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The Completeness Condition: Realisation of the Twelve-Axiom Framework at a Site of the Interior Chrysene Lattice, and the Genome Length It Requires

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Abstract

Article 79 delivers a substituted-chrysene stock to the protocell and Article 78 places an interior lattice of that stock inside the vessel, reporting that the lattice restores a three-dimensional representation lost at origination. Neither paper asks the prior question: at which site of the dividing vessel, if any, do the twelve axioms of Article 40 have a model at all? The axioms are realised by contractions between *adjacent* cells, so the question is settled by neighbour relations and not by how much chrysene a phase holds.

We answer it. A site realises every contraction of Article 40, Definition 19 only if it has both stacking neighbours, both lateral neighbours in each planar direction, and a covalent linker to each of them. No phase of the arc satisfies all three: the stock columns of Article 79 and the strand realise the stacking contraction and have no lateral extent; the bilayer has lateral extent, no covalent lateral bridge, and at two leaflets no interior stacking site. Fibre and base are realised in different phases and at no common site outside the interior lattice, so without it the base–fibre decomposition is realised nowhere in the vessel: the axioms have no geometric realisation at any site, which is what survival through division has to mean for a framework stated over a network.

The lattice that carries it has a size. The least occupied *box* with a complete site is $3 \times 3 \times 3 = 27$ nodes, which is exactly the configuration Article 78 tabulates as $\kappa_{\mathrm{ICL}} = 1$ and defers as a target; under Article 78's midplane cleavage the least such box whose halves are each complete is $3 \times 3 \times 6 = 54$ nodes. Over arbitrary configurations the minima are 7 and 14, and we give three reasons for taking the box figures as governing. Article 78's minima of 8 and 16 carry no complete site, so within boxes the heritable minimum rises by 3.38, which is affordable at 0.128 per cent of the per-period pool. The binding constraint is geometric, and it is settled exactly: fitting one complete lattice in the lumen is containment of a convex body in a ball, not packing, so no packing fraction enters and the circumradius of the wrapped unit decides it. The least genome length admitting a heritable complete lattice is $L_{\mathrm{crit}} = 222$ bp, 11 per cent above Article 78's minimum viable length; at $L = 200$ none fits. The two inheritance routes then separate — Type A needs 222 bp and Type B, needing two units in one lumen, at most 637 — so over some four hundred base pairs the framework survives only by cleavage of a single lattice.

What the lattice does not achieve is stated before it is mistaken for a result: it does not make the generated algebra Morita equivalent to $\mathfrak{A}_{\mathfrak{C}}$, since an approximately finite algebra has vanishing K_1 and no finite box or limit of boxes is equivalent to a crossed product with $K_1 \neq 0$. Enlarging the lattice changes nothing, so Article 78's corner is not an artefact of its minimality.

Keywords: Chrysene Formalism; interior chrysene lattice; axiomatic realisation; stacking contraction; exact-closure connector; Morita equivalence; K-theory; packing bound; genome length; cell division.

1 Introduction

1.1 What the arc carries, and what it loses

The corpus now runs from a polyaromatic bulk to a dividing vessel without a gap in the chemistry. Article 79 [13] brominates chrysene in the bulk, installs hydroxyethoxy arms through an oxalate, and exports a substituted stock whose composition is matched to demand by a single design equation. Article 77 [6] admits that stock by substitution family and traces its carbon. Article 65 [9] allocates one pool among the demands of the cycle.

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Articles 62 and 76 [11, 10] enclose the strand and divide the vessel. Article 78 [12] places an interior lattice of tetrasubstituted units in the lumen and reports that it restores a three-dimensional representation which undifferentiated origination had lost.

What the arc does not yet carry is an account of its own mathematics. Article 40 [8] axiomatises the formalism on a $\mathbb{Z}^3$-fibrated tensor network and is scrupulous about what the axioms do not give: its Remark 12 reserves bridge and stacking interactions to a physical Hamiltonian introduced later and declines to implement them by quotienting the quasi-local algebra; its Remark 18 declines to infer disjoint exclusion volumes from combinatorial disjointness; its Remark 59 declines to infer a rotational operator, a folding law, a stacking offset, a steric potential, an electronic coupling or an energetic preference from the named perimeter motifs. Each is deferred to "the relevant realisation." The arc is that realisation, and it has never been asked whether it supplies what was deferred.

1.2 Realisation, not transfer

The question is usually put as whether the twelve-axiom framework *transfers* from the bulk to the dividing cell. Put that way it has a known negative answer, and the corpus supplies it: Article 78 establishes $P\mathfrak{A}_{\mathfrak{C}}P \nsucc_M \mathfrak{A}_{\mathfrak{C}}$, so what the protocell interior carries is a corner and not the algebra. Asking again whether the framework transfers intact is asking to contradict a published theorem.

The productive question is different and is local. Article 40 builds its structures by *contractions between adjacent cells*: the directional contractions C_x and C_y of its Definition 19, across the exact-closure connectors of its equation (107); and, for each registry state $\eta \in \mathbb{Z}^2$, the stacking contraction $C_z^{(\eta)}$ of its equation (128), between the two interfaces V_{z^-}, V_{z^+} that its equation (118) assigns to a local cell. Whether those contractions have values is a question about neighbour relations in matter. It is therefore a question each phase of the arc answers differently, and it can be answered exactly.

This paper asks it. The object is a site, not an algebra; the predicate is that every contraction at that site has a value; and the finding is that the predicate holds in one phase of the arc and no other.

1.3 What is and is not claimed

Scholium 1.1 (The shape of the claim). **Claimed.** That the contractions of [8, Def. 19] have values at a site of the vessel if and only if that site sits inside a laterally cross-linked three-dimensional chrysene network of extent at least three in each direction; that the least such network has 27 nodes and the least heritable one 54; that this is affordable on the material ledger and not on the geometric one; and that the least genome length admitting it is 222 bp.

Not claimed. That the generated algebra is Morita equivalent to $\mathfrak{A}_{\mathfrak{C}}$. It is not, by Proposition 8.2, and no enlargement of the lattice alters that. A finite vessel cannot contain $\mathbb{Z}^3$; what it can contain is a site at which the axioms hold. The distinction is drawn in Scholium 8.4 and is the difference between this paper's claim and one that [12] has already refuted.

Load-bearing. Theorem 4.1 rests on Hypothesis 3.5, that the exact-closure connector requires a covalent linker and is not realised by lateral van der Waals contact. A reader who rejects that hypothesis recovers the bilayer as a candidate and should read Remark 4.5 before doing so. Theorem 6.4 rests on no hypothesis of this paper: its geometry is exact and its vessel-radius law is imported in closed form from [7, Prop. 4.1].

2 Conventions and imports

Convention 2.1 (Status labels)**.** Every numbered statement carries one of four labels. [A] finite, or derived without appeal to a hypothesis of this paper. [D] derived under hypotheses named at the point of use. [C] imported from a cited source and not re-derived here. [E] a molecular reading, which enters no proof. Scholia are unlabelled and enter no proof. Every quantitative claim in the body is a macro emitted by `verify_a80.py`; the emitting block is cited as W1–W5 and the blocks are described in Appendix A.

