ANNALS OF THE CHRYSENE FORMALISM

AI-AUGMENTED PEER-REVIEWED | VOL. 1, NO. 4 | ISSN 3144-2021 | OCTOBER 2026

ORTHOGONAL REVIEW RECORD

The Interior Chrysene Lattice: Mathematical Necessity, Distributed Inheritance, and Catalytic Organization in the Protocell

Compiled by the Editorial Office
Annals of the Chrysene Formalism
Schaper Systems Press

Paper ID:	ACF-1.4-078
Issue:	Volume 1, Number 4
ISSN:	3144-2021
Review architecture:	Same-model independent-agent review followed by independent-model referee report
Review focus:	Interior chrysene lattice, Morita inequivalence, UHF algebra, redundancy theorem, Tarski fixed point
Acceptance date:	2026-10-25
Online publication:	2026-10-25
Pages:	1606–1615
Document version:	v1.0
Last audited:	25 October 2026

Citation: Schaper, C. D. (2026). The Interior Chrysene Lattice: Mathematical Necessity, Distributed Inheritance, and Catalytic Organization in the Protocell. *Annals of the Chrysene Formalism*, 1(4), 1606–1615. ISSN 3144-2021. `https://chrysene.com/index.php/acf/article/view/78` •
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Declared conflict

The first report was prepared by the same system that drafted the manuscript, but by an independent agent. A subsequent independent-model referee report was prepared for the Editorial Board. There is no author-reply section.

> **Scope of Review**
>
> The central result — that the fiber corner $P\mathfrak{A}_{\mathfrak{C}}P$ is not Morita equivalent to the full crossed product $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$ — is correct in direction, and the consequence that chain expression

alone cannot close the gap is an important observation for the corpus. The three-dimensionality lower bound is tight. The Redundancy Theorem is clean. The identification of the minimal Bratteli diagram with the UHF algebra of type $\overline{8^\infty}$ is correct. The independent-model review confirms five principal results and notes residual unpropagated revisions, a negated-symbol rendering error, and typographical items.

Compiled disposition

Accept with minor revisions. The first-round required corrections (K-theory proof for Theorem 3.1, Connes-distance argument, packing approximation) were incorporated. The independent-model referee confirms the principal results and recommends acceptance after propagation of remaining Appendix C items, clarification of the UHF supernatural number, reconciliation of the enclosed-radius figures, and clearance of typographical items. No further computation is required. The Morita inequivalence, the UHF identification, and the Redundancy Theorem stand.

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1 Stage summary of the first report

R1 — Theorem 3.1: Picard groups should be K-groups

[**MAJOR**] The theorem claims the two algebras have unequal Picard groups, citing the Künneth formula. This is imprecise. The Picard group of a unital C^*-algebra is the group of Morita self-equivalences, and for a simple unital C^*-algebra it equals the outer automorphism group. The intended argument is correct but requires K-theory. The precise claim is that $K_0(\mathfrak{Q}_{\mathfrak{C}} \rtimes_\alpha \mathbb{Z}) \ncong K_0(\mathfrak{Q}_{\mathfrak{C}} \rtimes_\alpha \mathbb{Z}^3)$ via Pimsner–Voiculescu and the Künneth theorem. Since Morita equivalent C^*-algebras have isomorphic K-groups, non-isomorphic K-groups imply non-Morita-equivalence. Required: replace the proof with the K-theory argument. The conclusion is unchanged.

R2 and R3 — Moderate items

R2: Proposition 4.3 invokes "continuity in the strong topology" for the Connes distance. The Connes distance is not in general continuous in the strong operator topology. Required: replace with the commutator-norm argument and add a forward reference to the spectral triple of Article 39, Axiom A_9. R3: Proposition 4.2 uses the sphere-packing fraction $\varphi = 0.64$ for a flat-slab ICL unit (aspect ratio 4.1). Required: note that the sphere-packing formula is an approximation and that the count is conservative. The conclusion that $L = 200$ is tight and $L \geq 500$ is comfortable is supported by either model.

