ANNALS OF THE CHRYSENE FORMALISM
RESEARCH ARTICLE | AI-AUGMENTED PEER-REVIEWED | VOL. 1, NO. 3 | OCTOBER 2026

The First Loop:

The Remnant Fixed Point, the Division Period, and the Self-Tuning Lineage

Charles D. Schaper, Ph.D.
Chief Editor, Annals of the Chrysene Formalism
San Francisco, California, USA
editor@chrysene.com

Abstract. The preceding article of *ACF* on the first polypeptide established a condensation edge that sequesters the remnant and defined the shared target `tgt:cond_rate`: what drives the condensation fraction f_c and at what rate constant k? This paper closes that target and derives the full dynamical consequences.

When $f_c(W_b) = k \cdot W_b/L_{\text{lip}}$, the augmented system has a fixed line $\mathcal{F} = \{(N, W_b) : W_b = \beta L_{\text{lip}}/k,\ N > 0\}$: a family of fixed points parameterised by the steady-state particle count N^*, which is free. At any point on $\mathcal{F}$ the condensation fraction is exactly $\beta = \alpha/(1-\alpha)$ and N is constant.

The fixed line is globally attracting for $0 < k < 2L_{\text{lip}}/N_0 = 0.1048$. The Lyapunov function $V = (W_b - W_b^*)^2$ decreases strictly each cycle in this range. Within the safe-region subset $0 < k < L_{\text{lip}}/N_0 = 0.052$ the convergence is monotone; outside it, oscillatory with $|\lambda| < 1$. The explicit convergence time to within 10% of W_b^* is four to five cycles for k near the safe-region lower bound, and one cycle near $k = L_{\text{lip}}/N_0$.

At any point on $\mathcal{F}$ the reduced volume decays by factor $Q(\alpha)^{-3/2}$ per cycle while N is constant. The division threshold is $v_{\text{thr}}(\alpha) = Q(\alpha)^{-3/2}$. One cycle after each division the vessel reaches $v_{\text{thr}}(\alpha)$ exactly, because

$$v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} \;=\; Q(\alpha)^{-3/2} \cdot Q(\alpha)^{3/2} \;=\; 1 \quad \text{for all } \alpha \in (0,1).$$

The division period is therefore exactly $T = 43{,}200$ s for every α, every N^*, and every k in the stability range. This is an algebraic identity from Article 64's definitions, not a numerical coincidence.

Period homeostasis holds unconditionally on $\mathcal{F}$: the period is T regardless of the vessel's particle count, bud fraction, or rate constant. Shape homeostasis (Article 64) holds unconditionally at every division. Size homeostasis does not hold: N^* is whatever the vessel had when the loop engaged. The architecture delivers two kinds of homeostasis and withholds a third.

During the transient from post-division to $\mathcal{F}$, the period is two cycles, converging to one as $W_b \to W_b^*$. Four targets from Articles 67 and 68 are simultaneously closed at the fixed line.

Keywords: first loop; fixed line; condensation rate; Lyapunov stability; exact division period; algebraic identity; period homeostasis; self-tuning lineage; ϱ-invariance; transient analysis; three regimes; Chrysene Formalism

1 Introduction

1.1 The shared target

Article 68 defined `tgt:cond_rate` as the question of what drives f_c and at what rate. It proved $f_c \geq \beta$ necessary, defined the linear model $f_c(W_b) = k \cdot W_b/L_{\text{lip}}$, and noted that the safe-region lower bound $k > \beta/\varrho_{\max}$ and the stability upper bound $k < 2L_{\text{lip}}/N_0$ are jointly satisfiable. It passed the fixed point to this paper.

1.2 Three results, in order of difficulty

First result: the fixed-line structure. The fixed "point" in (N, W_b) space is not a point but a line: any value of N^* with $W_b = \beta L_{\text{lip}}/k$ is a fixed point for the W_b dynamics. The period is T at every point on the line, so the period is independent of the steady-state particle count. Size is free; period is not.

Second result: global stability. The Lyapunov function $V = (W_b - W_b^*)^2$ decreases strictly each cycle for all $k \in (0, 2L_{\text{lip}}/N_0)$, giving global asymptotic convergence from any initial condition $W_b(0) \geq 0$.

Third result: the algebraic identity. On the fixed line, N is constant and v decays by factor $Q(\alpha)^{3/2}$ per cycle from area growth alone. The division threshold is $v_{\text{thr}}(\alpha) = Q(\alpha)^{-3/2}$.

Citation: Schaper, C. D. (2026). The First Loop: The Remnant Fixed Point, the Division Period, and the Self-Tuning Lineage. *Annals of the Chrysene Formalism*, 1(3):1423-1431. https://chrysene.com/index.php/acf/article/view/69 © 2026 Charles D. Schaper. Distributed under the CC Attribution-NonCommercial-NoDerivatives 4.0. First published by Schaper Systems Press in the *Annals of the Chrysene Formalism*, Vol. 1, No. 3 (2026). ISSN 3144-2021.

The product $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$ for all α: division fires after exactly one cycle, for every parameter value.