Notation 2.2 (Imports)**.** From [8]: the locant sets and attachment vertices of Axiom A_1; the exact-closure connector $(\mathbf{k}, v_{+\alpha}) \rightharpoonup (\mathbf{k}+e_\alpha, v_{-\alpha})$ of equation (107) for $\alpha \in \{x, y\}$; the stacking relation $\mathbf{k} \sim_z \mathbf{k}+e_z$ of equation (108); the registry label $\eta_{\mathbf{k}} \in \mathcal{R}_z := \mathbb{Z}^2$ of equation (109) and its translation transport, equations (112)–(113); the interface spaces V_{z^-}, V_{z^+} of equation (118); the local tensor space $\mathsf{T}_{\mathfrak{C}}$ of equation (119); the contractions C_x, C_y of equations (126)–(127) and $C_z^{(\eta)}$ of equation (128), all of Definition 19; the base–fibre decomposition of §4.4 and Proposition 3; the tensor-space fibration of §4.6; the quasi-local algebra $\mathfrak{Q}_{\mathfrak{C}}$ of equation (155) and the crossed product $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_\alpha \mathbb{Z}^3$ of Axioms A_7–A_9.
From [12]: the lattice geometry of Definition 5.2; the packing bound of Proposition 5.3 and its tabulated rows; the configuration table, including the entry $\kappa_{\mathrm{ICL}} = 1$; the three-dimensionality lower bound of Theorem 6.3; the midplane cleavage of Hypothesis 6.2; the supply formula, equation (1); the corner statement $P\mathfrak{A}_{\mathfrak{C}}P \nsucc_M \mathfrak{A}_{\mathfrak{C}}$.
From [9]: the per-period demand of Theorem 8.5 and its verification block. From [13]: the stock phase σ and its columnar sub-phase κ.

Remark 2.3 (A citation that resolves elsewhere)**.** C [12] cites the stacking operator $\hat{\Pi}$ and the threshold $\Gamma_{\text{stack}} \le \Lambda_{\text{crit}}$ to "Article 39, Lemma 11," and separately cites a distance to the same lemma. Lemma 11 of [8] is *Disjoint primary and dual locant supports*, $A^{\text{pri}}_{\mathfrak{C}} \cap A^{\text{dual}}_{\mathfrak{C}} = \emptyset$, equation (211): a graph-theoretic statement, and neither a distance nor an operator. The operator that [12] needs does exist in [8]: it is the stacking contraction $C_z^{(\eta)}$ of Definition 19, equation (128), together with the registry field of equations (109)–(113). The threshold does not, and cannot be derived there, since Remarks 12 and 59 of [8] expressly reserve stacking offsets and energetics to a realisation. Both corrections are recorded in Appendix B; the realisation that discharges the reservation is [13], whose columnar free energy Δ_π and registry suppression $e^{-\nu\Delta_{\text{rgy}}}$ are an energetic model of the registry field.

3 The realisation predicate

Definition 3.1 (Occupied box)**.** For $n_x, n_y, n_z \ge 1$ let

$$B(n_x, n_y, n_z) := \prod_{\alpha \in \{x,y,z\}} \{0, 1, \dots, n_\alpha - 1\} \subset \mathbb{Z}^3,$$

with one chrysene unit at each site, each unit carrying the local tensor space $\mathsf{T}_{\mathfrak{C}}$ of [8, eq. (119)]. Write B for such a box when its extents are not in question, and $\iota(n) := \max(0, n-2)$.

C The restriction to boxes is [12]'s. Its Definition 5.2 defines a lattice unit as "$n_x \times n_y \times n_z$ chrysene nodes" and gives its spans as functions of those extents, so the configurations the corpus admits are boxes. Remark 5.2 shows that the restriction is not merely inherited: the completeness predicate of Definition 3.2 is local, and relaxing it to arbitrary finite subsets of $\mathbb{Z}^3$ admits configurations that satisfy the predicate without being lattices in the sense [8]'s apparatus requires.

Definition 3.2 (Site classes)**.** Let $\mathbf{k} \in B$. Then $\mathbf{k}$ is

(i) *z-engaged* if $\mathbf{k} \pm e_z \in B$, so that both interfaces of [8, eq. (118)] are contracted by $C_z^{(\eta)}$;

(ii) *laterally bridged* if $\mathbf{k} \pm e_x \in B$ and $\mathbf{k} \pm e_y \in B$ and each of the four lateral neighbours is joined to $\mathbf{k}$ by an exact-closure connector in the sense of [8, eq. (107)];

(iii) *complete* if both.

Write $Z(B), X(B), K(B)$ for the three sets.

Hypothesis 3.3 (H_{sat}, saturated bridging)**.** D Every lateral adjacency within an occupied box carries an installed exact-closure connector. At the heritable minimum of Theorem 5.1(b) this is 72 bridges over 216 arm terminals, 72 of which remain free, alongside 45 stack relations, which require no installation.

E The hypothesis is one of synthesis, not of combinatorics: whether an assembly of this size can be built with every lateral adjacency bridged is not addressed here and is Target 11.3. It is used by Proposition 3.4, Theorem 4.1 and Theorem 5.1; under partial bridging the counts of (3) and (4) are upper bounds and the minima of Theorem 5.1 rise.

Proposition 3.4 (Site counts)**.** A *For $B = B(n_x, n_y, n_z)$, under Hypothesis 3.3,*

$$|B| = n_x n_y n_z, \tag{1}$$
$$|Z(B)| = n_x n_y\, \iota(n_z), \tag{2}$$
$$|X(B)| = \iota(n_x)\, \iota(n_y)\, n_z, \tag{3}$$
$$|K(B)| = \iota(n_x)\, \iota(n_y)\, \iota(n_z). \tag{4}$$

Proof. A site $\mathbf{k} = (k_x, k_y, k_z)$ has both α-neighbours in B precisely when $1 \le k_\alpha \le n_\alpha - 2$, which admits $\iota(n_\alpha)$ values and none when $n_\alpha \le 2$. The three coordinate conditions are independent, so each class is a product of the corresponding ranges, giving (2)–(4). Equation (1) is the cardinality of the box. All four are exact integer identities; block W2 evaluates them. □

Hypothesis 3.5 (H_{cov}, the covalent lateral bridge)**.** D The exact-closure connector of [8, eq. (107)], which joins the attachment vertex $v_{+\alpha}$ of one cell to the vertex $v_{-\alpha}$ of its neighbour, is realised between two chrysene units if and only if those vertices are joined by a covalent linker. In the interior lattice the linker is the arm

-O-CH_2CH_2-O-P(=O)(O^-)-O-CH_2CH_2-O-

of [12, §4], installed on the primary locants $\Lambda = \{3, 6, 9, 12\}$ by the route of [13]. In a fluid bilayer, lateral neighbours are in van der Waals contact and carry no such linker.

E The chemical reading is that a connector is a bond and not a proximity. The hypothesis is load-bearing for Theorem 4.1 and for nothing else in this paper; Remark 4.5 states what follows if it is rejected.

D The corpus makes it natural without establishing it. [6, Def. 3.1] separates, in defining a constitutional stock type, "constituent copies, covalent bridges, non-covalent relations, orientation reference and order," and states expressly that *a stack relation is not a covalent bond and adds no atoms*. So the two relations are already distinct fields of the admitted

object.
That separation is made for atom inventory, which is a different purpose, and it does not say which field [8, eq. (107)]'s connector reads. Hypothesis 3.5 says it reads the covalent one. The inference is a reasonable one across two papers and it is an inference; it is marked [D] accordingly, not imported.

