ANNALS OF THE CHRYSENE FORMALISM

AI-AUGMENTED PEER-REVIEWED | VOL. 1, NO. 3 | ISSN 3144-2021 | OCTOBER 2026

ORTHOGONAL REVIEW RECORD

The First Loop: W_b^*, the Division Period, and the Self-Tuning Lineage

Compiled by the Editorial Office
Annals of the Chrysene Formalism
Schaper Systems Press

Paper ID: ACF-1.3-069
Issue: Volume 1, Number 3
ISSN: 3144-2021
Review architecture: Author-side check followed by independent referee report
Review focus: Stability table errors, global stability claim
Acceptance date: 2026-10-18
Online publication: 2026-10-18
Pages: 1423–1431
Document version: v1.0
Last audited: 18 October 2026

Citation: Schaper, C. D. (2026). The First Loop: W_b^*, the Division Period, and the Self-Tuning Lineage. *Annals of the Chrysene Formalism*, 1(3), 1423–1431. ISSN 3144-2021. `https://chrysene.com/index.php/acf/article/view/69` • © 2026 Charles D. Schaper. Distributed under the CC Attribution-NonCommercial-NoDerivatives 4.0.

Declared conflict

This report was prepared by the same system that drafted the manuscript. All quantitative points are verifiable by running `acf-69-blocks.py` or substituting into the formulas in the body. There is no author-reply section.

Scope of Review

The paper closes the shared target `tgt:cond_rate` from Article 68 and derives three results of increasing strength: the fixed line $\mathcal{F}$, its stability, and the exact period identity $v_{\text{thr}} \cdot Q(\alpha)^{3/2} = 1$. The exact period result is the strongest. Its proof is one line from Article 64's own definitions, and the algebraic identity is verified to twelve significant figures across seven values of α in block V3. The derivation of the ϱ-invariance through the loop (Lemma 4.2) generalises Article 67's result correctly and cleanly. The three required corrections are all in the stability

table of Theorem 6.2. They do not affect any other result in the paper. The fixed line, the exact period, and the four closures are unaffected.

Compiled disposition

Minor revision. Three numerical errors in the stability table must be corrected before acceptance. Two of the three are consequential. One entry claims a point is stable when it is not, and another understates the recovery time by a factor of roughly one-quarter. A fourth point, the global characterisation of the Lyapunov argument, should be downgraded or its proof extended.

Required before print

Required before acceptance. Replace the $k = 0.105$ table row with a representative interior point (e.g. $k = 0.095$) and correct $k_{\max}$ from 0.105 to 0.1048. Correct $k^* = 22$ to $k^* = 25$ for the $k = 0.100$ row. Correct $k^* = 4$ to $k^* = 3$ for the $k = 0.075$ row. Downgrade or extend the global stability claim. The reporter would re-review items 1 through 4 only.

Contents

1 Error 1 (stability table, $k = 0.105$): point is unstable, not stable **3**

2 Error 2 (stability table, $k = 0.100$): recovery time is understated **3**

3 Error 3 (stability table, $k = 0.075$): minor recovery time discrepancy **4**

4 The "global" stability claim should be qualified or proved properly **4**

5 Further points (at the author's discretion) **5**

1 Error 1 (stability table, $k = 0.105$): point is unstable, not stable

Required — $k = 0.105$ exceeds $k_{\max}$

This item is required before acceptance. The table in Theorem 6.2 states, for $k = 0.105$: $\lambda = -0.999$, $|\lambda| < 1$ yes, 2300 cycles, oscillatory. This entry is wrong in every column. The stability boundary is $k_{\max} = 2L_{\text{lip}}/N_0$. With $L_{\text{lip}} = 13{,}200$ and $N_0 = 2.52 \times 10^5$:

$$k_{\max} = \frac{2 \times 13200}{252000} = 0.104762\ldots$$

The value $k = 0.105$ exceeds $k_{\max}$ by 0.000238. The eigenvalue at $k = 0.105$ is $\lambda = 1 - 2.0045 = -1.0045$, giving $|\lambda| = 1.0045 > 1$. The point is unstable. A recovery time of 2300 cycles is not defined, and the entry $|\lambda| < 1$ yes is false. The confusion arises because the text earlier rounds $k_{\max}$ to 0.105, which is fine as a boundary approximation but should not then be used as a representative interior point for the oscillatory regime.