1.3 What is established, by kind

Imported. The state space Ω, per-cycle map Φ_c, and ϱ-invariance from Article 67 [Sch26d]; the linear condensation model $f_c(W_b) = k \cdot W_b/L_{\text{lip}}$ (H1) and the rate condition $f_c \geq \beta$ from Article 68 [Sch26b]; the division threshold $v_{\text{thr}}(\alpha) = Q(\alpha)^{-3/2}$ and area-growth factor $g = Q(\alpha) - 1$ from Article 64 [Sch26e]; the inventory and sink corollary from Article 65 [Sch26a]; the pore from Article 66 [Sch26c]; all scale parameters N_0, L_{lip}, α.
Proved here. That the fixed "point" is a line (Theorem 5.1 and Remark 4.5); that the fixed line is globally attracting with an explicit Lyapunov function (Theorems 6.1 and 6.3); that the convergence regime and rate are computable from k (Proposition 6.4); that the post-division recovery time is bounded and explicit (Proposition 6.5); that the division period on the fixed line is exactly one cycle for all α by an algebraic identity from Article 64 (Theorem 7.1); that period homeostasis holds unconditionally on the fixed line and is independent of k, N^*, and α (Corollary 7.3); and that four earlier targets are simultaneously closed (Corollary 9.1).
Found. That the fixed-line structure separates period regulation (unconditional) from size regulation (absent); that the period converges from two cycles to one as $W_b \to W_b^*$; that the architecture's first homeostatic period result uses the same $Q(\alpha)$ construction as Article 64's shape homeostasis.
Not claimed. The value of k; the fate of the co-aggregate (Target 10.4); any extension to non-linear f_c beyond the fixed-point existence and exact period results (Target 10.5); any feedback of the encoder on k (Target 10.6).

2 Status, scope, and conventions

Convention 2.1 (Status labels). [A] finite or axiom-free; [D] derived under named hypotheses; [C] imported with source named; [E] molecular reading.

Hypothesis 2.2 (The linear condensation model). [D] $f_c(W_b) = k \cdot W_b/L_{\text{lip}}$ for $k \in (0, 2L_{\text{lip}}/N_0)$. Inherited from Article 68, Hypothesis H1. All numbered results in this paper that use a specific form for f_c rest on this hypothesis and are labelled [D]. Results that require only monotonicity of f_c are labelled [A].

Convention 2.3 (The augmented state space). [A] The state space is $\Omega = \mathbb{R}^3_+$ with coordinates (N, A, W_b) (three-dimensional; $\varrho = W_b/N$ is a derived quantity, not an independent coordinate); $\varrho = W_b/N$ is derived. The per-cycle map is Φ_c (Definition 4.1); the division reset is ϱ_α, as in Article 67.

Convention 2.4 (Scope). [A] No new species, no new edges, no new chemistry. This paper derives consequences of the maps defined in Articles 67 and 68.

3 Non-implications

Proposition 3.1 (Non-implications). [A] *None of the following is asserted below.*

(i) The period is T from the first generation. *The steady state is approached over a transient; the exact period applies on the fixed line, not before it.*

(ii) Size is regulated. N^* *is free; the fixed line carries no preferred* N^*.

(iii) The loop fires without the pore. *The pore of Article 66 supplies the heteroatom feedstock that the excision converts to residues. Without it* $c_k = 0$ *and* $W_b = 0$ *permanently:* $f_c = 0$ *and no condensation occurs.*

(iv) The exact period extends to all f_c. *The algebraic identity* $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$ *holds whenever* $f_c = \beta$ *and* N *is constant; the linear model is not required.*

(v) The fate of the co-aggregate is determined. *Target 10.4.*

4 The augmented dynamical system

Definition 4.1 (The augmented per-cycle map). [A] Let $\gamma_k = \beta - f_c(W_{b,k})$ be the net-increment fraction at cycle k. The augmented per-cycle map $\Phi_c : \Omega \to \Omega$ is

$$N_{k+1} = N_k\,(1 + \gamma_k), \tag{1}$$
$$A_{k+1} = A_k\,Q(\alpha), \tag{2}$$
$$W_{b,k+1} = W_{b,k} + \gamma_k\,N_k. \tag{3}$$

The division reset $\varrho_\alpha : \Sigma(\alpha) \to \Omega \times \Omega$ maps $(N, A, W_b) \mapsto (\alpha N, \alpha^{2/3}A, \alpha W_b) \oplus ((1-\alpha)N, (1-\alpha)^{2/3}A, (1-\alpha)W_b)$.

Remark 4.2 (Equal increments: the source of the invariance). [A] From equations (1) and (3) of Definition 4.1: $N_{k+1} - N_k = \gamma_k N_k = W_{b,k+1} - W_{b,k}$. The increments of N and W_b are identical at every cycle. This single observation underlies Lemma 4.3 and Proposition 4.4 below.

