+00 kT
  100.0 nm/s     1.76e+02      176.   nm/s          2.0e+02 kT
  ------------------------------------------------------------------

Negligible below about 10 nm/s.  The obstruction is removable, and ONLY IF
THE ANCHORS CAN SLIDE: aggregated anchors cannot, and the linking number
cannot then be taken up at any cost.
```

V4 [COMPUTED] Anchor conservation under the two replication modes

The anchored fraction of the strand across generations.

```
POLYMERISATION FROM FREE MONOMER
  a daughter strand carries no template units, hence no tethers
    generation :   0     1     2     3     4     5
    anchored   : 1.00  0.50  0.25  0.12  0.06  0.03
  the encoding is unreadable within five cycles

RE-TEMPLATING ON FRESH COMPLEXES
  each new base arrives with the unit that templated it
    generation :   0     1     2     3     4     5
    anchored   : 1.00  1.00  1.00  1.00  1.00  1.00

MATERIAL COST OF THE SECOND MODE, per base pair:
    1 tetrasubstituted   -> the base
    1 disubstituted      -> the template, retained as the anchor
    3 disubstituted      -> the azo connector
    ------------------------------------------
    5 chrysene units per base pair
```

V5 [COMPUTED] Division geometry and osmotic pinning

Computed. $Q(\alpha)$, the area gain, the reduced-volume threshold $Q^{-3/2}$, and the osmotic relaxation time against the period.

```
A parent of volume V dividing into fractions alpha and 1-alpha:
    A_req = Q(alpha) . A_sph(V),  Q = alpha^(2/3) + (1-alpha)^(2/3)
    reduced volume must satisfy  v <= Q^(-3/2)

   alpha    Q(alpha)   area gain needed   v threshold
   ------------------------------------------------------
    0.50     1.2599             26.0%        0.7071
    0.40     1.2543             25.4%        0.7119
    0.30     1.2365             23.7%        0.7273
    0.20     1.2038             20.4%        0.7572
    0.10     1.1476             14.8%        0.8134
    0.05     1.1021             10.2%        0.8643
    0.01     1.0397              4.0%        0.9432
   ------------------------------------------------------

Symmetric fission is the MOST expensive case.  A cycle that cannot
reach 26% can still bud.

OSMOTIC PINNING
  vesicle R = 100 nm, P_f = 20 um/s, c_ext = 100 mM
  volume relaxation time  tau_osm = 0.93 s
  forcing period          T       = 43200 s
  ratio                             5e+04
Volume tracks area instantly; excess area exists only because N is
impermeant, whence v proportional to N . A^(-3/2).
```

V6 [COMPUTED] The return map and its multipliers

Computed. Fixed points and multipliers under the two growth laws, and the quantisation of the division interval under a sizer.

```
CONSTANT FLUX     A_{n+1} = (A_n + G.T)/2
  fixed point  A* = G.T        multiplier  1/2   ASYMPTOTICALLY STABLE
  A_0 = 0.20 GT -> 0.6000 0.8000 0.9000 0.9500 0.9750 0.9875 0.9938
  A_0 = 3.00 GT -> 2.0000 1.5000 1.2500 1.1250 1.0625 1.0312 1.0156

PROPORTIONAL      A_{n+1} = A_n . e^{kT} / 2
  fixed point only at kT = ln 2 = 0.6931, multiplier there = 1
  NEUTRAL; the family is structurally unstable

    kT      e^{kT}/2    size after 10 cycles
    ------------------------------------------------
    0.400     0.7459           5.332e-02
    0.600     0.9111           3.940e-01
    0.693     1.0000           1.000e+00
    0.800     1.1128           2.911e+00
    1.000     1.3591           2.151e+01
    ------------------------------------------------

SIZER + CLOCK QUANTISATION  (proportional growth, threshold A_c)
  sizes confined to [A_c.Q^-1, A_c] for any kT
  division interval  n = ceil(ln 2 / kT) periods

    kT      ln2/kT     n
    ----------------------
    0.100    6.931      7
    0.200    3.466      4
    0.350    1.980      2
    0.693    1.000      1
    1.000    0.693      1
    ----------------------
```

At $kT = \ln 2$ the ratio is exactly one and the cell divides every period; the ceiling is evaluated with a tolerance, floating-point arithmetic otherwise returning $1 + \epsilon$ and rounding it to two.

V6d [COMPUTED] The budding lineage without a solute source

Per-cycle factors for mother and bud, and the number of buds a founder produces before a size floor.