Required: replace the $k = 0.105$ row with a representative interior point, e.g. $k = 0.095$:

$$\lambda = 1 - 0.095 \times 19.09 = -0.814, \quad |\lambda| < 1, \quad k^* = \left\lceil \frac{\log 0.1}{\log 0.814} \right\rceil = 12 \text{ cycles.}$$

Revise the stated $k_{\max}$ from 0.105 to the exact value 0.1048 (four significant figures) wherever it appears in the body and in the stability boundary text of Theorem 6.2.

2 Error 2 (stability table, $k = 0.100$): recovery time is understated

Required — recovery time understated

This item is required before acceptance. The table gives $k^* = 22$ cycles for $k = 0.100$. The eigenvalue is $\lambda = 1 - 0.100 \times 19.09 = -0.909$ and the recovery time is

$$k^* = \left\lceil \frac{\log 0.1}{\log 0.909} \right\rceil = \lceil 24.2 \rceil = 25 \text{ cycles.}$$

The body states 22 cycles. Block V2 also gives 25 cycles for $k = 0.090$ at $|\lambda| = 0.718$. The body's value of 22 at the larger $|\lambda| = 0.909$ is therefore inconsistent with the block.

Required: correct $k^* = 22$ to $k^* = 25$ for the $k = 0.100$ row. A reader using the formula $k^* = \lceil \log 0.1 / \log |\lambda| \rceil$ can verify this directly.

3 Error 3 (stability table, $k = 0.075$): minor recovery time discrepancy

Required — ceiling-function rounding

This item is required before acceptance. The table gives $k^* = 4$ cycles for $k = 0.075$. The eigenvalue is $\lambda = -0.432$ and the correct value is

$$k^* = \left\lceil \frac{\log 0.1}{\log 0.432} \right\rceil = \lceil 2.748 \rceil = 3 \text{ cycles.}$$

This is a rounding error in the ceiling function. The body appears to have used $|\lambda| = 0.432^2 = 0.187$ or a similar mis-step.

Required: correct $k^* = 4$ to $k^* = 3$ for the $k = 0.075$ row.

4 The "global" stability claim should be qualified or proved properly

Recommended — title overstates the proof

This item is recommended before acceptance. Theorem 6.1 is titled Global asymptotic stability and Lemma 6.1 asserts V decreases strictly for all $\delta_k \neq 0$. Both rest on the linearised recursion $\delta_{k+1} = \lambda \delta_k$ with $\lambda = 1 - kN^*/L_{\text{lip}}$, which uses the approximation $N_k \approx N^*$. Away from the fixed line $\mathcal{F}$, N is not constant. When $W_b < W_b^*$, $\gamma > 0$ and N grows each cycle. If N grows to $N_{\max}$, the effective eigenvalue becomes $1 - kN_{\max}/L_{\text{lip}}$, and stability requires $k < 2L_{\text{lip}}/N_{\max}$, a stricter condition than the stated $k < 2L_{\text{lip}}/N_0$.

The practical concern is bounded. Division resets N every two to three cycles during the transient (Remark 6.4), so N cannot grow without limit. Over three cycles at full β growth it reaches at most $N_{\max} = N_0(1+\beta)^3$. At $\alpha = 0.10$ this gives $N_{\max} \approx 1.37\, N_0$ and a revised stability boundary $k < 2L_{\text{lip}}/N_{\max} \approx 0.076$. The safe-region lower bound ($k > \beta/\varrho_{\max} \leq 0.022$) remains inside this revised range, so the physically relevant regime is still stable. The theorem as written asserts more than the proof supports. Three resolutions are available: (1) downgrade the title to Asymptotic stability (local) and clarify that the Lyapunov argument holds for $N_k \approx N^*$; (2) extend the proof by bounding N using the division events; (3) split the theorem into local stability and a proposition on global convergence under bounded N. Option 1 is the minimum change. Option 3 is the most rigorous. Required action: adopt one of the three resolutions above. Annotate the stability condition $k < 2L_{\text{lip}}/N_0$ with a note that it is exact only when $N \approx N_0$.

5 Further points (at the author's discretion)

> **5.1 — State-space notation is inconsistent**
>
> Convention 2.3 defines $\Omega_4 = \mathbb{R}^3_+$ with coordinates (N, A, W_b). The subscript 4 is inconsistent with the superscript 3. $\mathbb{R}^3_+$ is three-dimensional, and there are three coordinates. The subscript appears to be a carry-over from Article 68's four-coordinate version. In Article 69, ϱ is derived and does not appear as an independent coordinate. Suggestion: drop the subscript (write Ω) or use Ω_3 to reflect the actual dimension. If the subscript is intended to name the system from Article 68 (not its dimension), add a parenthetical explaining the convention.

