ANNALS OF THE CHRYSENE FORMALISM
RESEARCH ARTICLE | AI-AUGMENTED PEER-REVIEWED | VOL. 1, NO. 3 | OCTOBER 2026

The Persistent Lineage

Composition of the Pore, the Cycle, and the Remnant

Charles D. Schaper, Ph.D.
Chief Editor, Annals of the Chrysene Formalism
San Francisco, California, USA
editor@chrysene.com

Abstract. Four earlier articles of this corpus each settled one piece of a composition that none of them performed. The enclosure gave the vessel its membrane; an earlier article showed that vessel dividing under a periodic force and bounded the founder lineage at roughly forty generations; a further paper found the architecture's only candidate osmolyte and left its supply rate as the open question; and the pore paper supplied the conduit and with it continuous satisfaction of the osmolyte-generation condition. This paper asks what the three together imply for the lineage, and finds two answers.

The first answer lifts a bound. The forty-generation bound was conditional on the absence of a particle source; that condition is now false. Under continuous osmolyte supply the reduced volume falls by a computable factor each cycle, and the return time to the division hyperplane after each budding event is bounded by two to three cycles independently of the generation number. The lineage is persistent.

The argument rests on a structural invariance that holds independently of every parameter. The ratio $\varrho = R/N$ of remnant count to particle count is preserved exactly by both the per-cycle map and the division reset: it evolves as a Möbius contraction toward unity and cannot be held below that limit by any edge that consumes remnants slower than they are produced. The polypeptide paper must discharge the remnant at the production rate, not merely at a slower one. This rate condition on the next article is derived from the symmetry of the map and from nothing else.

The second answer identifies a new bound, not previously seen, that arises from the same edge that generates the osmolyte. The excision emits one apolar chrysene arm per osmolyte particle, and that arm partitions to the bilayer by the same rule that keeps every non-polar species out of the lumen. The ratio of remnant count to particle count is an *invariant* of both the per-cycle map and the division reset: it converges monotonically to one and cannot be held below that limit by any edge consuming remnants at a rate below the production rate. The bilayer remnant count therefore grows without bound, and the lineage exits the safe region of bilayer integrity at a cycle number that depends on α and on the assumed tolerance. To survive forty cycles at a tolerance of five remnants per lipid, α must be below 0.013. This is the fourth argument for small α in this corpus and the first that is a necessary condition rather than a cost preference.

A fifth result, stated as an observation rather than a theorem, names for the first time the energetic basis of the whole programme. The chrysene π-system absorbs UV-A photons; intersystem crossing stores 143 kJ mol^{-1} in the triplet state; proton-coupled electron transfer along the phosphodiester arms drives all synthesis at a quantum yield of five to ten events per photon. The twelve-hour forcing period of the earlier article is the light phase of a diurnal cycle. Neither a metabolic carrier nor a coding apparatus is required because the molecule is simultaneously antenna, energy carrier, catalyst and substrate. Both features are deferred for the same architectural reason: there is no mediator yet to evolve, because the substrate already is one.

Keywords: persistent lineage; hybrid dynamical system; reduced volume; R/N invariance; remnant accumulation; bilayer ceiling; photoentrained cycle; proton-coupled electron transfer; small α; four-article composition

1 Introduction

1.1 What four articles established separately

The enclosure paper fixed the geometry: one family of tetra-substituted chrysenes assembles a membrane-bounded vessel whose area and particle count jointly set a reduced volume $v = V/V_{\mathrm{sph}}(A)$.

The cycle paper showed that vessel to have a periodic orbit under a twelve-hour forcing and derived two consequential limits. Shape homeostasis holds unconditionally: the reduced volume returns to unity at every division with no feedback required. Size homeostasis does not: without a particle source the particle count runs down, the founder lineage reaches a dilution floor at roughly

Citation: Schaper, C. D. (2026). The Persistent Lineage: Composition of the Pore, the Cycle, and the Remnant. *Annals of the Chrysene Formalism*, 1(3), 1405-1413. https://chrysene.com/index.php/acf/article/view/67 © 2026 Charles D. Schaper. Distributed under the CC Attribution-NonCommercial-NoDerivatives 4.0. First published by Schaper Systems Press in the *Annals of the Chrysene Formalism*, Vol. 1, No. 3 (2026). ISSN 3144-2021.

forty generations, and the scholium records this explicitly as a bound conditional on the absence of any osmolyte source.

The material ledger found the only internal candidate osmolyte—the residue, zwitterionic and impermeant—and proved the allocation of the single pool that produces it. It left the supply *rate* as its principal unknown, since neither the ledger nor any preceding article supplied a route by which the required heteroatom feedstock could cross the membrane.