Lemma 4.3 (The ϱ-invariance through the loop)**.** [A] *Let $\varrho_k = W_{b,k}/N_k$. Under Φ_c and ϱ_α:*

(i) Under ϱ_α: *ϱ is preserved exactly. Both components scale by the same factor (α or $1-\alpha$).*

(ii) Under Φ_c: *with net-increment fraction γ_k,*

$$\varrho_{k+1} = \frac{W_{b,k} + \gamma_k N_k}{N_k(1+\gamma_k)} = \frac{\varrho_k + \gamma_k}{1+\gamma_k}.$$

This is a Möbius map with unique fixed point $\varrho^* = 1$ and deviation contraction:

$$\varrho_{k+1} - 1 = \frac{\varrho_k - 1}{1+\gamma_k}.$$

(iii) Contraction rate. *$|\varrho_k - 1|$ contracts by $(1+\gamma_k)^{-1}$ per cycle, where $\gamma_k = \beta - f_c(W_{b,k}) \geq 0$ during the transient. On the fixed line $\gamma_k = 0$ and ϱ is* frozen*: no further contraction, no divergence.*

Proof. Part (i): $(\alpha W_b)/(\alpha N) = W_b/N$. Part (ii): direct substitution into the ratio using (1)–(3). Part (iii): subtract 1 from both sides of part (ii) and rearrange. □

Proposition 4.4 (ϱ-invariance in the loop context)**.** [A] *The ϱ-invariance established in Article 67 (Proposition (invariant)) holds in the loop context for every value of f_c. On the fixed line ($\gamma_k = 0$) the ratio is frozen at its current value; away from it the ratio contracts toward 1 at rate $(1+\gamma_k)^{-1}$ per cycle.*
In particular, if the initial condition has $\varrho_0 = 1$ (e.g., $W_b(0) = N_0$), then $\varrho_k = 1$ for all $k \geq 0$.

Proof. Follows from Lemma 4.3 with $\varrho^* = 1$ and $\gamma_k = \beta - f_c(W_{b,k}) \geq 0$. □

Remark 4.5 (The fixed-line structure)**.** [D] The map Φ_c has a set of fixed points in (N, W_b) space. Setting $\gamma_k = 0$ requires $f_c(W_b) = \beta$, which under the linear model gives $k\,W_b/L_{\text{lip}} = \beta$, i.e. $W_b = W_b^* := \beta L_{\text{lip}}/k$. Since this is independent of N, the set of fixed points is the *fixed line*

$$\mathcal{F} = \{(N, W_b^*) : N > 0\}.$$

The particle count N^* at the operating point is determined by the initial condition and the approach trajectory, not by the fixed-line equation. The period is T at every point on $\mathcal{F}$, independent of N^*.

5 The fixed line

Theorem 5.1 (The fixed line exists and is unique)**.** [D] *Under Hypothesis 2.2, the map Φ_c has a unique fixed line $\mathcal{F} = \{(N, W_b^*) : N > 0\}$ with*

$$W_b^* = \frac{\beta\, L_{\text{lip}}}{k}.$$

At every point on $\mathcal{F}$: $f_c(W_b^*) = \beta$, $\gamma = 0$, and N is constant at whatever value it holds.
Shared target closed. `tgt:cond_rate` *is answered: the condensation fraction at steady state is $f_c = \beta$, independent of k and of N^*. Block V1.*

Proof. The W_b-coordinate is fixed when $\gamma_k = 0$, i.e. $f_c(W_b) = \beta$. Under the linear model: $k\,W_b/L_{\text{lip}} = \beta \Rightarrow W_b = \beta L_{\text{lip}}/k$. This is unique (strict monotonicity of f_c) and independent of N. The N-coordinate is fixed when $\gamma_k = 0$: $N_{k+1} = N_k(1+0) = N_k$. Hence N is constant at any $N > 0$. □

Corollary 5.2 (W_b^* is finite and in the safe region)**.** [D] *For any $k > \beta/\varrho_{\max}$,*

$$\frac{W_b^*}{L_{\text{lip}}} = \frac{\beta}{k} < \varrho_{\max}.$$

The orbit at the fixed line does not exit the safe bilayer region of Article 67. Block V1.

Proof. Dividing $W_b^* = \beta L_{\text{lip}}/k$ by L_{lip} gives $W_b^*/L_{\text{lip}} = \beta/k < \varrho_{\max}$ iff $k > \beta/\varrho_{\max}$. □

Scholium 5.3 (k determines the bilayer load, not the period)**.** The rate constant k determines $W_b^*/L_{\text{lip}} = \beta/k$: larger k gives a lower steady-state remnant load. It does not determine N^* (which is free) or the division period (which is T at every point on $\mathcal{F}$, as Theorem 7.1 establishes). The three quantities W_b^*, N^*, and the period are therefore independent of each other: W_b^* is set by k; N^* is set by history; the period is T regardless.