Minor items and sound findings

Minor: secondary concordance uses the wrong dimension (cavity width 2.90 nm vs. centre-to-centre 3.60 nm); flip-flop rate deserves quantitative discussion; Theorem 6.3 requires fixed $n_{\max}$; PDG arm PCET claim unsourced; cleavage geometry requires opposite-face tethering; failure-probability formula is a product of non-independent events. Sound: Theorem 3.1 correct in direction; Theorem 4.3 (UHF algebra) correct; Theorems 5.1 and 5.4 correct; Theorem 6.3 (Tarski) correctly applied. Ledger supply formula corrected. Connection to Article 3 correctly stated.

2 Stage summary of the independent-model report

Five principal results confirmed

(i) Algebraic necessity via K-theory (Theorem 3.1). (ii) AF-algebra approximation and the Bratteli diagram of division (Theorem 4.4, UHF of type 8^∞). (iii) Geometry, lumen packing, and the three-route Redundancy Theorem. (iv) Environmental reconstitution, the fifth constraint functional C_5, and the Knaster–Tarski fixed point. (v) Interstitial geometry and secondary-structure concordance. Recommendation: Accept with Minor Revisions.

Residual items M1–M4 and minor corrections

M1: Unpropagated revisions from Appendix C (Theorem 6.5 failure probability, Scholium 10.2 cavity-width comparison, Falsification Register Picard-group language). M2: Clarify finite-stage matrix dimensions and UHF(8^∞) vs. UHF($8 \cdot 2^\infty$) in Theorem 4.4; note that $8 \cdot 2^\infty = 2^\infty = 8^\infty$ as supernatural numbers. M3: Reconcile the enclosed ICL radius (6.15 nm vs. 7 nm vs. 2 nm) and the ledger table range (0.54% vs. 0.76%). M4: Synchronize cross-references to Articles 76, 72, 39/40, and 3/4. Minor: restore the missing slash on the negated Morita symbol in Theorem 3.1; move the misplaced proof of Proposition 5.3; clear remaining typographical items.

3 Full first report (same-model independent agent, verbatim)

The following is the complete text of the first referee report.

> **Recommendation: minor revision.** The central result — that the fiber corner $P\mathfrak{A}_{\mathfrak{C}}P$ is not Morita equivalent to the full crossed product $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$ — is correct in direction, and the consequence that chain expression alone cannot close the gap is an important observation for the corpus. The three-dimensionality lower bound is tight and simply proved. The Redundancy Theorem is clean. The identification of the minimal Bratteli diagram with the UHF algebra of type $\overline{8^{\infty}}$ is correct and gives the paper a strong connection to classical C^*-algebra theory. The Tarski fixed-point application is correct.
>
> The revision required is confined to one mathematical item (R1) and two moderate items (R2, R3), none of which affect the main theorems. R1 asks that the Picard group argument in Theorem 3.1 be replaced by an explicit K-theory argument, which the paper's own citation already supports; the direction of the result is not changed. R2 asks that the packing proof acknowledge the flat-slab geometry of the ICL and its effect on the count at $L = 200$. R3 asks that the secondary-structure concordance in Scholium 8.3 clarify which dimension is being compared to the β-strand repeat.
>
> The remaining items are minor. The flip-flop concern is the most practically consequential: for a large amphiphilic cage it may be much slower than the paper implies, and this should be stated rather than deferred entirely to a target.
>
> ### Mathematical stage
>
> - **[MAJOR] Theorem 3.1: Picard groups should be K-groups.** The theorem claims the two algebras have unequal Picard groups, citing the Künneth formula. This is imprecise. The Picard group of a unital C^*-algebra is the group of Morita self-equivalences (invertible C^*-correspondences), and for a simple unital C^*-algebra it equals the outer automorphism group $\mathrm{Out}(A)$, which is generally much larger than any K-group. The Künneth formula applies to K-theory, not to Picard groups directly. The intended argument is correct but requires K-theory, not Picard groups. The precise claim is $K_0(\mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}) \ncong K_0(\mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3)$ via the Pimsner–Voiculescu six-term sequence (for $\mathbb{Z}$) and the Künneth theorem for K-theory (Rosenberg–Schochet, already cited). Since Morita equivalent C^*-algebras have isomorphic K-groups, non-isomorphic K-groups imply non-Morita-equivalence. This replaces Picard groups entirely. Required: Replace the proof with: (i) apply Pimsner–Voiculescu to $\mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}$ and the Künneth theorem to $\mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$; (ii) show the resulting K-groups are non-isomorphic (they differ in rank); (iii) conclude via the K-theory obstruction to Morita equivalence. The conclusion of the theorem is unchanged.
> - **[MODERATE] Proposition 4.3: "continuity in the strong topology" is incorrect.** The proof states that $d_{\mathrm{ICL}}(t) \to 0$ follows from "continuity of the distance in the strong operator topology". The Connes distance is not in general continuous in the strong operator topology: the spectral triple (A, H_{ω}, D) must be specified to evaluate d_{ICL}, and the convergence of the distance as boxes grow requires that the Dirac operator D be compatible with the exhaustion, which is an additional condition.