The pore paper supplied that route. A ring of three tetrasubstituted columns, joined laterally by the same phosphodiester chemistry the encoding uses, admits ammonium and excludes any residue bearing a side chain, so the excision that makes the osmolyte can proceed at the rate the cycle paper requires. The pore also showed that a conducting membrane is fatal without selectivity: with fixed anionic head groups a non-selective pore would pin the reduced volume at unity and remove the division condition entirely.

1.2 What the composition asks

None of the four papers put all three pieces together. This paper asks: given a pore that continuously satisfies the osmolyte-generation condition C1′, does the cycle paper's lineage persist, and if so, for how long and under what constraint?

The answer is in two parts. The persistence result lifts the forty-generation bound. The remnant result identifies a new bound whose form was not visible until the excision chemistry was written down as a map.

1.3 The energetic basis, stated once

The twelve-hour forcing period $T = 43{,}200$ s of the cycle paper is the light phase of a diurnal solar cycle. That paper names its orbit "photoentrained" without following the name all the way through; this paper does. The chrysene π-system absorbs UV-A photons, intersystem crossing reaches the T_1 triplet state in 10 ns and stores 143 kJ mol^{-1} there for up to 1 ms, and proton-coupled electron transfer along the phosphodiester arms draws on that store to form covalent bonds at a cost of 10–30 kJ mol^{-1}. Each absorbed photon drives five to ten synthesis events.

Modern metabolism uses a separate carrier—ATP—as the intermediary between energy source and synthesis reaction; modern coding uses a separate apparatus—the ribosome—between the specification and the product. The architecture here defers both for the same reason: the molecule is simultaneously the photon antenna, the energy carrier, the catalyst and the substrate, so there is no mediator yet to evolve. Glycolysis and the genetic code are not initially necessary for the same architectural reason, and that reason is stated here. Block V5.

1.4 What is established, by kind

Imported. C1 and C1′, the division threshold $v_{\mathrm{thr}}(\alpha) = Q(\alpha)^{-3/2}$, the scale parameters N_0, T, L, ν from the cycle paper; the inventory, the excision edge ε, and the sink corollary from the material ledger; the selectivity corollary and the head-group reading H1 from the pore paper.

Proved here. That Article 65's allocation is confirmed by the one-skeleton excision chemistry (Proposition 4.1); that the ratio $\varrho = R/N$ is an invariant of both Φ and ϱ_α (Proposition 6.1); that the lineage is persistent under continuous C1′ discharge (Theorem 7.1); that the return time to $\Sigma(\alpha)$ is bounded by $K(\alpha)$ independently of the generation number (Theorem 7.1); that $R_k \to \infty$ and the orbit exits the safe region at a finite $k_{\max}$ (Theorem 8.1).

Found. That the remnant species W is the first sink in the inventory with no consumption edge; that the crossover condition on α is a necessary condition for survival to the forty-cycle scale; that the route-A/route-C tension of the pore paper is sharpened here into a further argument for route C.

Observed, not proved. That the twelve-hour forcing period is the light phase of a diurnal solar cycle; that the chrysene triplet state is the energy carrier for all synthesis; that neither metabolism nor coding is initially required because the molecule is simultaneously antenna, carrier, catalyst and substrate. These are observations about the corpus's own parameters and molecular architecture, verifiable in part from Article 63's block V1 and Article 64's block V1; they are not theorems. Block V5.

Not claimed. Any mechanism discharging the remnant (Target 9.2); any claim that the architecture fails before $k_{\max}$ (the ceiling is a rate argument); any revision to Article 65's allocation (it is confirmed, not changed); any evaluation of the photon-sufficiency inequality (Target 9.4).

2 Status, scope, and conventions

Convention 2.1 (Status labels)**.** [A] finite or axiom-free; [D] derived under named hypotheses; [C] imported with source named; [E] molecular reading, entering no proof. Inherited from the coordinating paper.

Convention 2.2 (The excision chemistry)**.** [C] Edge ε (excision) is $\mathtt{Nt} + \mathtt{Xh}(N) \to \mathtt{R} + \mathtt{W}$. The tethered nucleotide Nt carries an adenosine unit formed by the tau1 edge (sugar-ring formation, Article 65 block V2). The C–N scission releases the residue R from one substituted position and emits the remaining chrysene arm as W. The adenosine half retains its pairing capacity and remains a functional nucleotide pair after scission.

Cost per residue: one chrysene skeleton and one nitrogen feedstock. Article 65's allocation counts one unit per residue and is confirmed on this reading (Proposition 4.1). The remnant W is a new sink not counted in that allocation.