Proposition 3.6 (What each structure requires at a site)**.** [C], [D] *Let* $\mathbf{k} \in B$*. Then, at* $\mathbf{k}$*:*

(a) the stacking relation of [8, eq. (108)] is realised on at least one incident edge iff $n_z \geq 2$*;*

(b) the registry label of [8, eq. (109)] exists on that edge whenever (a) holds, and its translation transport, [8, eqs. (112)–(113)], is defined on two incident edges iff $n_z \geq 3$*;*

(c) both stacking contractions $C_z^{(\eta)}$ *of [8, eq. (128)] have values iff* $\mathbf{k}$ *is z-engaged;*

(d) both directional contractions C_x, C_y *of [8, eqs. (126)–(127)] have values iff* $\mathbf{k}$ *is laterally bridged, which under Hypothesis 3.5 requires a covalent linker to each of the four lateral neighbours;*

(e) the base–fibre decomposition of [8, §4.4] is realised at $\mathbf{k}$ *iff* $\mathbf{k}$ *is complete.*

Proof. (a)–(d) are the definitions of the cited objects together with Definition 3.2; the contractions are bilinear maps on tensor factors carried by *adjacent* cells, so each has a value exactly when the adjacency it requires lies in B. For (e): [8, Prop. 3] decomposes $\mathbb{Z}^3$ into the base $\mathbb{Z}^2$ under π_{xy} and the fibre $\pi_{xy}^{-1}(m,n) \cong \mathbb{Z}$. Realised in matter at $\mathbf{k}$, the fibre direction is the stacking direction and requires (c); the base directions are the planar directions and require (d). Both together are completeness. □

4 The first theorem: necessity

Theorem 4.1 (Necessity of a laterally bridged three-dimensional network)**.** [D] *Under Hypothesis 3.5, a site* $\mathbf{k}$ *of a phase of the arc realises every contraction of [8, Def. 19] only if*

(i) $n_z \geq 3$ *and* $\mathbf{k} \pm e_z$ *are occupied;*

(ii) $n_x \geq 3$ *and* $n_y \geq 3$ *and* $\mathbf{k} \pm e_x$, $\mathbf{k} \pm e_y$ *are occupied;*

(iii) the four lateral neighbours of $\mathbf{k}$ *are joined to it by covalent linkers.*

No phase of the arc other than the interior chrysene lattice satisfies (i)–(iii). Consequently, in a vessel containing no interior chrysene lattice, the base–fibre decomposition of [8, §4.4] is realised at no site and the tensor-space fibration of [8, §4.6] has no fibre over any occupied base point.

Proof. Conditions (i) and (ii) are Proposition 3.6(c)–(d) with Proposition 3.4: a z-engaged site exists only if $\iota(n_z) \geq 1$, and a laterally bridged site only if $\iota(n_x), \iota(n_y) \geq 1$. Condition (iii) is Hypothesis 3.5.
The phase classification is block W5, tabulated in Table 1. The bulk β and the free lumen carry no lattice and fail (i)–(iii). The stock phase σ and its columnar sub-phase κ of [13] are columns: $\iota(n_z) \geq 1$ holds and $n_x = n_y = 1$, so (i) holds and (ii) fails. The strand is tabulated at extent 1 laterally; a duplex carrying the two tethers of [11] and [5] would reach extent 2, which still fails (ii), since completeness requires 3. The row is therefore read as extent at most two, and the conclusion is the same either way. The bilayer has $n_x, n_y \gg 3$, so the extent part of (ii) holds; but its lateral neighbours carry no covalent linker, so (iii) fails, and at two leaflets $\iota(n_z) = 0$, so (i) fails as well. The interior lattice is the only phase of the arc in which the linker of Hypothesis 3.5 is present, and it satisfies (i)–(iii) exactly when its extents reach three, which is Theorem 5.1.
For the final clause, apply Proposition 3.6(e): if no site is complete then no point of the realised network carries the base–fibre decomposition, so in the fibration of [8, §4.6] the fibre $p_{\text{ten}}^{-1}(\mathbf{k})$ is unvalued at every occupied $\mathbf{k}$. □

Remark 4.2 (What does *not* follow, and why)**.** [C] An earlier form of Theorem 4.1 carried a third conclusion: that axioms A_7–A_9 have no model in a vessel without an interior lattice. It is withdrawn, and [8] is the reason.
Remark 12 of that paper states that the contraction maps of its Definition 19 "specify how the local tensors are geometrically connected", that the algebra $\mathfrak{Q}_{\mathfrak{C}}$ *instead* "records bounded operations on the local structural degrees of freedom", and that bridge and stacking interactions "are not implemented by quotienting the quasi-local algebra by the tensor-contraction relations". The local tensor space of its Definition 17 sits at every site irrespective of its neighbours. A vessel with no complete site therefore still carries the local degrees of freedom and the algebra built on them; what it does not carry is the geometric connection between them. This is the distinction the whole paper turns on, and it is drawn again at Scholium 8.4. What fails without the lattice is the *realisation* of the axioms' geometry, not the existence of the algebra. The withdrawn clause confused the two.

Corollary 4.3 (Complementarity of the fibre and the base)**.** [D] *The stock column realises the stacking contraction and no lateral connector; the bilayer has lateral extent and*

neither a lateral connector nor an interior stacking site. The fibre and the base of [8, §4.4] are therefore realised in different phases of the arc, and at no common site outside the interior lattice.

Proof. Immediate from Table 1, rows σ and bilayer, with Proposition 3.6(e). □

Scholium 4.4 (Why the lattice is the only candidate). Corollary 4.3 explains, in the arc's own terms, the observation [12] records without deriving: that the $\mathbb{Z}^2$ base is lost at undifferentiated origination and recovered by the lattice. The loss is not an accident of the origination step. π-stacking supplies the fibre wherever chrysene stacks, which is in the bulk, in the stock columns and along the strand; the base needs covalent connectors in two planar directions, and the only structure in the arc that installs them is the linker arm.

The two are not equally cheap, and the asymmetry is why it is the base that goes missing. A column of 3 units already carries a z-engaged site; a laterally bridged site needs nine units in the plane; a complete site needs 27. The fibre is almost free and the base is not. A phase can have one or the other. The lattice is where both occur at one site, and that is the whole of its mathematical function.

Remark 4.5 (Scope, and what would weaken this). D Three limits should be stated.
First, Theorem 4.1 is conditional on Hypothesis 3.5. If lateral van der Waals contact were held to realise the exact-closure connector, the bilayer would satisfy (ii) and (iii) and would fail only (i), which two leaflets cannot supply; the theorem's conclusion would survive but its margin would narrow to the stacking direction alone, and a three-leaflet or interdigitated membrane would then have to be excluded on other grounds. The hypothesis is the one place where a chemical judgement carries a mathematical conclusion, and it is registered accordingly — though, as Hypothesis 3.5 records, the judgement is [6]'s before it is this paper's.
Second, the theorem is a statement about sites of a phase as the arc currently describes those phases. It does not exclude a structure the corpus has not proposed.
Third, the necessity is of a network, not of [12]'s particular network. Any laterally cross-linked three-dimensional chrysene arrangement of sufficient extent would serve; that the corpus's candidate is the interior lattice is a fact about the corpus and not a consequence of Theorem 4.1.

5 The second theorem: the minimum

Theorem 5.1 (The completeness minimum). A *Let B range over occupied boxes, under Hypothesis 3.3.*

(a) $\min\{\,|B| : |K(B)| \geq 1\,\} = 27$, *attained only at* $3\times3\times3$.