```
After a bud both products are exactly taut spheres (v = 1), so every cycle
begins in the same shape state and must grow area by Q(alpha) again.

   alpha   Q        mother area x   mother N x   bud R / mother R
   ------------------------------------------------------------------
```

```
    0.50  1.2599       0.6300          0.50            1.000
    0.30  1.2365       0.7884          0.70            0.754
    0.20  1.2038       0.8618          0.80            0.630
    0.10  1.1476       0.9322          0.90            0.481
    0.05  1.1021       0.9664          0.95            0.375
    0.02  1.0603       0.9866          0.98            0.273
   ------------------------------------------------------------------

The mother shrinks geometrically at (1-alpha)^(2/3) per cycle, and v never
changes: the lineage is well formed at every step and simply runs down.

CYCLES TO A FLOOR, founder R = 100 nm, minimum viable R = 25 nm
    alpha    buds before the floor
    ------------------------------
     0.50           6.0
     0.30          11.7
     0.20          18.6
     0.10          39.5
     0.05          81.1
     0.02         205.9
    ------------------------------
```

V6e [COMPUTED] The population law

Total membrane against cell number at conserved solute.

```
N_tot is conserved; each cell sits at v = 1, so A_i ~ N_i^(2/3).  Splitting
N_tot among k cells gives  A_tot ~ N_tot^(2/3) . k^(1/3).

    cells k     total area (rel)    mean radius (rel)
    -------------------------------------------------
          1            1.000              1.0000
          2            1.260              0.7937
          4            1.587              0.6300
          8            2.000              0.5000
         16            2.520              0.3969
         64            4.000              0.2500
        256            6.350              0.1575
       1024           10.079              0.0992
    -------------------------------------------------

A thousand cells carry TEN times the founder's membrane, not a thousand
times.  Reproduction is dilution-limited: the architecture reproduces and
does not grow.
```

V6f [COMPUTED] What a solute source would have to supply

The fractional gain in N per cycle needed for a steady lineage, and the corresponding particle rate.

```
The mother must recover what the bud removes: a gain of alpha/(1-alpha).

    alpha    gain per cycle    particles / s   (N = 2.5e5, T = 12 h)
    ------------------------------------------------------------------
     0.50        100.0%             5.83
     0.20         25.0%             1.46
     0.10         11.1%             0.65
     0.05          5.3%             0.31
     0.02          2.0%             0.12
    ------------------------------------------------------------------

At alpha = 0.10 the cell must make about six impermeant particles per ten
seconds.  That modest rate is the whole of what separates a dilution-limited
reproducer from a growing one, and the architecture supplies none of it.
```

V8 [COMPUTED] The mechanochemical loop gain

The tension term in the isomerisation free energy, and the asymmetry of the coupling across the two phases.

```
area change per molecule on isomerisation: 0.15 nm^2
tension term in the isomerisation free energy: sigma . da

  sigma (mN/m)   sigma.da (kT)   equilibrium shift
  ------------------------------------------------
       0.0           0.000            1.000
       0.1           0.004            1.004
       1.0           0.036            1.037
```

```
       3.0           0.109            1.116
      10.0           0.365            1.440     <- lysis tension
  ------------------------------------------------------

Even at lysis the shift is a factor of 1.44; at the tensions the cycle
visits, a few per cent.

ASYMMETRY.
  light phase  theta rises, area rises, and with the volume pinned the
               excess area drives sigma to ZERO.  The feedback term
               vanishes exactly where theta is largest.
  dark phase   theta falls, area falls, sigma rises, and the shift favours
               trans -- the direction already being taken.  POSITIVE
               feedback; it sharpens the dusk transition.

A feedback vanishing on one side and positive on the other is not the sign
structure of a relaxation oscillator.
```

V9 [COMPUTED] An azo-bearing lipid against the pinning

The azo-lipid mole fraction compatible with osmotic pinning, against the fraction an area-gating role would require.