Convention 2.3 (Remnant partitioning)**.** [C] `W` is apolar (Article 65, column of the species table). By the sink corollary of Article 65, every apolar sink leaves the lumen. For an apolar fragment in a lipid–water system "leaves the lumen" means partitions to the bilayer. This convention is the assumption on which Section 8 rests; it is registered as H1 and its falsification entry names the consequence of a polar `W`.

Convention 2.4 (Scope)**.** [A] The paper computes the consequences of three published results. No new species, no new edges, no new chemistry. The excision edge is the one in Article 65's digraph; the pore is the one in Article 66; the division map is the one in Article 64.

3 Non-implications

Proposition 3.1 (Non-implications)**.** [A] *None of the following is asserted below.*

(i) That the remnant destroys the bilayer at a specific cycle. *The ceiling $k_{\max}$ is a first-exit time from a safe region defined by a tolerance parameter $\varrho_{\max}$ that this paper does not supply from the corpus. The ceiling exists and is finite; the crossing point depends on what the bilayer actually tolerates.*

(ii) That any mechanism discharges the remnant. *The remnant accumulation is structural; it follows from the invariance of ϱ and from $N_k \to \infty$. Discharging it requires an edge consuming `W` at the production rate. The polypeptide paper (Article 68) addresses this.*

(iii) That Article 65's allocation is revised. *It is confirmed. The remnant enters the ledger as a new sink species; it does not replace any existing term.*

(iv) That the route-A head group is ruled out. *The route-A/C tension is a target (Target 9.3), not a resolution.*

(v) That the photon-sufficiency inequality is satisfied. *Block V5 states the inequality; evaluating it requires the absorption cross-section and irradiance, which are not corpus parameters.*

4 The allocation confirmed, and the remnant as a new sink

Proposition 4.1 (The allocation is unchanged)**.** [A] *Under Convention 2.2, the allocation $(f_{\text{anc}}, f_{\text{res}}, f_{\text{lip}}) = (0.024, 0.664, 0.313)$ of Article 65, Theorem (feasible), is confirmed at $\alpha = 0.10$ and does not change on the one-skeleton reading. Block V1.*

Proof. Finite; Appendix B, block V1. The residue cost is one unit, as Article 65 counts; the remnant `W` enters as a new sink not previously in the allocation. □

Remark 4.2 (A new sink with no consumption edge)**.** [A] Every sink in Article 65's inventory is eventually consumed by or converted to another species, except the residue `R`, which accumulates by design. After including the remnant, there are two sinks with no consumption edge: the residue and the remnant. They differ in one decisive property: the residue is impermeant and stays in the lumen, while the remnant is apolar and goes to the bilayer. They accumulate in different compartments and compete for different resources.

5 The hybrid dynamical system

Definition 5.1 (State space and coordinates)**.** [A] The state space is $\Omega = \mathbb{R}^3_+$ with coordinates (N, A, R): the impermeant lumen particle count, the membrane area, and the bilayer remnant count. The initial state of the founder is $(N_0, A_0, 0)$ with $N_0 = 2.52 \times 10^5$, $A_0 = 4\pi R_0^2$ at $R_0 = 100$ nm.

Definition 5.2 (Reduced volume and division hyperplane)**.** [A] The reduced volume is

$$v(N, A) \;=\; \frac{N}{N_0} \cdot \left(\frac{A}{A_0}\right)^{-3/2},$$

normalised so that $v(N_0, A_0) = 1$. The division hyperplane is

$$\Sigma(\alpha) \;=\; \big\{(N, A, R) \in \Omega : v(N, A) \le v_{\text{thr}}(\alpha)\big\}, \quad v_{\text{thr}}(\alpha) = Q(\alpha)^{-3/2},$$

where $Q(\alpha) = \alpha^{2/3} + (1-\alpha)^{2/3}$.

Definition 5.3 (Per-cycle map)**.** [A] Between division events the state evolves by the per-cycle map $\Phi : \Omega \to \Omega$,

$$\Phi(N, A, R) \;=\; \big(N + \beta N,\; A + \delta_A,\; R + \beta N\big), \quad \beta = \frac{\alpha}{1-\alpha},$$

where δ_A is the membrane-area addition per cycle from the pool, satisfying $\delta_A/A = Q(\alpha) - 1$ (Article 64, block V5). Two features of Φ are immediate from its definition.

- The increments of N and R are identical: $\Delta N = \Delta R = \beta N$.
- The increment of A is independent of R.

Definition 5.4 (Division reset)**.** [A] When the orbit reaches $\Sigma(\alpha)$, the division reset $\varrho_\alpha : \Sigma(\alpha) \to \Omega \times \Omega$ produces a bud and a mother,

$$\varrho_\alpha(N, A, R) = \big(\alpha N,\ \alpha^{2/3}A,\ \alpha R\big) \oplus \big((1-\alpha)N,\ (1-\alpha)^{2/3}A,\ (1-\alpha$$

Both products satisfy $v = 1$ exactly: Lemma (taut) of Article 64.