The correct argument is that for a crossed product $\mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$ approximated by finite-box truncations, the commutator norms $\|[D, \pi_{\text{box}}(a)]\|/\|a\|$ converge uniformly on the unit ball as the box exhausts $\mathbb{Z}^3$, which follows from the local nature of D in the lattice; this is standard in noncommutative geometry (Connes, Chapter VI.4). Required: Replace the one-sentence proof with the commutator-norm argument, and add a forward reference to the spectral triple of Article 39, Axiom A_9, supplying the Dirac operator explicitly.

- **[MODERATE] Proposition 4.2: sphere packing does not apply to a flat slab.** The $2 \times 2 \times 2$ ICL unit has lateral span 4.30 nm and stacking span 1.04 nm, an aspect ratio of 4.1: it is a flat slab, not a sphere. The random close packing fraction $\varphi = 0.64$ applies to monodisperse spheres. For flat slabs the packing fraction is lower and the geometry-dependent formula gives different counts. The paper's formula uses the outer radius $r_{\text{ICL}} = 6.15$ nm, which is close to the lumen radius of 9.04 nm at $L = 200$. The conclusion that the configuration is tight at $L = 200$ is likely correct, but the proof does not support the specific count of 2. Required: Note that the sphere-packing formula is an approximation and state explicitly that the count is conservative. The conclusion that $L \geq 500$ is comfortable, and $L = 200$ admits only the minimum viable configuration, is supported by either model and should be retained.
- **[MINOR] Secondary concordance uses the wrong dimension.** Scholium 8.3 states that the ICL's centre-to-centre spacing 3.60 nm matches the β-strand repeat 3.47 nm to within 4%. The relevant quantity for β-strand templating is the cavity width — the space available between chrysene faces — which is $3.60 - 0.70 = 2.90$ nm. The β-strand repeat and the 2.90 nm cavity width differ by 16%, not 4%. Required: Replace the claim with the cavity-width comparison and state it as a weaker concordance.
- **[MINOR] Flip-flop rate deserves quantitative discussion.** Hypothesis H4 supposes entry by membrane partition and flip-flop. For a large, asymmetrically charged amphiphile, the flip-flop half-life can exceed the period by many orders of magnitude. Required: Add a remark after H4 noting the flip-flop concern quantitatively and naming alternative entry routes if the period does not provide enough time.
- **[MINOR] Theorem 6.3: fixed point requires fixed $n_{\max}$.** The proof assumes the lattice $\{0, \ldots, n_{\max}\}^3$ is fixed. But $n_{\max}$ depends on the vessel radius, which grows during the period. Required: State the theorem for the dusk vessel as the binding constraint, or note that the fixed point is conditional on a stationary vessel radius.

Sound findings

Theorem 3.1 is correct in direction. Theorem 4.3 (UHF algebra) is correct by Glimm's classification. Theorems 5.1 and 5.4 (Redundancy Theorem) are correct. Theorem 6.3 (Tarski) is correctly applied. Article 65 and Article 70 titles are now correct. The ledger supply formula is corrected. The connection to Article 3 is correctly stated. Article 76's supply condition is extended, not contradicted.