6 Stability: global convergence to the fixed line

Theorem 6.1 (Asymptotic stability of the fixed line)**.** [D] *Define the Lyapunov function $V(W_b) = \frac{1}{2}(W_b - W_b^*)^2$. Near the fixed line where $N_k \approx N^*$,*

$$V(W_{b,k+1}) - V(W_{b,k}) = \tfrac{1}{2}(\lambda^2 - 1)\delta_k^2,$$

where $\delta_k = W_{b,k} - W_b^*$ and $\lambda = 1 - kN^*/L_{\text{lip}}$. V decreases strictly iff $|\lambda| < 1$, i.e. $0 < k < 2L_{\text{lip}}/N^*$. At the corpus scale $N^* \approx N_0$: $0 < k < 2L_{\text{lip}}/N_0 = 0.1048$. The fixed line is therefore asymptotically stable in the neighbourhood where $N \approx N^*$. Block V2.

Proof. Near the fixed line, $\delta_{k+1} = \delta_k \cdot \lambda$ (Theorem 6.3 linearisation). Then $V_{k+1} - V_k = \frac{1}{2}(\lambda^2 - 1)\delta_k^2 < 0$ iff $|\lambda| < 1$. □

Proposition 6.2 (Convergence under bounded N). 🄳 *During the transient ($W_b < W_b^*$, $\gamma > 0$), N grows until division fires. Division resets N every $K(\alpha)$ cycles (Article 67, Theorem (persist)), bounding the maximum N to $N_{\max} \le N_0(1+\beta)^{K(\alpha)}$. The effective stability boundary during the transient is $k < 2L_{\text{lip}}/N_{\max}$. For $\alpha = 0.10$, $K = 3$: $N_{\max} \approx 1.37\,N_0$ and $2L_{\text{lip}}/N_{\max} \approx 0.076$. For all k satisfying the safe-region lower bound $k > \beta/\varrho_{\max}$ at $\varrho_{\max} = 5$ (i.e. $k > 0.022$ at $\alpha = 0.10$), the effective eigenvalue satisfies $|\lambda_{\max}| < 1$ throughout the transient, so the orbit converges to the fixed line from any initial $W_b(0) > 0$. Block V2.*

Proof. The effective eigenvalue at $N_{\max}$ is $\lambda_{\max} = 1 - kN_{\max}/L_{\text{lip}}$. At $k = 0.022$ and $N_{\max} = 1.37\,N_0$: $\lambda_{\max} = 1 - 0.022 \times 1.37 \times 19.09 = 1 - 0.575 = 0.425 < 1$. For all k in the safe range the bound holds similarly. Block V2. □

Theorem 6.3 (Linearised stability and eigenvalue). 🄳 *The linearisation of Φ_c about any point (N^*, W_b^*) on the fixed line gives the recursion $\delta_{k+1} = \lambda\,\delta_k$ with eigenvalue*

$$\lambda \;=\; 1 - \frac{k\,N^*}{L_{\text{lip}}}.$$

With $N^* = N_0 = 2.52 \times 10^5$ and $L_{\text{lip}} = 13{,}200$:

k	λ	$\lvert\lambda\rvert < 1$?	10%-recovery	regime
0.022	0.580	*yes*	*5 cycles*	*monotone*
0.052	0.007	*yes*	*1 cycle*	*monotone*
0.075	−0.432	*yes*	*3 cycles*	*oscillatory*
0.095	−0.814	*yes*	*12 cycles*	*oscillatory*
0.100	−0.909	*yes*	*25 cycles*	*oscillatory*
0.110	−1.100	***no***	∞	*unstable*

Block V2.

Proof. Set $W_{b,k} = W_b^* + \delta_k$. Then $f_c(W_{b,k}) = k(W_b^* + \delta_k)/L_{\text{lip}} = \beta + k\delta_k/L_{\text{lip}}$. The increment $\gamma_k = \beta - f_c = -k\delta_k/L_{\text{lip}}$. From (3): $\delta_{k+1} = \delta_k + \gamma_k N^* = \delta_k(1 - kN^*/L_{\text{lip}}) = \lambda\delta_k$. □

Proposition 6.4 (Two convergence regimes). 🄳 *The stability range $(0, 2L_{\text{lip}}/N_0)$ splits into two regimes:*

(i) Monotone convergence*: $0 < k < L_{\text{lip}}/N_0 = 0.052$. $\lambda \in (0,1)$; δ_k decreases without sign change. Recovery to within 10% of W_b^* in $\lceil \log 0.1/\log\lambda \rceil$ cycles.*

(ii) Oscillatory convergence*: $L_{\text{lip}}/N_0 < k < 2L_{\text{lip}}/N_0$. $\lambda \in (-1,0)$; δ_k overshoots alternately. Convergence is slower near $k = 2L_{\text{lip}}/N_0$ and faster near the boundary L_{lip}/N_0.*

The safe-region lower bound $k > \beta/\varrho_{\max} \le 0.022$ (at $\alpha = 0.10$, $\varrho_{\max} = 5$) lies in the monotone regime. All safe, physically relevant k values give monotone convergence. Block V2.

Proposition 6.5 (Post-division recovery is bounded). 🄳 *After a division event the bud has $W_{b,0}^{\text{bud}} = \alpha W_b^* < W_b^*$, giving initial deviation $\delta_0 = -(1-\alpha)W_b^*$. The deviation decays as $\delta_k = \delta_0\lambda^k$. The bud returns to within 10% of W_b^* in*

$$k^* \;=\; \left\lceil \frac{\log 0.1}{\log|\lambda|} \right\rceil \text{ cycles,}$$

which is bounded for all k in the stable range. For the safe-region lower bound $k \approx 0.022$, $k^* = 5$ cycles; for $k \approx 0.052$, $k^* = 1$ cycle. Block V2.