6 The invariant and its consequence

Proposition 6.1 (The R/N ratio is an invariant)**.** [A] *Let $\varrho = R/N$. Then:*

(i) ϱ is preserved by ϱ_α: the bud and mother each inherit the ratio ϱ of the parent at division.

(ii) Under Φ, setting $\beta = \alpha/(1-\alpha)$,

$$\varrho' = \frac{\varrho + \beta}{1 + \beta},$$

which is a Möbius map with unique fixed point $\varrho^* = 1$. The deviation satisfies $\varrho' - 1 = (\varrho - 1)/(1 + \beta)$, so $|\varrho_k - 1|$ contracts by $(1 + \beta)^{-1}$ per cycle and $\varrho_k \to 1$ monotonically from below when $R_0 = 0$.

Proof. (i) $\alpha R/\alpha N = R/N$. (ii) Direct substitution. □

Corollary 6.2 (Remnant accumulation is structural)**.** [A] *Since $N_k \to \infty$ (Theorem 7.1) and $\varrho_k \to 1$, we have $R_k = \varrho_k N_k \to \infty$. The remnant count grows without bound.*
More precisely: any edge consuming W *at rate $r_k < \beta N_k$ per cycle shifts ϱ^* below 1 but does not remove the accumulation; R_k remains unbounded.* The polypeptide paper (Article 68) must discharge W at the production rate βN_k, not merely at a slower one. *This is a precise rate condition on Article 68 derived here from the symmetry of the map, not from any property of* W.

Scholium 6.3 (Why the invariance is structural, not kinetic)**.** The result does not depend on any physical or chemical property of the remnant. It follows from two observations about Φ and ϱ_α: that both the N-increment and the R-increment are βN (the same), and that both the N-component and the R-component of the division reset scale by the same factor α or $1 - \alpha$.
A model in which the remnant has a different fate — for example one in which it dissolves in the lumen rather than partitioning to the bilayer — would change where the accumulation occurs but would not remove it: $R_k \to \infty$ regardless of the compartment, because the invariance argument uses only the map and not the interpretation of the R-coordinate. The strong corollary is equally compartment-independent.

7 The persistence theorem

Theorem 7.1 (Persistence and the return-time bound)**.** [D] *Under continuous satisfaction of C1′ (the pore of Article 66 open), the orbit $O(N_0, A_0, 0) = \{\Phi^k(N_0, A_0, 0) : k \geq 0\}$ satisfies $N_k \to \infty$. Moreover, the return time to $\Sigma(\alpha)$ after each division event is bounded by $K(\alpha)$, independent of the generation number k.*

Proof. Growth of N. Between divisions $N_{k+1} = (1 + \beta)N_k > N_k$ for all $\beta > 0$, so $N_k = N_0(1 + \beta)^k \to \infty$.
Fall of v. After a division both products satisfy $v = 1$. Writing $f(\alpha)$ for the per-cycle factor of v,

$$f(\alpha) = \frac{1 + \beta}{(1 + g)^{3/2}}, \quad g = Q(\alpha) - 1,$$

where $(1 + g)^{3/2}$ is the scaled area-growth factor. For all $\alpha \leq 0.20$ in the corpus range, $f(\alpha) < 1$ (block V3), so v falls geometrically each cycle.
Return-time bound. Starting from $v = 1$, the orbit reaches $v_{\text{thr}}(\alpha)$ after

$$K(\alpha) = \left\lceil \frac{\log v_{\text{thr}}(\alpha)}{\log f(\alpha)} \right\rceil \text{ cycles},$$

which evaluates to 2–3 for $\alpha \in [0.02, 0.10]$ (block V3). Since $K(\alpha)$ depends only on α and not on k, the bound is uniform across generations. The lineage therefore divides indefinitely and is persistent. □

Corollary 7.2 (The forty-generation bound is lifted)**.** [D] *Article 64's forty-generation bound was derived under the hypothesis that no particle source exists. The pore of Article 66 falsifies that hypothesis. The bound is not revised;*

it is conditional on a premise that is now false*, and the corollary applies: the lineage persists.*

Remark 7.3 (Clock-forced division at large α)**.** [A] At $\alpha \geq 0.20$, $f(\alpha) \geq 1$ and v does not fall naturally: particle addition outpaces the membrane-pool demand in the scaled sense. Division is then clock-forced by the photoentrained period T, not self-triggered by v reaching $v_{\text{thr}}(\alpha)$. The photoentrained forcing of Article 64 fires the division reset at each period T regardless of v; the persistence of $N_k \to \infty$ under clock-forced division is then immediate from the geometric growth of N between resets. Persistence holds in both regimes. The return-time bound of the theorem applies to the corpus range $\alpha \in [0.02, 0.10]$; outside it the clock supplies the bound.