(b) Under the midplane cleavage of [12, Hyp. 6.2], which separates B perpendicular to the stacking axis into halves B^-, B^+,

$$\min\{\,|B| : n_z \text{ even},\ |K(B^{\pm})| \geq 1\,\} = 54,$$

attained only at $3\times3\times6$. *Each half is* $3\times3\times3$ *and carries exactly one complete site.*

(c) The configurations of [12, Table 1] at 8 and 16 nodes have no complete site: $|K| = 0$ *for* $2\times2\times2$ *and* 0 *for* $2\times2\times4$. *The 16-node configuration has* $|Z| = 8$ *and* $|X| = 0$.

Proof. (a) By (4), $|K(B)| \geq 1$ forces $\iota(n_\alpha) \geq 1$, hence $n_\alpha \geq 3$, for each α. Therefore $|B| = n_x n_y n_z \geq 27$, with equality exactly when $n_\alpha = 3$ for all three, and in that case $|K| = 1$. The bound is analytic, so no search is required; block W2 confirms it by exhaustive enumeration over $n_\alpha \leq 8$.
(b) The halves are $B(n_x, n_y, n_z/2)$. Applying (a) to a half forces $n_x, n_y \geq 3$ and $n_z/2 \geq 3$, so $n_z \geq 6$ and $|B| \geq 54$, with equality exactly at $(3,3,6)$. Each half is then $(3,3,3)$ and has one complete site by (a).
(c) $\iota(2) = 0$, so $|K| = 0$ whenever any extent is 2; and for $(2,2,4)$, $|Z| = 2\cdot2\cdot\iota(4) = 8$ while $|X| = \iota(2)\iota(2)\cdot4 = 0$. □

Remark 5.2 (The unrestricted minima, and why boxes govern). A Definition 3.2's completeness predicate is local — six occupied neighbours — so over arbitrary finite subsets of $\mathbb{Z}^3$ the least configuration carrying a complete site is 7: a site and its six neighbours. The least heritable one is 14, two such crosses cleaving between them. Theorem 5.1 reports 27 and 54 because it minimises over boxes, and the reader is owed the difference and the reason for it.
Beyond [12, Def. 5.2], which defines a lattice unit as a box and so settles which configurations the corpus admits, the cross fails in three respects to be a lattice in the sense [8]'s apparatus requires. Block W8 computes each.

(a) *It is a star.* Its degree sequence is $[1,1,1,1,1,1,6]$: all 6 nodes but the centre have exactly one neighbour, against a minimum degree of 3 in the $3\times3\times3$ box. It realises the contractions at one site and at no other, and no enlargement of it along the same pattern changes that.

(b) *It carries almost no translation structure.* [8, eqs. (110)–

Table 1: Article 40 structures realised at a site, by phase of the arc. Computed at block W5 from Proposition 3.4 and Hypothesis 3.5. "η" is the existence of a registry label on an incident z-edge; "transport" its translation transport, [8, eqs. (112)–(113)]. The last column is completeness in the sense of Definition 3.2. Exactly 2 rows are complete, both of them interior lattices at extent three or more. The "3D" column is [12, Thm. 6.3]'s criterion, extent at least two in each direction; it is weaker than completeness and a row may satisfy it without carrying a single valued contraction.

Phase	extent	3D	η	transport	$C_z^{(\eta)}$	C_x, C_y	complete
bulk β	$1\times1\times1$	–	–	–	–	–	–
stock σ (column)	$1\times1\times n$	–	yes	yes	yes	–	–
columnar sub-phase κ	$1\times1\times n$	–	yes	yes	yes	–	–
bilayer	$m\times m\times2$	yes	yes	–	–	–	–
strand	$1\times1\times n$	–	yes	yes	yes	–	–
lumen (free unit)	$1\times1\times1$	–	–	–	–	–	–
interior lattice, $2\times2\times2$ [12]	8 nodes	yes	yes	–	–	yes	–
interior lattice, $2\times2\times4$ [12]	16 nodes	yes	yes	yes	yes	yes	–
interior lattice, $3\times3\times3$	27 nodes	yes	yes	yes	yes	yes	**yes**
interior lattice, $3\times3\times6$	54 nodes	yes	yes	yes	yes	yes	**yes**

(113)] build the network's symmetry from translations U_ℓ acting on the occupied set. The cross overlaps a nonzero translate of itself in at most 2 of its 7 nodes; the box, in 18 of 27. A configuration that barely meets any translate of itself realises no part of the periodic structure the axioms are stated over.

(c) *It does not cleave as the corpus's lattice cleaves.* [12, Hyp. 6.2] describes cleavage along a midplane perpendicular to the stacking axis under tether tension. The cross pair severs 1 contact at that plane; the $3\times3\times6$ box severs 9. A single bond is not a midplane.

Points (b) and (c) are the substantive ones: the completeness predicate is local by construction, and a purely local predicate cannot by itself exclude a configuration that is locally complete and globally degenerate. The box restriction supplies what the predicate omits. A sharper criterion — completeness together with a minimum degree, or with a translation condition — would exclude the cross directly and is Target 11.2.

Corollary 5.3 (The minimum is Article 78's $\kappa_{\mathrm{ICL}} = 1$). A *The box of Theorem 5.1(a) is precisely the configuration tabulated as $\kappa_{\mathrm{ICL}} = 1$ in [12, Table 1], at 27 nodes with lateral span 7.90 nm and stacking span 1.38 nm. That table introduces κ_{ICL} as a complexity parameter and defers its consequences, expecting $\kappa_{\mathrm{ICL}} > 0$ to template secondary structure and registering the expectation as a target. Independently of that expectation, the parameter acquires here a determination: $\kappa_{\mathrm{ICL}} \geq 1$ is necessary and sufficient for the occupied box to carry a complete site.*

A *The coincidence should not be read as more than it is. Within box configurations the smallest with an interior point is $3\times3\times3$, and $\kappa_{\mathrm{ICL}} = 1$ is that box; the agreement is close to forced once the restriction of Definition 3.1 is in place. What it does establish is that [12]'s complexity parameter, introduced there without a criterion, has one.*

Proof. Comparison of extents with [12, Table 1]; the spans are reproduced from [12, Def. 5.2] at block W3. Sufficiency and necessity are Theorem 5.1(a) and the monotonicity of (4) in each extent. □

Remark 5.4 (Two criteria, and a minimum that rises). A [12, Thm. 6.3] proves that a finite sub-lattice of $\mathbb{Z}^3$ carries a three-dimensional representation — extent at least two in each direction — if and only if it has at least 8 nodes, the minimum being the $2\times2\times2$ cube, and sets the minimal whole Type A at 16. Theorem 5.1 does not contradict either statement. The criteria differ: [12] asks for *extent*, and this paper asks for *valued contractions*. Extent two in a direction places a neighbour on one side of a site and not on both, so it supports a representation and not a contraction.

Under the stronger criterion, and within the box configurations [12, Def. 5.2] admits, the minima rise: from 8 to 27 nodes for a half unit, a factor of 3.38, and from 16 to 54 for a whole heritable one, a factor of 3.38. The increase is a refinement under a stronger criterion and not the correction of an error. Both qualifications matter: without the box restriction the minima would *fall*, to 7 and 14, for the reasons Remark 5.2 rejects. Its consequences are material — Proposition 7.2 — and geometric — Theorem 6.4 — and only the second binds.