Domain and verification

PDG arm geometry is plausible but unsourced. The bilayer enclosure hypothesis is speculative but labelled as such. The cleavage force direction needs a sentence. W-series block descriptions do not include outputs; the verification script is required. The failure-probability formula in Theorem 5.4 needs a correction (report the two one-daughter probabilities separately).

Meta-review

This report was prepared by the same system that drafted the article. Item R1 was found by this review and was not caught during drafting. The editor is asked to obtain an independent mathematical opinion on Theorem 3.1 before acceptance. The UHF identification is stronger than it appears and should be elevated.

4 Full independent-model referee report (verbatim)

The following is the complete text of the independent-model referee report.

Manuscript ID: ACF-2026-078 (Vol. 1, No. 4)
Title: *The Interior Chrysene Lattice: Mathematical Necessity, Distributed Inheritance, and Catalytic Organization in the Protocell*
Author: Charles D. Schaper, Ph.D.
Recommendation: Accept with Minor Revisions

1. Summary and Overall Assessment

This manuscript bridges the operator-algebraic foundations of the Chrysene Formalism (Articles 3/4 and 39/40) and the biophysical protocell division arc (Articles 62–76). Observing that origination transforms the $\mathbb{Z}^2$ base of the $\mathbb{Z}^3$-fibrated tensor network into an undifferentiated phospholipid bilayer, the author shows that the protocell of Articles 62–76 retains only the one-dimensional fiber corner $P\mathfrak{A}_{\mathfrak{C}}P \cong \mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}$ along $\mathbb{Z} \times \{0\} \times \{0\}$. The paper proves that this corner is not Morita equivalent to the full sequence algebra $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$ and constructs the minimal intracellular structure—the Interior Chrysene Lattice (ICL, or Abiotic Loom Unit, ALU)—capable of restoring a bounded three-dimensional representation inside the dividing vessel.

The manuscript establishes five principal results, supported by computational verification blocks W1–W4 (`acf-78-blocks.py`):

1. **Algebraic Necessity via K-Theory (Section 3).** Using the Pimsner–Voiculescu six-term exact sequence and the Rosenberg–Schochet Künneth theorem for KK-theory, Theorem 3.1 proves that $\operatorname{rank} K_0(P\mathfrak{A}_{\mathfrak{C}}P) = r$ whereas $\operatorname{rank} K_0(\mathfrak{A}_{\mathfrak{C}}) = 4r$ (with $r = \operatorname{rank} K_0(\mathfrak{Q}_{\mathfrak{C}}) + \operatorname{rank} K_1(\mathfrak{Q}_{\mathfrak{C}}) \geq 1$), establishing that $P\mathfrak{A}_{\mathfrak{C}}P \not\cong_M \mathfrak{A}_{\mathfrak{C}}$. Corollary 3.2 proves that no adjustment of the chain-expression parameters of Article 76 can close this algebraic deficit.

2. **AF-Algebra Approximation and the Bratteli Diagram of Division (Section 4).** A protocell carrying N ICL units supports the finite-dimensional C^*-algebra $\bigoplus_{i=1}^{N} M_{n_i}(\mathbb{C})$, and the generational lineage under cell division defines an AF system whose inductive limit approximates $\mathfrak{A}_{\mathfrak{C}}$. In the minimal viable configuration, Theorem 4.4 shows that the Bratteli diagram has uniform partial multiplicity $(1, 1)$ at each level, connecting the division rule to a UHF algebra. Proposition 4.5 uses locality of the lattice Dirac operator of Article 39 (Axiom A_9) to show that the Connes spectral distance is non-increasing and converges to 0 as the largest ICL sub-cube grows.

3. **Geometry, Lumen Packing, and the Three-Route Redundancy Theorem (Sections 5–6).** Under Hypothesis 5.1 each tetrasubstituted chrysene node carries PDG linker arms enclosed in a nucleated lipid bilayer. Proposition 5.3 shows that at $L = 200$ random close packing admits $n_{\max} = 2$ units. Theorem 6.3 proves that a 3D representation requires at least 8 nodes. Theorem 6.5 (the Redundancy Theorem) proves that $n_A \geq 1$ or ($n_{B,1} \geq 1$ and $n_{B,2} \geq 1$) guarantees both daughters inherit at least one 3D ICL unit with probability 1.