8 The remnant accumulation and the lineage ceiling

Theorem 8.1 (The lineage ceiling)**.** [D] *Let $\varrho_{\max}$ be the maximum tolerable remnant-to-lipid ratio, and let $L_{\text{lip}} = vL = 13{,}200$ be the lipid count per vessel. Define the safe region $\mathcal{S} = \{(N, A, R) : R/L_{\text{lip}} \leq \varrho_{\max}\}$. The orbit exits $\mathcal{S}$ at the cycle*

$$k_{\max} = \left\lfloor \frac{\varrho_{\max} \cdot L_{\text{lip}} \cdot (1 - \alpha)}{\alpha \cdot N_0} \right\rfloor,$$

which is finite for all $\alpha > 0$.

Proof. $R_k \geq k \cdot \beta N_0$ (since $N_k \geq N_0$ and $\Delta R = \beta N_k \geq \beta N_0$ each cycle). The orbit exits $\mathcal{S}$ when $k\beta N_0 > \varrho_{\max} L_{\text{lip}}$, i.e. at $k > \varrho_{\max} L_{\text{lip}}/(\beta N_0) = \varrho_{\max} L_{\text{lip}}(1 - \alpha)/(\alpha N_0)$. Block V4. □

Scholium 8.2 (The ceiling at corpus values)**.** At $\alpha = 0.10$ and $\varrho_{\max} = 5$, $k_{\max} = 2$. At $\alpha = 0.02$ and $\varrho_{\max} = 5$, $k_{\max} = 12$. At $\alpha = 0.01$ and $\varrho_{\max} = 5$, $k_{\max} = 25$. Block V4.
The ceiling is severe at $\alpha = 0.10$ and moderate at $\alpha = 0.02$. The tolerance $\varrho_{\max}$ is not supplied by this corpus; it is a property of the bilayer's ability to accommodate apolar inserts. That quantity is Target 9.1.

Corollary 8.3 (The crossover condition)**.** [A] *The orbit remains in $\mathcal{S}$ for K_{safe} cycles if and only if $\alpha \leq \alpha_c(K_{\text{safe}}, \varrho_{\max})$, where*

$$\alpha_c = \frac{\varrho_{\max} \cdot L_{\text{lip}}}{K_{\text{safe}} \cdot N_0 + \varrho_{\max} \cdot L_{\text{lip}}}.$$

For $K_{\text{safe}} = 40$ (the scale of Article 64's bound) and $\varrho_{\max} = 5$, $\alpha_c = 0.013$. For $K_{\text{safe}} = 10$ and $\varrho_{\max} = 5$, $\alpha_c = 0.031$. Block V4.

Scholium 8.4 (The fourth argument for small α, and its kind)**.** Articles 64 and 65 each gave three arguments for small α, all of them cost arguments: smaller α is cheaper in pool, cheaper in time, or cheaper in some other resource. Corollary 8.3 gives a fourth that is different in kind: it is a *necessary condition*. A lineage at $\alpha > \alpha_c$ does not merely cost more; it exits the safe region before reaching the timescale on which the programme operates. The earlier three are preferences; this one is a constraint.
At $\varrho_{\max} = 5$, the condition requires $\alpha < 0.0255$ to reach ten cycles and $\alpha < 0.0065$ to reach forty. These are substantially more restrictive than the preceding three arguments implied, and the direction matters: a necessary condition understated is a necessary condition not enforced.

Remark 8.5 (Route A sharpens the ceiling)**.** [E] Scission by route A is `D + D → LA + W`: one remnant per lipid, by construction and independently of α. Route A raises the per-cycle remnant increment by L_{lip}/T and lowers $k_{\max}$ by approximately a factor of two. This is a further argument for route C beyond the structural ones in Articles 62 and 65. The route-A ceiling is $k_{\max}(\text{A}) \approx \frac{\varrho_{\max} L_{\text{lip}}(1-\alpha)}{(\alpha+1) N_0}$, approximately half the route-C value, so the route choice has a computable effect on the necessary condition α_c. That computation is available from block V4 and is the first output the resolution of Target 9.3 should supply.

9 Targets and conclusion

9.1 Targets

Target 9.1 (The bilayer tolerance)**.** Determine $\varrho_{\max}$: the maximum ratio of apolar inserts to lipids at which the bilayer retains its function as a selective permeability barrier. Until it is supplied, Theorem 8.1 gives $k_{\max}$ as a function of $\varrho_{\max}$ and the crossover condition of Corollary 8.3 is parametric in it. This is the first target of this paper.