Remark 5.5 (The supply formula is not extent-free). C [12, eq. (1)] reads $n_{\mathrm{supply}} = 8n_A + 8(n_{B,1} + n_{B,2}) + 16n_C$,

in which the coefficients 8 and 16 are the node counts of that paper's minimal half and whole units and not constants of the construction. Restated with the node count explicit, one Type A founder draws 8 nodes at [12]'s minimum and 27 at the minimum of Theorem 5.1(b). Block W4 recovers both.

6 The third theorem: the genome bracket

6.1 Containment, not packing

[12, Prop. 5.3] counts lattice units in the lumen by the random close packing bound

$$n_{\max}(L) = \left\lfloor \varphi \left(R_{\text{lumen}}(L)/r_{\text{ICL}}\right)^3 \right\rfloor, \qquad \varphi = 0.64, \tag{5}$$

with $R_{\text{lumen}} = R_{62}(L) - d_{\text{bl}}$, $d_{\text{bl}} = 4.0$ nm, and r_{ICL} taken as half the lateral span plus d_{bl}. That paper records (5) as an approximation, noting that the true unit is a flat slab of aspect ratio about four.

For the question this paper asks, the approximation is unnecessary. The threshold is reached when *one* complete unit fits, and whether one convex body fits inside a ball is containment and not packing: it is decided exactly, with no packing fraction. Proposition 6.1 supplies the criterion and Theorem 6.4 applies it, so the threshold carries no hypothesis of this paper.

Proposition 6.1 (The wrapped unit and its circumradius)**.** [A] *Let K be the core lattice box of edge lengths (a_x, a_y, h), let $d_{\text{bl}} > 0$, and let $K_{d_{\text{bl}}} := K \oplus B(d_{\text{bl}})$ be the set of points within d_{bl} of K. Then*

$$\text{circumrad}(K_{d_{\text{bl}}}) = \tfrac{1}{2}\sqrt{a_x^2 + a_y^2 + h^2} \; + \; d_{\text{bl}}, \tag{6}$$

and $K_{d_{\text{bl}}}$ is contained in a lumen of radius R_{lumen} if and only if that quantity is at most R_{lumen}.

Proof. Let c be the circumcentre of K and $\rho = \text{circumrad}(K)$. Every point of $K_{d_{\text{bl}}}$ lies within d_{bl} of a point of K, hence within $\rho + d_{\text{bl}}$ of c; and the bound is attained by displacing a point of K at distance ρ from c by d_{bl} along that ray. So $\text{circumrad}(K_{d_{\text{bl}}}) = \rho + d_{\text{bl}}$. For a box, ρ is half the space diagonal, giving (6). The containment criterion is the definition of the circumradius together with the convexity of $K_{d_{\text{bl}}}$: the smallest ball containing a convex body has radius equal to its circumradius, so the body fits in a ball of radius R_{lumen} exactly when circumrad $\le R_{\text{lumen}}$. □

Remark 6.2 (Why the Minkowski sum is the physical shape)**.** [E] Two readings of "wrapped in a bilayer" must be distinguished, and they differ materially. Enlarging the core box by $2d_{\text{bl}}$ in each edge is *not* $K \oplus B(d_{\text{bl}})$: the enlargement keeps square corners, while the Minkowski sum rounds them at radius d_{bl}. At the $2 \times 2 \times 2$ unit the two differ by 2.72 nm of radius, which is the whole of the discrepancy reported in §6.2.

The Minkowski sum is the physical shape, not merely the smaller bound. A lipid bilayer cannot form a sharp edge; it rounds at a radius set by its own bending modulus and thickness, which for a membrane of thickness d_{bl} is of order d_{bl}. A bilayer following the slab at constant distance and rounding its edges is exactly $K \oplus B(d_{\text{bl}})$.

Independent support comes from [12] itself. Its Definition 5.2 states an outer radius of "3 + 4 = 7 nm" for the minimal $2 \times 2 \times 2$ unit. The circumradius convention gives 7.08 nm; that paper's own Proposition 5.3 gives 6.15 nm. The narrative figure agrees with (6) and not with the convention (5) is built on, so [12] contains both conventions and they disagree. Its further figure of "≈ 9 nm" for the $2 \times 2 \times 4$ unit matches neither, (6) giving 7.16 nm; that is recorded in Appendix B and not reconciled here.

Proposition 6.3 (The vessel radius law)**.** [C] *$R_{62}(L) = c\sqrt{L}$ with $c = 0.92195$ nm·bp$^{-1/2}$ and $\nu = 65.73$. [7, Prop. 4.1] gives*

$$\frac{8\pi R_{62}^2}{a} = \frac{\nu L}{2}, \qquad R_{62}^2 = \frac{L_p h L}{2E^*}, \qquad \nu = \frac{8\pi L_p h}{aE^*},$$

whence $c^2 = L_p h / 2E^*$.

[A] *This paper first fitted $c = 0.9219$ to [12, Prop. 5.3]'s four tabulated rows, with maximum relative residual 0.00015, [12] having cited R_{62} from [11] without a closed form. The fit agrees with [7]'s closed form to 0.000029 relative, so the law is imported and not assumed, and Theorem 6.4 carries no hypothesis of this paper at all. Block W3 checks both.*

Theorem 6.4 (The genome threshold)**.** [A] *With Proposition 6.3 and the criterion of Proposition 6.1:*

(a) The least genome length whose lumen admits one complete site is 219 bp, at the 27-node minimum of Theorem 5.1(a).

(b) The least genome length admitting the heritable minimum of Theorem 5.1(b) is

$$L_{\text{crit}} = 222 \text{ bp}, \tag{7}$$

exceeding [12]'s minimum viable length by 22 bp, or 11 per cent. At $L = 200$ no complete lattice fits.

(c) Parts (a) and (b) are exact: the criterion is necessary and sufficient, and no packing fraction enters.

Proof. Block W6. By Proposition 6.1 with the spans of [12, Def. 5.2], the wrapped unit has circumradius 9.63 nm at $3 \times 3 \times 3$ and 9.71 nm at $3 \times 3 \times 6$. It is contained in the lumen exactly when $R_{\text{lumen}}(L) \geq$ that radius; solving with Proposition 6.3 and taking the least integer gives 219 and 222. Exactness is Proposition 6.1: a convex body fits in a ball if and only if its circumradius does not exceed the radius of that ball, so for a single unit the condition is not merely sufficient. □

Corollary 6.5 (The two inheritance routes are not equally cheap)**.** D *[12, Thm. 6.5] secures the representation by either route: one Type A unit, which cleaves at division, or one Type B unit tethered to each strand. Under completeness the two routes have different costs. The Type A route requires $L_{\text{crit}} = 222$ bp by Theorem 6.4(b). The Type B route requires two complete units in one lumen, for which $R_{\text{lumen}} \geq 29.63$ nm suffices, giving $L \geq 637$ bp. Consequently, for*

$$222 \;\leq\; L \;\leq\; 636 \quad bp,$$

only the Type A route can carry a complete framework, and the midplane cleavage of [12, Hyp. 6.2] becomes load-bearing for the survival of the axioms and not only for the statistics of inheritance.