Remark 4.1 should cite equations explicitly. Remark 4.1 states: $N_{k+1} - N_k = \gamma_k N_k = W_{b,k+1} - W_{b,k}$. This follows directly from equations (1) and (3) of Definition 4.1, but the remark does not cite them. A reader who has just read the definition can verify it in one line, but the citation would make the remark self-contained. Suggestion: add (from equations (1) and (3)) after the statement.

The three-regime proposition could include a quantitative table. Proposition 8.1 compares the three regimes qualitatively. For the quantitative reader, the claim the period converges from 2–3 cycles (R2) to 1 cycle (R3) could be supported by the block V4 numbers directly in the proposition's body. Block V4 gives the comparison. A two-row table of (N-growth, v-decay, W_b-behaviour, period) for R2 and R3 side by side would make the transition quantitative. Suggestion: incorporate the block V4 comparison table into the proposition body or its scholium.

What should not be changed

Theorem 7.1 and Corollary 7.2 (exact period and period homeostasis). The proof $v_{\text{thr}} \cdot Q(\alpha)^{3/2} = Q(\alpha)^{-3/2} \cdot Q(\alpha)^{3/2} = 1$ is correct, complete, and one line. The distinction between period homeostasis (unconditional on $\mathcal{F}$) and size homeostasis (absent) is the right characterisation and should stay. Lemma 4.2 (ϱ-invariance through the loop). The three-part lemma is the cleanest result in the paper. Part (i) is two words, part (ii) is one line, part (iii) names the frozen state at the fixed line. The frozen-state observation (contraction factor = 1 when $f_c = \beta$) is new relative to Article 67 and important. It should remain prominent. Remark 6.4 (transient period). The derivation of the two-cycle transient period from $f_{\text{tr}} = (1 + \gamma_k)/Q(\alpha)^{3/2}$ is correct. Block V3 confirms $k_{\text{div}} = 2$ for all corpus values of α, and the convergence narrative (period decreasing from 2 to 1 as $W_b \to W_b^*$) is accurate. Scholium 9.1 (what the self-tuning means). The observation that the fixed-line equation is not a setpoint chosen but the unique solution of $f_c(W_b) = \beta$ is philosophically correct and scientifically important. It should stay. Scholium 10.2 (what the loop makes available next). The final paragraph correctly identifies that the encoder controls the amplitude W_b^* (through k), not the period T (which is α-independent). This connects directly to Target (encoder) and should remain.

Summary of required and recommended actions

1. **Required.** Replace the $k = 0.105$ table row with a representative interior point ($k = 0.095$ suggested). Correct $k_{\max}$ from 0.105 to 0.1048 throughout.
2. **Required.** Correct $k^* = 22$ to $k^* = 25$ cycles for the $k = 0.100$ row.

3. **Required.** Correct $k^* = 4$ to $k^* = 3$ cycles for the $k = 0.075$ row.
4. **Recommended.** Downgrade Global asymptotic stability to Asymptotic stability (or extend the proof as described).
5. *At discretion.* Fix $\Omega_4 = \mathbb{R}^3_+$ notation inconsistency.
6. *At discretion.* Add equation citations to Remark 4.1.
7. *At discretion.* Add a quantitative table to Proposition 8.1.

The reporter would re-review items 1 through 4 only.

Independent referee report (subsequent)

Independent assessment — Accept with minor revisions

A subsequent independent referee report was prepared for the Editorial Board after the author-side check. It recommends Accept with Minor Revisions. The report confirms the three principal results: existence of the fixed line $\mathcal{F}$, Lyapunov stability with the monotone and oscillatory regimes, and the exact period identity $v_{\mathrm{thr}}(\alpha) \cdot Q(\alpha)^{3/2} = 1$. It notes that the paper is mathematically crisp, structurally disciplined, and represents a major milestone in the protocell arc by showing how period homeostasis and shape homeostasis emerge unconditionally while size homeostasis is withheld.

M1 — Reconciling the fixed line with the division reset

The independent report requests clarification of whether W_b in the mother vessel is held at W_b^* because the condensation threshold is evaluated relative to the instantaneous membrane lipid pool $L_{\mathrm{lip}}(A_k)$ that scales alongside W_b during division, or whether L_{lip} is fixed and $\mathcal{F}$ is strictly the invariant manifold of the intra-cycle operator Φ_c, with the hybrid trajectory (Φ_c, ρ_α) executing a sawtooth reset around $\mathcal{F}$ after each division. Making this distinction explicit will harmonize Theorem 5.1 and Theorem 7.1 with Proposition 6.5 and Remark 6.6.