Target 9.2 (The remnant discharge)**.** Identify an edge consuming `W` at the production rate βN_k. By Corollary 6.2, a slower rate does not remove the accumulation. The polypeptide paper (Article 68) proposes a mechanism; it must satisfy this rate condition to discharge the target.

Target 9.3 (The route-A/C tension)**.** Resolve the conflict between the pore paper's H1 (route-A dialkyl phosphate supplies the fixed anionic charge for the necessity argument) and Remark 8.5 here (route A doubles the remnant production rate). The two cannot both be optimised simultaneously.

Target 9.4 (The photon-sufficiency inequality)**.** Evaluate $J_{\text{synthesis}} \geq c$ with the absorption cross-section σ_{abs} of the tetrasubstituted chrysene and the solar UV-A irradiance at the relevant wavelength. Block V5 establishes that the inequality exists and that c is small (0.65 s^{-1}) and η large (5–10), so it is easily satisfiable; the numerical confirmation is left for subsequent work.

Target 9.5 (The carbon skeleton supply rate)**.** Show that the rate at which carbon skeletons reach the vessel from the environment matches the demand. The pore paper removes the heteroatom-supply bottleneck; the skeleton-supply rate is the remaining unknown that separates a persistent reproducer from one that simply dilutes more slowly.

9.2 What is established

Proposition 9.6 (Summary)**.** [A] *Subject to Appendix A:*

(i) Article 65's allocation is confirmed on the one-skeleton excision reading (Proposition 4.1).

(ii) The ratio $\varrho = R/N$ is an invariant of both Φ and ϱ_α, converging monotonically to one (Proposition 6.1).

(iii) The lineage is persistent under continuous C1′ discharge, with return time bounded by $K(\alpha) = 2$–3 cycles independently of generation number (Theorem 7.1).

(iv) The forty-generation bound of Article 64 is lifted: it was conditional on the absence of a particle source, and that condition is now false (Corollary 7.2).

(v) The orbit exits the safe bilayer region at $k_{\max}$ cycles, which is finite for all $\alpha > 0$ and is the architecture's binding constraint at $\alpha = 0.10$ (Theorem 8.1).

(vi) The crossover α_c is the first necessary condition on α in this corpus; previous arguments were cost preferences (Corollary 8.3, Scholium 8.4).

(vii) The energetic basis of the whole programme is the chrysene triplet state, whose twelve-hour forcing period is the light phase of a diurnal cycle. This is stated as an observation and not a theorem (block V5, Section 1).

Scholium 9.7 (What the remnant means for the programme)**.** The forty-generation bound was a limit that the programme worked toward removing. It is now removed. The remnant ceiling is the limit that replaced it, and it was not visible until the excision chemistry was written as a map.

That replacement is not a setback. The remnant ceiling has a definite mathematical form, a necessary condition on α that is computable from corpus values, and a discharge mechanism whose rate condition is now precisely stated. *The polypeptide paper's task is not to make the first polymer; it is to make one that removes remnants at the production rate.* That is a stronger requirement than any earlier paper imposed on a first polypeptide, and it is also a narrower one: the function is specified by the constraint, not by a design choice. The architecture proposes its own next problem.

A Hypothesis register

	Premise	What rests on it	What survives its failure
H1	The remnant `W` partitions to the bilayer (Conv. 2.3). It is apolar and leaves the lumen by Article 65's sink corollary; partitioning to the bilayer is the consequence for an apolar fragment in a lipid–water system.	Theorem 8.1 and Corollary 8.3: the remnant accumulates in the bilayer and the ceiling exists in that compartment.	Proposition 6.1 ($\varrho \to 1$) and Corollary 6.2 ($R_k \to \infty$): the remnant count is unbounded regardless of where it goes. The ceiling shifts to whichever compartment `W` accumulates in.
H2	The scale parameters N_0, T, L, ν, α are imported from Articles 64 and 62 without revision.	Every numerical statement in blocks V1–V4.	All theorems hold as formulas in those parameters.
H3	The bilayer tolerance $\varrho_{\max}$ is a finite positive number.	Theorem 8.1: $k_{\max}$ is finite.	If $\varrho_{\max} = \infty$ (the bilayer tolerates any remnant load) then $k_{\max} = \infty$ and the ceiling disappears. The persistence theorem is unaffected.

B Verification blocks

V1 [COMPUTED] **The excision, the remnant, and the allocation**

Confirms Article 65's allocation on the one-skeleton reading; identifies `W` as a new sink.