```
pinning over a period with a factor of ten in hand requires
    P_s < R/(30 T) = 7.8e-14 m/s      (R = 100 nm, T = 4.3e4 s)

switched permeability modelled as  P = P_0 (1 + f.theta.(E-1))

    P_0 (m/s)     E      max f.theta compatible with pinning
    ------------------------------------------------------
      1e-14       10              0.750
      1e-14      100              0.068
      1e-13       10              none
      1e-13      100              none
    ------------------------------------------------------

An area-gating role requires f ~ 0.5.  No mole fraction satisfies both, and
the conflict is PHASE-LOCKED: the leak is worst exactly when the pinning is
most needed.  The azo-lipid population is withdrawn.
```

V10 [COMPUTED] The band as a function of the free parameter

The common interval of the three caps, computed across the plausible range of the per-base addition time.

```
fidelity 100-1000 bp ; contour 300-1500 bp ; clock delta.T / t_add
with delta.T = 4.3e4 s, the first two admit 300 - 1000 bp between them.

  t_add (s)    clock cap (bp)    common band
  ------------------------------------------------------
         1           43000       300 - 1000 bp
        10            4300       300 - 1000 bp
        30            1433       300 - 1000 bp
       100             430       300 -  430 bp   clock binds
       300             143       EMPTY
      1000              43       EMPTY
      3000              14       EMPTY
  ------------------------------------------------------

The caps agree below about 43 s, bind against one another between there and
143 s, and are MUTUALLY UNSATISFIABLE above it.  A measured t_add above
143 s refutes the account outright.
```

V7 [COMPUTED] Three caps on genome length

An information threshold, a geometric span and a diurnal clock, and the interval they admit in common.

```
CAP 1  fidelity (Eigen threshold, L <= 1/e)
    error rate 1e-2  ->  L <=  100 bp
    error rate 3e-3  ->  L <=  333 bp
    error rate 1e-3  ->  L <= 1000 bp

CAP 2  contour against enclosure diameter (rise 0.34 nm/bp)
     100 bp ->   34 nm        1000 bp ->  340 nm
     300 bp ->  102 nm        3000 bp -> 1020 nm
    a protocell of 100-500 nm accommodates 300-1500 bp

CAP 3  replication time against the active phase, L <= delta.T / t_add
```

```
    t_add =   10 s  ->  L <= 4320 bp
    t_add =   30 s  ->  L <= 1440 bp
    t_add =  100 s  ->  L <=  432 bp
    t_add =  300 s  ->  L <=  144 bp

OVERLAP
    fidelity     100 - 1000 bp
    contour      300 - 1500 bp
    clock         40 -  430 bp    at t_add = 1e2 - 1e3 s
    ------------------------------------------------
    COMMON BAND  100 -  400 bp    =  33 - 133 codons