Proof. Block W6. Two disjoint balls of radius r lie in a ball of radius R whenever $R \geq 2r$: place their centres at $\pm(R - r)$ along a diameter, which are at separation $2(R - r) \geq 2r$. The Type A figure is Theorem 6.4(b). □

Remark 6.6 (The Type B figure is sufficient and not exact)**.** D Corollary 6.5's Type B threshold bounds the requirement from above and is not claimed as the least such length. Replacing each wrapped unit by its circumsphere discards the fact that two oblate bodies may interlock — lying face to face, or offset — more efficiently than two spheres. The exact two-body threshold is therefore at most 637 bp and may be smaller; Target 11.4 asks for it. Nothing else in this paper depends on it, since Theorem 6.4 is a one-body statement and exact.

6.2 A superseded bound, and a withdrawn claim

Remark 6.7 (What the exact criterion corrects)**.** A An earlier draft of this paper read the geometry by circumscribing the *enlarged box*, of edges $(a_x + 2d_{\text{bl}},\ a_y + 2d_{\text{bl}},\ h + 2d_{\text{bl}})$, rather than the Minkowski sum, and so obtained the radii 9.80, 12.18 and 12.39 nm in place of 7.08, 9.63 and 9.71. Those overstate the wrapped unit, by 2.72 nm at the smallest, for the reason given in Remark 6.2: enlarging a box keeps its corners and the Minkowski sum rounds them.

Two consequences follow.
First, the threshold is a number and not an interval. The bracket that the enlarged-box radius and (5) produced between them, [206, 316] bp, is superseded by (7), which lies inside it. Neither of the two crude criteria is needed and the packing fraction φ plays no part.
Second, a claim must be withdrawn. Under the enlarged-box radius, [12]'s own minimal 8-node unit appeared not to clear $L = 200$, requiring 225 bp, which would have put that paper in conflict with itself. Under the exact criterion it clears $L = 200$ comfortably: one such unit fits from $L = 145$ bp. The apparent conflict was an artefact of the superseded bound and is withdrawn.
What survives is smaller and is a counting matter. Two 8-node units require $L \geq 389$ bp by the argument of Corollary 6.5, so at $L = 200$ exactly one fits, against the $n_{\max} = 2$ that [12, Prop. 5.3] states. That paper's viability conclusion is unaffected, since its §6 requires one unit and not two; only the count is high, and (5) is the reason. Recorded in Appendix B.

Scholium 6.8 (A lower bound on genome length). Read forward, Theorem 6.4 says that completeness is bought with vessel volume, and vessel volume with genome length. A vessel too small to hold a complete interior lattice still carries chrysene, still carries a strand, still carries a bilayer — and carries no site at which the twelve axioms have a model. Whether such a vessel should be called a protocell of the formalism is a question this paper raises and does not answer. What it answers is narrower: the formalism does not survive in it.

The bound is worth comparing with the design intent of [10], which asks for a genome that is self-filtering — one that does not divide unless it is rich enough to be worth copying. Theorem 6.4 supplies a second filter at the same place, derived from the mathematics rather than imposed from the biology, and (7) is now sharp enough to compare: 222 bp against [12]'s minimum viable 200. The two filters do not coincide, and the mathematical one binds first by 22 bp. Whether [10]'s expression threshold also sits in that interval is not settled here.

7 The material ledger

Ruling 7.1 (The two pool figures)**.** A [9, Thm. 8.5] gives the per-period demand

$$R \;\geq\; 5L + \frac{\alpha}{1-\alpha} N + \nu L, \tag{8}$$

and its verification block evaluates (8) at $L = 200$, $\nu = 66$, $N = 2.52 \times 10^5$ and $\alpha = 0.10$, obtaining 42, 200. [12, §9] quotes a per-period pool of 14, 705 at the same genome length, a factor of 2.87 smaller.

The two are the same expression. The L-proportional part $5L + \nu L$ is 14, 200; the residue that [12]'s figure leaves for the osmolyte term is 505, which is that term at $\alpha = 0.002$. Back-substitution into (8) reproduces 14, 705 exactly, at block W1.

Ruling. [9] derives (8) and [12] values it. By corpus seniority on the object, [9]'s derivation governs the formula, and [12]'s figure stands as a valuation at a smaller bud fraction. Both are retained and neither is withdrawn. [12]'s conclusion that the pool does not bind holds *a fortiori* at the larger pool: its two tabulated demands fall from 0.054 and 0.54 per cent to 0.019 and 0.19 per cent.

Proposition 7.2 (The stronger lattice is affordable)**.** A *The heritable-complete Type A of Theorem 5.1(b) draws* 54 *chrysene nodes per founder. As a fraction of the per-period pool this is* 0.128 *per cent at [9]'s bud fraction and* 0.367 *per cent at [12]'s, both below the ceiling of* 0.76 *per cent that [12, Rem. 9.2] states for the maximal lattice demand. The binding constraint on the lattice is therefore geometric, by Theorem 6.4, and not material.*

Proof. Block W4, with the node count of Theorem 5.1(b) and the pool figures of Ruling 7.1. □

Remark 7.3 (An internal disagreement in Article 78)**.** C [12] gives the lattice demand as 0.05–0.76 per cent of the pool in its abstract, and its §9 table lists a largest configuration of 80 nodes at 0.54 per cent. A demand of 0.76 per cent of 14, 705 is 112 nodes, a configuration that appears nowhere in that paper. The abstract's range and the §9 table therefore disagree. Proposition 7.2 is stated against the 0.76 per cent ceiling because that is the more permissive of the two and the conclusion survives it; the discrepancy is recorded in Appendix B for [12] to settle.

8 Non-implications

The results above are easily over-read, and the over-reading has already been refuted in the corpus. This section states what the lattice does not achieve, before Section 12 draws what it does.

Hypothesis 8.1 (H_{K_1}, non-vanishing of $K_1(\mathfrak{A}_{\mathfrak{C}})$)**.** D $K_1(\mathfrak{A}_{\mathfrak{C}}) \neq 0$ for $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$ as constructed in [8, A_7]. [8] computes no K-groups. The Pimsner–Voiculescu exact sequence [4], applied three times, contributes to K_1 of a crossed product by $\mathbb{Z}^3$ from the K_0 of the coefficient algebra, and non-vanishing is expected for the translation action; it is not established in the corpus. Registered because Proposition 8.2 is conditional on it. Computing $K_1(\mathfrak{A}_{\mathfrak{C}})$ is Target 11.6.

Proposition 8.2 (No finite or approximately finite realisation is Morita equivalent to $\mathfrak{A}_{\mathfrak{C}}$)**.** D *Let B be a finite occupied box, $\mathfrak{B}(B)$ the algebra generated by the structures realised over it, and $\mathfrak{B}_{\infty}$ any inductive limit of such algebras along unital embeddings. Under Hypothesis 8.1, neither $\mathfrak{B}(B)$ nor $\mathfrak{B}_{\infty}$ is Morita equivalent to $\mathfrak{A}_{\mathfrak{C}}$.*

Proof. $\mathfrak{B}(B)$ is finite-dimensional, being generated by finitely many local tensor factors, and $\mathfrak{B}_{\infty}$ is approximately finite by construction. An approximately finite algebra has $K_1 = 0$ [3, 1]. K-theory is a Morita invariant [1], so Morita equivalence with $\mathfrak{A}_{\mathfrak{C}}$ would force $K_1(\mathfrak{A}_{\mathfrak{C}}) = 0$, contradicting Hypothesis 8.1. □

Corollary 8.3 (Enlargement does not close Article 78's gap)**.** D *[12]'s conclusion $P\mathfrak{A}_{\mathfrak{C}}P \nsim_M \mathfrak{A}_{\mathfrak{C}}$ is not a consequence of the minimality of its configuration. By Proposition 8.2 it is unchanged at* 27 *nodes, at* 54*, and at any finite or approximately finite size. The increase of Theorem 5.1 buys completeness of the realisation, and buys nothing in the Morita class.*

Scholium 8.4 (What survival through division should be taken to mean). Two statements are separated by Corollary 8.3 and are easy to conflate.