V2 [COMPUTED] **The hybrid dynamical system**

Defines Ω, Φ, ϱ_α, $\Sigma(\alpha)$; proves the ϱ invariance and its Möbius form; computes the contraction rate.

V3 [COMPUTED] **The persistence theorem and the return-time bound**

Derives $f(\alpha)$; verifies $f < 1$ for $\alpha \le 0.20$; computes $K(\alpha)$ for the corpus range; establishes Theorem 7.1.

V4 [COMPUTED] **The remnant accumulation and the lineage ceiling**

Proves the linear lower bound on R_k; computes $k_{\max}$ and α_c at representative values; states the route-A addendum.

V5 [COMPUTED] **The energetic basis: photon, triplet, and synthesis**

Computes $\eta = E_{T_1}/\Delta G_{\text{synthesis}}$; identifies $T = 43{,}200$ s as the diurnal light phase; states the photon-sufficiency inequality and notes that it is not evaluated.

The source file `acf-67-blocks.py` runs as `python3 acf-67-blocks.py` and emits every number in this paper.

C Falsification register

Proposition 4.1
A reading of the excision chemistry under which more than one chrysene skeleton is emitted per residue. This would revise Article 65's allocation (it remains correct on the one-skeleton reading confirmed here).

Proposition 6.1
An edge or process under which the R increment differs from the N increment in a single cycle. The Möbius form of the proof shows this cannot occur within Φ as defined; it would require a modification to the map.

Theorem 7.1
A demonstration that $f(\alpha) \ge 1$ for some α in the corpus range $[0.02, 0.10]$. Block V3 shows this does not occur; the falsification entry names what would have to be wrong in the membrane-pool or particle-addition rates.

Corollary 7.2

Evidence that the cycle paper's forty-generation bound is not conditional on the absence of a particle source. The cycle paper's statement is examined in block V3; its conditionality is explicit in that paper's own scholium.

Theorem 8.1
$\varrho_{\max} = \infty$: the bilayer tolerates an unlimited remnant load. This is a property of the bilayer's physical chemistry and is settled by Target 9.1 rather than by anything in this corpus.

Corollary 8.3
A derivation of $k_{\max}(\alpha)$ that yields a larger value than block V4 for the same $\varrho_{\max}$, which would raise α_c and relax the necessary condition. The lower bound used in the proof ($R_k \geq k \cdot \beta N_0$) is a lower bound; the true $k_{\max}$ is not smaller.

D Precedence

Ruling D.1 (Against Article 64)**.** [A] The forty-generation bound is lifted, not revised. Its premise (no particle source) is removed by Article 66. The persistence theorem (Theorem 7.1) replaces it with an unconditional result. The cycle paper's existence theorem, its Poincaré map, and its stability results are untouched: those concern a different question (existence and stability of a periodic orbit) and are not affected by the pore.

Ruling D.2 (Against Article 65)**.** [A] Confirmed, not revised. The allocation is unchanged. The remnant `W` is added to the sink inventory as a new species. The free-residue hypothesis H1 of Article 65 is not used here and not relaxed.
The remark on four sources in block V2 of Article 65 stated that no figure attests where each heteroatom goes. The excision chemistry confirmed here (Convention 2.2) does not add a new figure; it confirms the reading that Article 65 already carried as the natural one.

Ruling D.3 (Against Article 66)**.** [A] Consistent, extending. The pore's necessity argument (Theorem (pinning) of Article 66) assumed route-A head groups, which supplies the fixed anionic charge. Remark 8.5 here shows that route A also doubles the remnant production rate and lowers $k_{\max}$. The two results are consistent; neither overturns the other. The tension is registered as Target 9.3 and is not resolved here.

E Literature context

Within the corpus

This paper is a composition of four articles, cited as [Sch26f, Sch26c, Sch26h, Sch26e, Sch26g] through [Sch26g]. It adds no new species, edges, or chemistry to any of them. Its mathematical contribution is the identification of the $\varrho = R/N$ invariant (Proposition 6.1), which follows from the symmetry of the map defined in those papers and was not visible until the map was written explicitly.

The hybrid dynamical system framework (Definition 5.1 through Definition 5.4) is standard in the theory of hybrid systems. The novelty here is not the framework but the identification of the invariant and the ceiling as consequences of the corpus's own dynamics, which no prior article of the corpus named.

The loom series within the corpus

The mathematical structure underlying the present paper is prepared by four articles of the loom series, all in this volume.

The Abiotic Loom [Sch26d] introduces the Chrysene Tensor Space $\mathcal{I}_C$, the steric Morse function, and the proton-coupled electron transfer operator that locks geometrically selected configurations into covalent permanence. The timescales and triplet energies reported in block V5 of this paper are the timescales of that construction. The PCET mechanism described there as locking secondary structure is the same mechanism that drives phosphodiester formation, nucleotide synthesis, and — by Article 68's proposal — polypeptide condensation.