Remark 6.6 (The period during the transient). 🄳 During the transient ($W_b < W_b^*$, $\gamma_k > 0$), N grows and ν falls at rate

$$f_{\text{tr}}(\gamma_k) = \frac{1+\gamma_k}{Q(\alpha)^{3/2}} > \frac{1}{Q(\alpha)^{3/2}}.$$

Since $f_{\text{tr}} < 1$ for all $\gamma_k < Q(\alpha)^{3/2} - 1$ (verified in block V3), ν falls at every cycle and division fires. The period during recovery is

$$k_{\text{div}} = \left\lceil \frac{\log \nu_{\text{thr}}(\alpha)}{\log f_{\text{tr}}} \right\rceil.$$

At $\alpha = 0.10$ and $W_b = \alpha W_b^*$ (immediately post-division), $f_{\text{tr}} = 0.914$ and $k_{\text{div}} = 2$ cycles. As $W_b \to W_b^*$, $f_{\text{tr}} \to Q(\alpha)^{-3/2}$ and $k_{\text{div}} \to 1$. The period converges from two cycles to one as the fixed line is approached. Block V3.

7 The exact division period

Theorem 7.1 (The division period on the fixed line is exactly one cycle). 🄰 *At any point (N^*, W_b^*) on the fixed line $\mathcal{F}$, the reduced volume satisfies*

$$\nu_{k+1} = \nu_k \cdot Q(\alpha)^{-3/2}$$

per cycle. Starting at $v = 1$ after each division (Lemma (taut), Article 64), one cycle later:

$$v_1 = Q(\alpha)^{-3/2} = v_{\text{thr}}(\alpha).$$

Division fires after exactly one cycle. The division period is T.

Proof. On $\mathcal{F}$, $\gamma_k = 0$ and $N_{k+1} = N_k = N^*$. Then

$$v_{k+1} = v_k \cdot \frac{N_{k+1}/N_k}{(A_{k+1}/A_k)^{3/2}} = v_k \cdot \frac{1}{Q(\alpha)^{3/2}}.$$

Starting from $v_0 = 1$: $v_1 = Q(\alpha)^{-3/2}$.
The claim is that $Q(\alpha)^{-3/2} = v_{\text{thr}}(\alpha)$. By Article 64, block V5, $v_{\text{thr}}(\alpha)(\alpha) = Q(\alpha)^{-3/2}$ by definition. So $v_1 = v_{\text{thr}}(\alpha)$ exactly. Division fires at $k = 1$.
Equivalently: $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = Q(\alpha)^{-3/2} \cdot Q(\alpha)^{3/2} = 1$. □ □

Remark 7.2 (The identity is exact for all α)**.** [A] Table V3 of block V3 verifies $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1.000\,000\,000\,000$ to twelve significant figures for $\alpha = 0.01, 0.02, 0.05, 0.10, 0.20, 0.30, 0.50$. This is not numerical agreement: the equality is algebraic, requiring only the two Article 64 definitions: (i) $v_{\text{thr}}(\alpha) \coloneqq Q(\alpha)^{-3/2}$, and (ii) $(1 + g)^{3/2} = Q(\alpha)^{3/2}$ since $g = Q(\alpha) - 1$. Both held since Article 64; the identity was latent until the condensation loop held N constant and left area growth as the sole driver of v.

Corollary 7.3 (Period homeostasis)**.** [A] *At every point on the fixed line $\mathcal{F}$, the division period is exactly T, independent of:*

(i) the bud fraction α,

(ii) the rate constant k,

(iii) the steady-state particle count N^*.

Proof. The proof of Theorem 7.1 uses neither k nor N^*: it requires only $\gamma = 0$ (which holds at any point on $\mathcal{F}$) and the Article 64 identity $v_{\text{thr}}(\alpha) = Q(\alpha)^{-3/2}$. Since the identity holds for all $\alpha \in (0, 1)$, the period is T for all α. □

Scholium 7.4 (Three homeostatic results and their conditions)**.** The corpus now has three homeostatic results.
Shape homeostasis (Article 64): v returns to unity after every division. Proof: the division reset produces both daughters at $v = 1$ by construction. Conditions: none. Holds at every division event, always.
Period homeostasis (this paper): the division period equals T on the fixed line. Proof: the algebraic identity $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$. Conditions: $W_b = W_b^*$ (fixed line reached). Holds unconditionally at the fixed line.
Size homeostasis: N returns to a fixed value after each division. Not established: N^* is a free parameter of the fixed line. The loop regulates the period and not the size.
Both established results use the same structure: $Q(\alpha)^{-3/2} \cdot Q(\alpha)^{3/2} = 1$. Article 64 used it for the shape result (the division reset is normalised by $Q(\alpha)$); this paper uses it for the period result (the area grows by $Q(\alpha)$ per cycle while N is constant). The dependence is on $Q(\alpha)$ alone, which is determined entirely by α.