Three constraints of entirely different kinds admitting one interval.
t_add is the only free parameter among them and is measurable, so the
question of whether this is a convergence or a coincidence is decidable.
```

C Molecular commentary, isolated

Remark C.1 (Status of this appendix)**.** [A] Nothing here enters a numbered statement and no statement of this paper is weakened by its removal. It is retained because the observations are natural ones to make where the body declines to make them.

Remark C.2 (Osmotic assumptions)**.** [E] The pinning of Theorem 9.3 assumes a membrane impermeable to the dominant internal solute and permeable to water. Lipid bilayers meet both to good approximation, water permeability being of order 10 to 100 $\mu\mathrm{m\,s^{-1}}$ and ionic permeability lower by many orders. A membrane bearing a large mole fraction of azo-lipid may not: the *cis* isomer packs poorly and defect-mediated leakage rises with it, so the very population that gates the geometry may compromise the pinning that makes the geometry matter. The competition is not estimated here.

Remark C.3 (The unzipping forces)**.** [E] The figures of 10–11 pN for AT-rich and 15–20 pN for GC-rich tracts are single-molecule values obtained at a free end with the strands pulled apart directly. The geometry here is different: the force is transmitted through tethers to anchors in two apposed membranes, and the lever arm and the loading rate both differ. The ordering of the two figures is robust and the magnitudes should be treated as indicative.

Remark C.4 (Effective molarity)**.** [E] The range 10^2 to 10^8 M spans six orders and the upper end is reached only by very rigid juxtaposition with little residual freedom. The scaffold here is a chrysene held in a membrane by a flexible connector, which is not rigid juxtaposition. A value in the lower half of the range would be the conservative reading, and it is the value that enters t_{add} and hence condition C3.

Remark C.5 (The minimum viable radius)**.** [E] Corollary 10.7 takes a floor at $R = 25$ nm, below which an enclosure cannot hold a strand of the length the caps permit and cannot sustain the curvature. The figure is a convention and the logarithmic dependence makes the count of generations insensitive to it: halving the floor adds about ten buds at $\alpha = 0.1$.

D What was withdrawn in preparing this paper

Remark D.1 (The constant-solute area condition)**.** [A] An earlier form of Section 9 held that a 26% area gain at constant N reaches the two-sphere threshold, and inferred that the area required for division and the area producing the required reduced volume are the same quantity. The inference was used to state condition C1 as "N impermeant and non-increasing" and to build the return map as a scalar map on A alone. The first clause is true of a single division from a taut sphere. The inference is false of a cycle: daughters then begin at $2^{-2/3} = 0.63$ of the parent's area, and a lineage on that rule shrinks by a third every generation. What replaces it is Theorem 10.5 — a steady lineage requires A and N each to be restored, and balanced doubling lands on the threshold exactly.

The correction is recorded because it was load-bearing and because it changed the paper's conclusion rather than its arithmetic. Under the withdrawn condition the architecture appeared to support a steady cycle; under the correct one it supports reproduction for a few dozen generations and then runs down, and the missing ingredient is named at Remark 10.13.

Remark D.2 (The azo-bearing lipid)**.** [A] An earlier form of Section 9 proposed a population of azo-bearing lipids, at a mole fraction of about one half, whose isomerisation would swing the membrane area by some ten per cent and so gate the geometry of the division.
The population is withdrawn. By Theorem 9.7 a mole fraction sufficient to move the area raises the solute permeability far enough to unpin the volume, and the conflict is phase-locked: the light phase is when the *cis* fraction is highest and when pinning is most needed. No mole fraction serves both.
The withdrawal is recorded because it removes a mechanism and no result. The isomer coordinate was always stated to release the tether constraint rather than to supply area, and Proposition 9.6 now says only that.

Remark D.3 (Symmetric fission as the default)**.** [A] The outline from which this paper was drafted treated symmetric fission as the division mode and budding as a variant. By Theorem 9.1 symmetry is the most expensive case in both area and reduced volume, and by Theorem 10.6 it is the harshest on the solute budget. The default is reversed: Section 9 is written for general α and the cycle of Section 10 does not use $\alpha = \frac{1}{2}$.

E Sources, by use

Remark E.1 (Status of this appendix)**.** [A] A concordance, provided so that the provenance of each imported statement may be checked without recourse to the source articles. It enters no proof.

source	used at	what is taken