M2 — Linearized versus nonlinear transient stability

The independent report asks that the Abstract, Section 1.2, and Section 10.1 briefly qualify that $0 < k < 2L_{\mathrm{lip}}/N_0 = 0.1048$ is the linearized stability range at corpus scale $N^* = N_0$ (Theorem 6.1), whereas global monotone decrease of V across the bounded-growth transient holds for $0 < k < 2L_{\mathrm{lip}}/N_{\max} \approx 0.076$ (Proposition 6.2). It also asks that the prefactor of the Lyapunov function V be unified: the Abstract and Section 1.2 write $V = (W_b - W_b^*)^2$, whereas Theorem 6.1 defines $V(W_b) = \frac{1}{2}(W_b - W_b^*)^2$.

M3 — Sign of γ_k in the contraction of ρ_k

Lemma 4.3(iii) states that $|\rho_k - 1|$ contracts by $(1+\gamma_k)^{-1}$ per cycle, where $\gamma_k = \beta - f_c(W_{b,k}) \geq 0$ during the transient. While $\gamma_k \geq 0$ holds throughout any transient approaching W_b^* from below in the monotone regime, in the oscillatory regime or for initial conditions $W_b(0) > W_b^*$ the trajectory overshoots so that $\gamma_k < 0$ on alternating cycles. The independent report asks that the qualifier for $W_{b,k} \leq W_b^*$ in the monotone regime be added to Lemma 4.3(iii) and Proposition 4.4.

Minor and typographical items from the independent report

1. Unrendered status labels (`[A]`, `[D]`, `[E]`) appear as blank spaces in Convention 2.1, Remark 4.5, Proposition 6.2, Theorem 6.3, Section 10.1, and Targets 10.4–10.6. These should be restored.
2. Unresolved cross-reference placeholders (`tgt:cond_rate`, `tgt:remnant`, Proposition (invariant), Theorem (persist), Lemma (taut), and others) should be replaced with their published theorem or target numbers.
3. In the final row of the Theorem 6.3 table ($k = 0.110$, $\lambda = -1.100$), the entries under 10%-recovery and $|\lambda| < 1$? are swapped. They should read ∞ (or —) under recovery and no under the stability column.
4. In the proof of Theorem 7.1, $v_{\mathrm{thr}}(\alpha)(\alpha) = Q(\alpha)^{-3/2}$ has a duplicated (α) argument and should read $v_{\mathrm{thr}}(\alpha) = Q(\alpha)^{-3/2}$.
5. In Remark 6.6, At t $\alpha = 0.10$ should be At $\alpha = 0.10$.
6. In Scholium 10.8, the symbol T is missing before the parenthesis in a division period that is still (since T is independent of k).

The independent report finds that these items do not affect the principal results. The fixed line, the exact period identity, and the four closures remain intact.

References

[Art68] C. D. Schaper. The first polypeptide. *Annals of the Chrysene Formalism* 1(3):1414–1422, 2026. Article 68. DOI: `https://doi.org/10.68366/jw1axc69`.

Revision history

Version	Date	Notes
v1.0	2026-10-18	Review record published with Article 69.

Living Document Notice

As with any actively developing field, the mathematical and applied claims in this article are subject to periodic re-evaluation as the surrounding formalism matures and as the journal's AI-augmented review tooling advances. Consistent with standard scientific practice for provisional or fast-moving frameworks, this article and its review record may be re-audited by subsequent review systems, and a revised version bearing an updated review history may be issued to reflect developments in the field. Readers should consult the Revision History table above, and the journal's records, for the current version number and audit date of this article.

Editorial Disclaimer

The *Annals of the Chrysene Formalism* aims to develop the Chrysene Formalism through the application of advanced mathematics, using AI systems selected as well suited to this task. In pursuing this aim, it is possible that errors occur during the mathematical progression and analysis of the formalism, including within the review and editing process itself. However, the formalism is constructed within a multidisciplinary, horizontal framework that is grounded in a specific instantiation of molecular matter and its associated networks, an instantiation that is invariant and falsifiable. For this reason, the editorial office considers the formalism well suited to an AI-augmented approach, and anticipates that this approach will continue to be adapted as AI models and the surrounding mathematical understanding advance.