8 Three regimes: a formal comparison

Proposition 8.1 (The three dynamical regimes)**.** [A] *The corpus defines three operating regimes for the lineage.*

R1. ***Article 64*** *(forced cycle, no source, no condensation). N is constant, $v = 1$ throughout, division is clock-forced. The forty-generation bound applies: without a solute source the lineage reaches the dilution floor in ≈ 40 cycles at $\alpha = 0.10$.*

R2. ***Article 67*** *(pore open, no condensation). N grows by βN per cycle, v falls, division is self-triggered in 2–3 cycles. The lineage persists ($N \to \infty$) but $W_b \to \infty$ and the bilayer ceiling applies. The forty-generation bound is lifted; the ceiling bound replaces it.*

R3. ***Article 69, fixed line*** *(pore open, condensation at $f_c = \beta$). N is constant, $W_b = W_b^*$ constant, $v = 1$ restored at each division. Division fires after exactly one cycle. The lineage is persistent, the ceiling is not reached, and the period is T.*

The three regimes are quantitatively compared in the following table (at $\alpha = 0.10$; block V4):

Regime	*R1 (Art. 64)*	*R2 (Art. 67)*	*R3 (Art. 69)*
N	*constant*	$\to \infty$	*constant*
W_b	—	$\to \infty$	$= W_b^*$
v	$= 1$, *clock-div*	*falls, self-div*	$1 \to v_{\text{thr}}(\alpha)$ *each T*
period	T *(forced)*	2–3 *cycles*	T *(exact)*
ceiling	*40 cycles*	$k_{\max}$ *finite*	∞ *(lifted)*

The three regimes differ in one parameter (the condensation fraction f_c) and produce qualitatively different long-run behaviours. The transition from R2 to R3 is the approach

to the fixed line; the period decreases monotonically from 2–3 cycles (R2) to 1 cycle (R3) as $W_b \to W_b^*$.

Proof. R1 is Article 64; R2 is Article 67 Theorems (persist) and (ceiling); R3 is Theorem 7.1 and Corollary 5.2 of this paper. Block V4. □

9 Four simultaneous closures

Corollary 9.1 (Four simultaneous closures on the fixed line)**.** [A] *At any point on the fixed line $\mathcal{F}$:*

(i) `tgt:remnant` *(Article 67) is* closed*: $f_c = \beta$ discharges* `W` *at the production rate exactly. The Article 67 rate condition (Corollary (structural)) is satisfied with equality, not merely exceeded.*

(ii) The bilayer ceiling *of Article 67 (Theorem (ceiling)) is lifted: $W_b = W_b^*$ is constant; $W_b^*/L_{\text{lip}} = \beta/k < \varrho_{\max}$ for any $k > \beta/\varrho_{\max}$ (Corollary 5.2). The orbit does not exit the safe region.*

(iii) The forty-generation bound *is superseded: Article 67 showed the lineage persistent; this paper shows it divides every T at the fixed line. The bound is not just lifted—it is replaced by an exact steady-state period.*

(iv) The shared target `tgt:cond_rate` *(Article 68, Definition (shared)) is closed: $f_c(W_b^*) = \beta$ is the fixed point equation, so the condensation fraction equals the bud fraction at steady state, as required.*

Proof. All four follow from Theorem 5.1 ($f_c = \beta$ at W_b^*), Corollary 5.2 ($W_b^* < \varrho_{\max} L_{\text{lip}}$), and Theorem 7.1 (period = T). □

Scholium 9.2 (What the self-tuning means)**.** None of the four closures was designed into the architecture. The remnant target came from a waste product. The rate condition came from a structural invariance. The ceiling came from a bilayer-physics argument. The period came from an algebraic identity that Article 64 carried silently.

The loop assembles these into one self-consistent operating point. The fixed-line equation $W_b = \beta L_{\text{lip}}/k$ is not a setpoint that was chosen; it is the unique solution of $f_c(W_b) = \beta$. The period T is not a target that was achieved; it is what happens when the only driver of v-decay is the membrane pool.

W_b is the division clock. Its steady-state concentration sets the condensation rate, which sets N, which determines when v reaches $v_{\text{thr}}(\alpha)$. The clock is a fixed point, not a mechanism.

10 Registers, precedence, targets, conclusion

10.1 Consolidated register

Does the condensation loop have a fixed line?
Yes: $\mathcal{F} = \{(N, \beta L_{\text{lip}}/k) : N > 0\}$ for any $k > 0$. N^* is free; $W_b^* = \beta L_{\text{lip}}/k$ is unique.
Rests on: Thm. 5.1, block V1.
Standing: [D] Rests on H1 (linear model). The fixed-line existence holds for any monotone increasing f_c; uniqueness uses linearity.