4. **Environmental Reconstitution, the Fifth Constraint Functional C_5, and the Knaster–Tarski Fixed Point (Sections 7–9).** Stationarity across generations requires an environmental supply of PDG-chrysene monomers per period. Section 8 extends Article 76's admissible domain with a fifth constraint functional $C_5(x) = n_{\mathrm{ICL}}(x) \geq 1$ per daughter. Theorem 9.3 applies the Knaster–Tarski theorem to establish a unique least and greatest fixed-point inventory.

5. **Interstitial Geometry and Secondary-Structure Concordance (Section 10).** Proposition 10.1 and Scholium 10.2 show that a $3 \times 3 \times n_z$ ICL forms interstitial cavities whose lateral repeat matches an extended β-strand repeat to 4%, registered as Target 11.6.

As documented in Appendix C, the manuscript has already undergone a productive first round of revisions. However, a few of the edits recorded in Appendix C were only partially propagated into the body text, and there is a crucial negated-symbol rendering error ($\cong_M$ instead of $\ncong_M$) in Theorem 3.1. I recommend acceptance once the technical and editorial items below are resolved.

2. Major Technical Comments

- **M1. Unpropagated Revisions from Appendix C.** Item 5.2 (Type C failure probability in Theorem 6.5): the statement of the theorem still multiplies the two daughter failure probabilities; update to state them separately. Item 2.4 (cavity width vs. centre-to-centre in Scholium 10.2): the body still quotes only the 4% centre-to-centre match; add the 16% cavity-width comparison. Item R1 (Picard groups vs. K-theory in the Falsification Register): replace "equal Picard groups" with "isomorphic K_0-groups".

- **M2. Clarifying the Finite-Stage Matrix Dimensions and UHF(8^∞) vs. UHF($8 \cdot 2^\infty$) in Theorem 4.4.** The sequence $M_8(\mathbb{C}) \hookrightarrow M_{16}(\mathbb{C}) \hookrightarrow M_{32}(\mathbb{C}) \hookrightarrow \cdots$ doubles the matrix size at each step, so its supernatural number is $8 \cdot 2^\infty = 2^\infty$. Because $8 = 2^3$, the supernatural numbers 2^∞ and 8^∞ are equal, so by Glimm's classification this inductive limit is indeed isomorphic to the UHF algebra of type

8^∞. Adding one sentence noting that $8 \cdot 2^\infty = 2^\infty = 8^\infty$ as supernatural numbers will make the proof transparent.

- **M3. Reconciling the Enclosed ICL Radius and the Ledger Table Range.** Harmonize the outer-radius figures (6.15 nm vs. 7 nm) and clarify that the 77-lipid figure is evaluated at the cage contact surface. Either add the fourth configuration (112 chrysene, 0.76%) to the table or change the text to match the printed 0.54%.
- **M4. Synchronizing Cross-References.** Correct the Article 76 and Article 72 locators (Definition 8.1, Remark 9.4, Ruling 11.1). Harmonize article numbers with reference keys for Articles 3/4 and 39/40.

3. Minor and Typographical Corrections

Restore the missing slash on the negated Morita symbol in Theorem 3.1. Move the misplaced proof of Proposition 5.3 so that it follows the proposition. Add a pointer under the orphaned Appendix B heading. Clear the remaining formula, symbol, and typography fixes listed in the full report (Abstract, Notation 2.2, Definition 4.3, Hypothesis 5.1, tables, Theorem 9.3, status labels, references).

Recommendation

Accept with minor revisions. Propagate the remaining Appendix C items, clarify the UHF supernatural-number identity, reconcile the enclosed-radius figures, and clear the typographical items. No further computation is required. The Morita inequivalence, the UHF identification, and the Redundancy Theorem stand.

References

[Art77] C. D. Schaper. Carbon traceable protocell construction. *Annals of the Chrysene Formalism* 1(4):1567–1605, 2026. Article 77.

Revision history

Version	Date	Notes
v1.0	2026-10-25	Review record published with Article 78.

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