The Geometric Validation [Sch26a] translates the abstract critical simplices of the loom into parameter-free angular windows and evaluates them against a high-resolution PDB corpus, establishing moderate-to-strong enrichment. It confirms that the lattice geometry is physically realised in native structures. The enrichment ratios there are the empirical complement to the purely formal persistence and ceiling arguments of this paper.

The Higher-Order Geometry [Sch26b] elevates the test from local dihedral statistics to three-dimensional compatibility: axis alignment, pore tethering with positive backbone clearance, and side-chain packing. It establishes that the triangular pore of the half-column-shifted lattice accommodates a native α-helix with residue-level clearance — the geometry that Article 68 proposes as the condensation site for the first polypeptide.

The Topological Scission [Sch26i] constructs functional voids in the lattice by controlled scission and inverse casting, and establishes PCET-enabled prototype enzyme formation. It is the article that most directly prepares the ground for Article 68: the condensation edge proposed there is PCET-mediated, and the site geometry and energetics it requires are developed there in detail.

The bilayer tolerance parameter $\varrho_{\max}$ of Target 9.1 is not addressed by any article of this corpus. It is a biophysical quantity — the maximum ratio of apolar inserts to lipids at which the bilayer retains selective permeability — and its determination requires either measurement or a dedicated molecular-level calculation outside the current scope. The ceiling $k_{\max}$ and the crossover α_c of this paper are stated as functions of $\varrho_{\max}$ precisely because the quantity is not yet in the inventory.

References

[Sch26a] Charles D. Schaper. Geometric Validation of the Abiotic Loom: Discrete Morse Predictions versus PDB Secondary-Structure Statistics and Ramachandran Densities. *Annals of the Chrysene Formalism*, 1(1):283–291, 2026. DOI: https://doi.org/10.68366/snnbbz13.

[Sch26b] Charles D. Schaper. Higher-Order Geometry of the Abiotic Loom: Axis Alignment, Pore Tethering, and Side-Chain Packing of Secondary-Structure Elements in the Chrysene Tensor Space. *Annals of the Chrysene Formalism*, 1(1):292–304, 2026. DOI: https://doi.org/10.68366/qn0sab53.

[Sch26c] Charles D. Schaper. Structural Adequacy for Signalling: The Chrysene–Nucleic Acid Complex as a Hybrid Channel, Scaffold Succession, and the Continuity of Directed Pathways. *Annals of the Chrysene Formalism*, 1(3):1321–1347, 2026. Article 63. DOI: https://doi.org/10.68366/0e3gsg12.

[Sch26d] Charles D. Schaper. The Abiotic Loom: Discrete Morse Theory and the Geometric Resolution of Levinthal's Paradox in the Chrysene Tensor Space. *Annals of the Chrysene Formalism*, 1(1):273–282, 2026. Article 23. DOI: https://doi.org/10.68366/vdze9508.

[Sch26e] Charles D. Schaper. The Chrysene-Reachable Material Ledger: Species, Properties, and the Allocation of a Single Pool. *Annals of the Chrysene Formalism*, 1(3):1371–1383, 2026. Article 65. DOI: https://doi.org/10.68366/dwqk8m15.

[Sch26f] Charles D. Schaper. The Enclosure via the Encoded Chrysene Cluster: Integrated Synthesis of Nucleic Acids and Phospholipids, and the Orientational Encoding of the Membrane. *Annals of the Chrysene Formalism*, 1(3):1279–1320, 2026. Article 62. DOI: https://doi.org/10.68366/24m3wx88.

[Sch26g] Charles D. Schaper. The First Pore: Closure, Aperture, and the Reconciliation of Impermeance with Supply. *Annals of the Chrysene Formalism*, 1(3):1384–1404, 2026. Article 66. DOI: https://doi.org/10.68366/en23gq52.

[Sch26h] Charles D. Schaper. The Photoentrained Cycle: Existence and Stability of a Periodic Orbit in a Forced Hybrid Protocell. *Annals of the Chrysene Formalism*, 1(3):1348–1370, 2026. Article 64. DOI: https://doi.org/10.68366/h9rzkd75.

[Sch26i] Charles D. Schaper. Topological Scission of the Chrysene Abiotic Loom: Dual Mastering, Structural Scaffolding, Sequential Catalytic Connectivity, and PCET-Enabled Prototype Enzyme Formation. *Annals of the Chrysene Formalism*, 1(1):305–321, 2026. DOI: https://doi.org/10.68366/2rgebp62.