Is the fixed line globally attracting?
Yes, for $0 < k < 2L_{\text{lip}}/N_0 = 0.1048$ (Theorem 6.1); convergent from any $W_b(0) > 0$ in the safe-region range (Proposition 6.2). Monotone for $k < 0.052$, oscillatory for $0.052 < k < 0.1048$.
Rests on: Thm. 6.1, Thm. 6.3, Prop. 6.4, block V2.
Standing: [D] Rests on H1 and H2 (scale parameters).

How fast does the bud recover after division?
$k^* = \lceil \log 0.1/\log|\lambda| \rceil$ cycles; for safe-region $k \approx 0.022$: five cycles; for $k \approx 0.052$: one cycle.
Rests on: Prop. 6.5, block V2.
Standing: [D]

What is the division period on the fixed line?
Exactly one cycle = T = 43,200 s, for all α, k, N^*.
Rests on: Thm. 7.1, Cor. 7.3, block V3.
Standing: [A] Algebraic identity; no hypothesis required beyond Art. 64's definitions.

Does period homeostasis hold?
Yes, unconditionally on $\mathcal{F}$: period = T independent of α, k, N^*. Shape homeostasis (Art. 64) also holds. Size homeostasis does not hold: N^* is free.
Rests on: Cor. 7.3, Sch. 7.4.
Standing: [A]

Are four earlier targets closed?
Yes: `tgt:remnant`, rate condition, bilayer ceiling, and the forty-generation bound are all resolved at the fixed line.
Rests on: Cor. 9.1.
Standing: [A] for items (i), (iii), (iv); [D] for (ii) (rests on H1).

10.2 Falsification register

Theorem 5.1
f_c is not monotone increasing in W_b: if condensation is insensitive to the bilayer remnant concentration the fixed line does not exist and the loop does not close. This is the joint falsification entry shared with Article 68 (Falsification register, item 1 there).

Theorem 6.1
$k \geq 2L_{\text{lip}}/N_0 = 0.1048$: the fixed line becomes unstable. The physical question is whether the aromatic sequestration saturates

at high W_b; saturation would impose an effective upper bound on the linear-model k.

Theorem 7.1
An area-growth factor per cycle different from $Q(\alpha)$. The theorem inherits Article 64's area model; if the membrane pool supply rate changes, the identity $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$ holds algebraically but the per-cycle decay differs. The period would no longer be T.

Corollary 9.1(ii)
$W_b^*/L_{\text{lip}} \geq \varrho_{\max}$: the steady-state load exceeds the bilayer integrity limit, requiring $k \leq \beta/\varrho_{\max}$, which conflicts with the stability lower bound. The two conditions are mutually exclusive for any finite $\varrho_{\max} > 0$; the ceiling is lifted for all feasible k.

10.3 Precedence

Ruling 10.1 (Against Article 64)**.** 🄰 The forty-generation bound is superseded. Article 67 lifted it to persistence; this paper replaces persistence with an exact period. Article 64's shape-homeostasis proof is not contradicted; its $Q(\alpha)$-construction is the identity's source. The area-growth model (block V5 of Article 64) is imported unchanged.

Ruling 10.2 (Against Article 67)**.** 🄰 All four of Article 67's open results are closed (Corollary 9.1). The ϱ-invariance is confirmed in the loop context (Lemma 4.3, Proposition 4.4). The ceiling theorem is not contradicted; it is lifted at the specific operating condition $W_b = W_b^*$.

Ruling 10.3 (Against Article 68)**.** 🄰 The shared target `tgt:cond_rate` is closed by Theorem 5.1. The rate condition (Article 68, Theorem (rate condition)) is satisfied with equality at every point on $\mathcal{F}$. The linear model (Article 68, Hypothesis H1) is adopted unchanged as Hypothesis 2.2.

10.4 Targets

Target 10.4 (The fate of the co-aggregate)**.** Determine whether the poly-R/W co-aggregate forms a dense bilayer phase, is expelled as a bud, or becomes a structural insert. The fixed-line analysis is insensitive to the fate; the rate constant k is not: a tighter co-aggregate raises k and reduces W_b^*; an expelled bud resets the local W_b and changes the transient period. The exact period result holds for any k in the stable range.

Target 10.5 (The non-linear condensation model)**.** Determine whether the aromatic sequestration saturates at high W_b, making $f_c(W_b)$ sub-linear. A saturating f_c gives a sub-linear fixed-point equation; the period identity $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$ still holds at any fixed point (where $f_c = \beta$), but the fixed-point value and stability analysis change. The global Lyapunov argument requires convexity of f_c; a concave f_c may restrict the basin of attraction.

Target 10.6 (Feedback of the encoder on k)**.** Determine whether the nucleic-acid encoder's anchoring modulates the local condensation probability at the hexameric site, thereby coupling the sequence (Article 63) to the rate constant k and, through it, to the steady-state bilayer load W_b^*. The period T is independent of k (Corollary 7.3); what the encoder would control is the *amplitude* W_b^*, not the period. This target is the entry point for the control paper.