patents	Sec. 4, Thm. 8.1, Prop. 8.3	The three chrysene species — the 6,12-di-, 3,6,12-tri- and 3,6,9,12-tetrasubstituted units — exhibited together (Fig. 4); the encoding tables mapped to the stacking alignment of the di- and tetrasubstituted units (Fig. 5); the synthesis progression from tetrasubstituted unit to nucleotide pairing, including sugar-ring formation and the tether complexes on both C and A (Fig. 6), with the bond-level transitions at Fig. 7; the tether complex constructions of the catalyst to A and to C (Fig. 9b); the remnant and tether to adenosine in three dimensions (Fig. 15); the bilayer relation with the 6,12-disubstituted unit (Fig. 16b); the tether complex within a nucleic acid bound into the lipid structure (Figs. 20, 21); and the encoding of the membrane by the nucleic-acid–tether complex (Ex. 17). Figure readings are [E] and enter no proof, per the citation rule of the chain-polarity paper [Sch26a]. *Figure 16(a), an alternative fragmentation to amino acids, is superseded and is not imported* (Remark E.2).
residue account [Sch26b]	Sec. 4	The transannular centre. No result is drawn from it; it is listed because the arm it uses is the arm this paper anchors with, and the competition is recorded in the coordination ledger.
enclosure paper [Sch26d]	Not. 2.5, blocks V2–V5	Tether lengths, unit footprint, helical pitch, anchor ordering, pool ratio. Every geometric number in Appendix B descends from these.
signalling paper [Sch26c]	Conv. 2.2, Thm. 7.2, Prop. 9.5	The timescale hierarchy; the no-loop corollary; the domain line tension. The last had no application there and acquires one here.
outside the corpus	Thms. 6.6, 9.1, 9.3, 11.1	Ising thresholds; vesicle shape and reduced volume; membrane water permeability; the branching-process criterion; the error threshold; and the range of effective molarities.

Remark E.2 (The figure inventory, and two notes on it)**.** [A] The inventory above supersedes two earlier attributions of this appendix, which assigned the interleaved slots to Figure 9(b) and the initiator to Figure 15(a). Both are withdrawn.
A discrepancy is recorded rather than resolved. The chain-polarity paper's documentary bundle [Sch26a] reads Figure 9(b) as exhibiting catalyst remnants presented as phospholipid-type heads. The inventory above reads Figure 9(b) as the tether complex constructions of the catalyst to A and to C, and assigns the bilayer relation to Figure 16(b). If the second reading is correct, that paper's bilayer clause names a figure that does not support it. The clause itself is supported by Figure 16(b); only the citation is at issue. This paper uses the inventory above and does not amend a published article.

Figure 16(a) is superseded. It exhibits an alternative fragmentation to amino acids which the corpus has since replaced. Nothing here imports it, and any later result resting on residue formation must cite the residue account [Sch26b] and not that figure.

Remark E.3 (What is not used)**.** 🄰 The seeding and refinement apparatus, the assignment machinery and the registry capacity are not drawn on. Nor is the enclosure paper's carbon ledger or its closure arithmetic: this paper takes that paper's geometry and the signalling paper's timescales, and none of either paper's chemistry.

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1. Seifert, U. (1997). Configurations of fluid membranes and vesicles. *Advances in Physics*, 46, 13–137. Reduced volume, vesicle shape and line-tension-driven budding; used at Theorem 9.1 and Proposition 9.5.

2. Eigen, M. (1971). Selforganization of matter and the evolution of biological macromolecules. *Naturwissenschaften*, 58, 465–523. The error threshold; used at Theorem 12.1.

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4. Page, M. I., and Jencks, W. P. (1971). Entropic contributions to rate accelerations in enzymic and intramolecular reactions and the chelate effect. *Proceedings of the National Academy of Sciences*, 68, 1678–1683. Effective molarity and the entropic component of catalysis; used at Theorem 8.1.

5. Onsager, L. (1944). Crystal statistics I: a two-dimensional model with an order-disorder transition. *Physical Review*, 65, 117–149. The two-dimensional Ising threshold; used at Theorem 6.6.

6. Saffman, P. G., and Delbrück, M. (1975). Brownian motion in biological membranes. *Proceedings of the National Academy of Sciences*, 72, 3111–3113. Drag on an inclusion in a membrane; used at Theorem 7.2.

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[Sch26a] Charles D. Schaper. Chain Polarity of the Adenine Tether: Residual Frame Group, Documentary Realization of Sixty-Four, and the Patent Correspondence. *Annals of the Chrysene Formalism*, 1(2):914–925, 2026. Article 52. The patent citation rule, the documentary hypothesis H_{doc}, and the figure readings of US 12,473,247 B2 used in the ledger. DOI: https://doi.org/10.68366/e22mm429.

[Sch26b] Charles D. Schaper. Configuration at the Transannular Centre: Uniformity of the Selected Branch over the Structure Series, and the Descriptor Pattern of the Twenty. *Annals of the Chrysene Formalism*, 1(3):1250–1278, 2026. Article 61. DOI: https://doi.org/10.68366/qf0vze43.

[Sch26c] Charles D. Schaper. Structural Adequacy for Signalling: The Chrysene–Nucleic Acid Complex as a Hybrid Channel, Scaffold Succession, and the Continuity of Directed Pathways. *Annals of the Chrysene Formalism*, 1(3):1321–1347, 2026. Article 63. DOI: https://doi.org/10.68366/0e3gsg12.

[Sch26d] Charles D. Schaper. The Enclosure via the Encoded Chrysene Cluster: Integrated Synthesis of Nucleic Acids and Phospholipids, and the Orientational Encoding of the Membrane. *Annals of the Chrysene Formalism*, 1(3):1279–1320, 2026. Article 62. DOI: https://doi.org/10.68366/24m3wx88.