Realisation of the axioms at a site. Every contraction of [8, Def. 19] has a value at that site, so the local structures of §§4.3–4.7 of that paper exist in the matter of the vessel. This is what Theorems 4.1 and 5.1 govern. It depends on the interior lattice, and it is available in a finite vessel.

Isomorphism of the generated algebra. The algebra generated is Morita equivalent to $\mathfrak{A}_{\mathfrak{C}}$. This is a statement about the infinite crystal. It is not available in a finite vessel, by Proposition 8.2, and its unavailability has nothing to do with the lattice.

A finite vessel cannot contain $\mathbb{Z}^3$, so the second was never the right requirement. What a vessel can contain is a site at which the axioms hold, and the claim of this paper is the first. Read as the second, the paper would assert what [12] has already disproved.

9 What the raised minimum obliges elsewhere

Theorem 5.1 raises a figure that two other papers have already used, and the consequences are recorded here rather than left for a reader to find.

Remark 9.1 (The stock witness no longer has room)**.** A [6, Fig. 1] exhibits an explicit stock-reservation witness of 810 carbon atoms, partitioned into four disjoint roles as 40 + 162 + 168 + 440. [13]'s formula reproduces that partition exactly: 162 is 9×18, the nine bare-core pore units that [13] draws once per founder; 168 is $3(22+26)+24$ at the disubstituted, tetrasubstituted and trisubstituted carbon counts; and 440 is twenty disubstituted copies. The two papers therefore agree on the molecules, and block W7 confirms each identity.

None of the four roles is an interior lattice. A lattice node is a tetrasubstituted unit of 26 carbon; its bridges are phosphate diesters, which by [6, §3.1]'s balance add no carbon, and its stack relations add no atoms by [6, Def. 3.1]. So the lattice of Theorem 5.1(b) costs 1404 carbon against a witness of 810, a factor of 1.73.

The obligation falls on this paper and not on [6]. At [12]'s minimum the lattice costs 416 carbon and the witness accommodates it; the shortfall is introduced by Theorem 5.1. What fails is capacity and not chemistry, and [6, Table 1] describes the witness as distinct from the operating model. An enlarged witness exists. Adjoining the lattice as a fifth disjoint role, its 54 copies named apart from the three tetrasubstituted copies of the pair reservation and the twenty disubstituted copies of the module budget, gives a witness of 2214 carbon, a growth factor of 2.73. Its lateral bridges are phosphate diesters carrying no carbon and its stack relations carry no atoms, so the whole increase is in the nodes. Whether to adopt it in place of [6]'s is that paper's decision; Target 11.8 records the construction rather than the problem.

Remark 9.2 (The manifest bound on stack height)**.** A [6, §3.1] requires a finite initial manifest declaring, among other bounds, an upper bound on stack height. [12]'s whole minimum is $2\times2\times4$ and needs 4; the lattice of Theorem 5.1(b) is $3\times3\times6$ and needs 6. The declared bound must rise accordingly, or the manifest excludes the only configuration that carries the axioms. This is the smallest of the obligations in this section and the easiest to overlook, since nothing in the carbon ledger records it.

Remark 9.3 (The admissible-domain predicate admits too much)**.** A [12, Thm. 8.2] extends [10]'s admissible domain by the clause $C_5 = n_{\mathrm{ICL}}$ per daughter ≥ 1, counting units of 8 nodes. By Theorem 5.1(c) an 8-node unit has no complete site. The clause as written is therefore satisfied by a daughter carrying no model of the axioms, which is the condition Theorem 4.1 rules out.

Restating it requires a choice this paper does not make for [10]: whether C_5 should count complete sites, complete units, or heritable units, which give thresholds of one site, 27 nodes and 54 nodes respectively. Target 11.9.

10 Hypothesis register

Theorem 5.1(a) and (c), Proposition 3.4 and Proposition 6.1 carry no hypothesis of this paper and are finite or elementary identities. Theorem 4.1 carries one, and Theorem 6.4 carries none, its radius law being imported in closed form. A reader allocating scepticism should spend nearly all of it on Hypothesis 3.5, which converts a chemical judgement into a mathematical conclusion and on which the necessity theorem wholly rests; Hypothesis 8.1 affects only the non-implication of §7.

11 Targets

Remark 11.1 (A target discharged)**.** C The closed form of $R_{62}(L)$ was registered here as a target while this paper's radius law was a fit. [7, Prop. 4.1] supplies it, as Proposition 6.3 records, and the target is discharged. The thresholds of Theorem 6.4 are unchanged, the correction entering in the fifth figure.

Target 11.2 (A criterion that excludes the degenerate case directly)**.** Strengthen Definition 3.2's completeness predicate so that it excludes the configurations of Remark 5.2 without appeal to the box restriction — by a minimum-degree clause, or by requiring the occupied set to meet a nonzero translate of itself in a stated fraction of its nodes. The restriction is warranted as it stands; a predicate that did the work itself would be better.

Target 11.3 (Saturated bridging)**.** Decide whether an assembly at the minimum of Theorem 5.1(b) can be built with every lateral adjacency bridged, as Hypothesis 3.3 assumes. Under partial bridging the minima rise and Theorem 5.1(b) becomes a lower bound.

Target 11.4 (The exact two-body threshold)**.** Determine the least genome length whose lumen admits *two* complete lattice units, and so the exact Type B threshold of Corollary 6.5. Replacing each wrapped unit by its circumsphere gives 637 bp as an upper bound; two oblate bodies may interlock more efficiently, so the true figure may be smaller and the interval of Corollary 6.5 correspondingly narrower. Counting three or more units requires the sphere-in-sphere packing literature [2] in place of the elementary two-body

Table 2: Hypotheses of this paper, where each is used, and what fails without it.

Tag	Statement	Used in	Without it
H_{cov} (Hyp. 3.5)	the exact-closure connector requires a covalent linker	Thm. 4.1	the bilayer satisfies (ii)–(iii) and fails only (i); the conclusion survives with a narrower margin (Rem. 4.5)
H_{K_1} (Hyp. 8.1)	$K_1(\mathfrak{A}_{\mathfrak{C}}) \neq 0$	Prop. 8.2, Cor. 8.3	the non-implication of §7 is not established; [12]'s corner statement is unaffected, being proved there by other means
[12, Hyp. 6.2] (imported)	midplane cleavage perpendicular to the stacking axis	Thm. 5.1(b)	the heritable minimum is not 54; part (a) is unaffected

argument used here.

Target 11.5 (The count, as against the threshold)**.** Replace (5) by an exact count of wrapped slabs in a spherical lumen, superseding [12, Prop. 5.3]'s $n_{\max}$ column. Theorem 6.4 does not need it, being a one-body statement, but [12]'s inheritance statistics are stated in terms of $n_{\max}$ and would move with it.

Target 11.6 (The K-theory of the crossed product)**.** Compute $K_0(\mathfrak{A}_{\mathfrak{C}})$ and $K_1(\mathfrak{A}_{\mathfrak{C}})$ for the translation action of [8, A_7], discharging Hypothesis 8.1 or refuting it. If $K_1(\mathfrak{A}_{\mathfrak{C}}) = 0$, Proposition 8.2 fails and the question of Scholium 8.4 reopens.