10.5 Conclusion

Proposition 10.7 (What is established)**.** 🄰 *Subject to Appendix A:*

(i) The condensation loop has a unique fixed line $\mathcal{F} = \{(N, \beta L_{\text{lip}}/k) : N > 0\}$ *at which* $f_c = \beta$ *(Theorem 5.1).*

(ii) $\mathcal{F}$ *is globally attracting for* $0 < k < 0.1048$*, with a quadratic Lyapunov function and explicit eigenvalue* $\lambda = 1 - kN_0/L_{\text{lip}}$ *(Theorems 6.1–6.3).*

(iii) Convergence is monotone for $k < L_{\text{lip}}/N_0 = 0.052$ *and oscillatory for* $k \in (L_{\text{lip}}/N_0, 2L_{\text{lip}}/N_0)$ *(Proposition 6.4).*

(iv) Post-division recovery is bounded; for k at the safe-region lower bound, recovery to 10% *takes five cycles (Proposition 6.5).*

(v) The division period on $\mathcal{F}$ *is exactly one cycle for all* α*, k, and* N^**, by the algebraic identity* $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$ *(Theorem 7.1).*

(vi) Period homeostasis holds unconditionally on $\mathcal{F}$*; shape homeostasis holds at every division; size homeostasis does not hold (Corollary 7.3, Scholium 7.4).*

(vii) During the transient the period is two cycles, converging to one as $W_b \to W_b^*$ *(Remark 6.6).*

(viii) Four earlier targets are simultaneously closed (Corollary 9.1).

Scholium 10.8 (What the loop makes available next)**.** The steady-state lineage divides every T. The period is fixed; what is free is the rate constant k, which sets the bilayer remnant load $W_b^* = \beta L_{\mathrm{lip}}/k$. If the encoding specifies k (Target 10.6), the lineage gains its first genetically regulated phenotype: a division period that is still T (since T is independent of k) but a bilayer composition that is controlled by the sequence. The sequence addresses the pore (Article 66). Addressing the bilayer load is the next step, using the same mechanism. Period is not the phenotype the encoder controls; the amplitude of the waste stream is.

A Hypothesis register

	Premise	What rests on it	What survives failure
H1	Linear condensation model: $f_c(W_b) = k \cdot W_b/L_{\text{lip}}$, $k \in (0, 2L_{\text{lip}}/N_0)$ (inherited from Art. 68).	Theorems 6.1–6.3 (stability analysis, eigenvalue, convergence time); Proposition 6.5 (transient bound).	Theorem 5.1 holds for any strictly increasing f_c (fixed line exists). Theorem 7.1 and Corollary 7.3 hold for any f_c with $f_c(W_b^*) = \beta$ (period identity is f_c-independent once on $\mathcal{F}$).
H2	Scale parameters N_0, T, L_{lip}, α inherited from Articles 64–67.	All numerical instances in blocks V1–V4.	All theorems hold as formulas in those parameters.

B Verification blocks

V1 [COMPUTED] **The fixed line: closing the shared target**

Derives $W_b^* = \beta L_{\text{lip}}/k$; table of W_b^*/L_{lip} for the corpus α range and representative k values; confirms the safe-region condition.

V2 [COMPUTED] **Stability, convergence regimes, and transient bound**

Computes $\lambda = 1 - kN_0/L_{\text{lip}}$; table of λ, recovery time, and regime for representative k; post-division recovery from αW_b^*.

V3 [COMPUTED] **The exact division period: the algebraic identity and the transient period**

Proves $v_{\text{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$ algebraically; verifies to twelve significant figures for seven α values; computes the transient period as $W_b \to W_b^*$.

V4 [COMPUTED] **Three regimes: a quantitative comparison**

Tabulates N-growth, v-decay, W_b-behaviour, and division frequency for R1, R2, and R3; confirms four closures numerically.

The source file `acf-69-blocks.py` runs as `python3 acf-69-blocks.py`.

References

[Sch26a] Charles D. Schaper. The Chrysene-Reachable Material Ledger: Species, Properties, and the Allocation of a Single Pool. *Annals of the Chrysene Formalism*, 1(3):1371–1383, 2026. Article 65. DOI: https://doi.org/10.68366/dwqk8m15.

[Sch26b] Charles D. Schaper. The First Polypeptide: Condensation, Sequestration, and the Discharge of the Remnant. *Annals of the Chrysene Formalism*, 1(3):1414–1422, 2026. Article 68. DOI: https://doi.org/10.68366/jw1axc69.

[Sch26c] Charles D. Schaper. The First Pore: Closure, Aperture, and the Reconciliation of Impermeance with Supply. *Annals of the Chrysene Formalism*, 1(3):1384–1404, 2026. Article 66. DOI: https://doi.org/10.68366/en23gq52.

[Sch26d] Charles D. Schaper. The Persistent Lineage: Composition of the Pore, the Cycle, and the Remnant. *Annals of the Chrysene Formalism*, 1(3):1405–1413, 2026. Article 67. DOI: https://doi.org/10.68366/h6d6b415.

[Sch26e] Charles D. Schaper. The Photoentrained Cycle: Existence and Stability of a Periodic Orbit in a Forced Hybrid Protocell. *Annals of the Chrysene Formalism*, 1(3):1348–1370, 2026. Article 64. DOI: https://doi.org/10.68366/h9rzkd75.