Target 11.7 (Whether proximity can realise a connector)**.** Decide whether lateral van der Waals contact in a fluid bilayer realises the exact-closure connector of [8, eq. (107)], discharging Hypothesis 3.5 or refuting it. [8, Rem. 18] leaves the analogous steric question to a later molecular instantiation theorem; this is the same question for connectors.

Target 11.8 (The witness, adopted or superseded)**.** Remark 9.1 constructs a witness of 2214 carbon reserving disjoint material for the lattice alongside [6, Fig. 1]'s four roles. What remains is for [6] to adopt it, or to show that the operating model of its Table 1 already covers the lattice and that no enlargement is needed. The construction does not settle which.

Target 11.9 (The admissible-domain predicate)**.** Restate [12, Thm. 8.2]'s clause C_5 so that it reads completeness rather than node count, and propagate the restatement to [10]'s admissible domain. Remark 9.3.

Target 11.10 (The lattice demand ceiling)**.** Settle the disagreement recorded in Remark 7.3 between [12]'s abstract and its §9 table, and identify the configuration at 112 nodes if one is intended.

12 Conclusion

The twelve axioms are realised by contractions between adjacent cells, so the question of their survival through division is a question about neighbour relations and is answerable exactly. The answer is that one phase of the arc realises them and no other. π-stacking supplies the fibre wherever chrysene stacks — in the bulk, in the stock columns of [13], along the strand — and the base requires covalent connectors in two planar directions, which only the linker arm installs. A phase can have one or the other; the interior lattice is where both meet at a site. Without it the base–fibre decomposition is realised nowhere in the vessel, and the axioms, which are stated over a network, have no realisation at any site in it.

The lattice that carries the framework has a least size, 27 nodes, and a least heritable size, 54. Both exceed [12]'s minima, which were set by a weaker criterion, and the first coincides with the configuration that paper had already tabulated as $\kappa_{\mathrm{ICL}} = 1$ and deferred. The increase costs 0.128 per cent of the per-period pool and is affordable. What it costs in vessel volume is settled exactly, because fitting one unit is containment and not packing: the genome must reach 222 bp, 11 per cent above [12]'s minimum viable length, and below that no site in the vessel realises the axioms. The two inheritance routes part company at the same point, the Type B route costing up to 637 bp, so over an interval of some four hundred base pairs the framework survives division only by cleavage of a single lattice.

What the lattice does not do is return the vessel to $\mathfrak{A}_{\mathfrak{C}}$. No

finite or approximately finite realisation can, by Proposition 8.2 and under its registered hypothesis that $K_1(\mathfrak{A}_{\mathfrak{C}}) \neq 0$, and enlarging the lattice does not help. That is not a deficiency of the construction but a misreading of the requirement: a finite vessel cannot contain $\mathbb{Z}^3$, and survival of a framework through division is the realisation of its axioms at a site, which is available, and not the reconstruction of the infinite crystal, which is not.

A Verification blocks

Every number in the body is a macro emitted by `verify_a80.py`, which uses the standard library only and exact rational arithmetic except where noted. The blocks are:

W1 Pool reconciliation.
Evaluates (8) at both bud fractions, solves for the fraction implied by [12]'s figure, back-substitutes to confirm it, and recomputes that paper's lattice percentages at both pools. Exact.

W2 Completeness geometry.
Evaluates (1)–(4) for [12, Table 1] and for the configurations of Theorem 5.1, and confirms the two minima by exhaustive enumeration over extents up to eight. Exact.

W3 Vessel geometry.
Reproduces [12, Def. 5.2]'s spans and [12, Prop. 5.3]'s $n_{\max}$ column exactly, fits the radius law and checks the fit against [7, Prop. 4.1]'s closed form. Retained because it validates the imported geometry against that paper's published table; its threshold figures are superseded by W6. Floating point; cube roots and the fit require it.

W4 Exact containment.
Computes the circumradius (6) of the wrapped unit for each configuration, compares the three conventions, checks the agreement with [12, Def. 5.2]'s narrative figure, and solves Theorem 6.4 and Corollary 6.5. Floating point.

W5 Supply and affordability.
Recovers [12, eq. (1)] and its restatement, and the pool fractions of Proposition 7.2. Exact.

W6 Realisation by phase.
Evaluates the predicates of Proposition 3.6 for each phase of the arc, producing Table 1, and asserts that only the complete configurations satisfy all of them. Exact.

The script asserts each published value rather than printing it, and exits non-zero on any failure; it also asserts that every macro it emits is defined once.

B Precedence rulings and recorded conflicts

R1. The pool figures.
Resolved; Ruling 7.1. One expression at two bud fractions. Both retained; [12]'s conclusion holds *a fortiori* at the larger pool.

R2. The stacking operator citation.
[12]'s $\hat{\Pi}$ resolves to the stacking contraction $C_z^{(\eta)}$ of [8, Def. 19, eq. (128)] with the registry field of [8, eqs. (109)–(113)], not to Lemma 11 of that paper, which is the disjointness of the primary and dual locant supports. Citation corrected; Remark 2.3.

R3. The stacking threshold.
$\Gamma_{\text{stack}} \leq \Lambda_{\text{crit}}$ does not occur in [8] and cannot be derived there: Remarks 12 and 59 of that paper reserve stacking offsets and energetics to a realisation. The mark on [12]'s use of it is downgraded from imported to derived, and the realisation that discharges the reservation is [13]. Recorded; not restated as a theorem here.

R4. The lattice minima.
[12, Thm. 6.3] and Theorem 5.1 answer different questions and both stand; the minima for *completeness* are 27 and 54 nodes. Remark 5.4.

R5. The supply coefficients.
[12, eq. (1)]'s coefficients 8 and 16 are its own minima and not constants; restated with the node count explicit. Remark 5.5.

R6. The demand ceiling.
[12]'s abstract and §9 table disagree on the maximal lattice demand. Recorded, unresolved; Remark 7.3 and Target 11.10.

R7. The outer radius.
[12] carries two conventions. Its Definition 5.2 states $3 + 4 = 7$ nm for the $2 \times 2 \times 2$ unit, agreeing with the circumradius (6), which gives 7.08 nm; its Proposition 5.3 uses half the lateral span plus d_{bl}, giving 6.15 nm. Ruling: the Definition 5.2 convention is correct and governs, by Proposition 6.1; Proposition 5.3's radius is superseded. Definition 5.2's further figure of ≈ 9 nm for the $2 \times 2 \times 4$ unit matches neither convention, (6) giving 7.16 nm, and is recorded unresolved.

R8. The unit count.
With the radius of R7 and the two-body argument of Corollary 6.5, two 8-node units require $L \geq 389$ bp, so exactly one fits at $L = 200$ against the $n_{\max} = 2$ of [12, Prop. 5.3]. The count is high; the viability conclusion of that paper's §6, which requires one unit, is unaffected. Recorded; Target 11.5.

R9. A withdrawal by this paper.
An earlier reading here circumscribed the enlarged box rather than the Minkowski sum and concluded that [12]'s minimal unit fails at $L = 200$. That conclusion was an artefact of the superseded bound and is withdrawn; the unit clears $L = 200$ from $L = 145$ bp. Remark 6.7.

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