ACF 80 — Lean 4 Proof Dossier

The Completeness Condition

Realisation of the Twelve-Axiom Framework at a Site of the Interior Chrysene Lattice, and the Genome Length It Requires

Charles D. Schaper, Ph.D., Schaper Systems Press

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Proof file: `acf-80-proof.lean` Verification harness: `verify_A80.py`

Kernel Certification

Item	Value
Proof file	`acf-80-proof.lean`
Lean version	`4.12.0` (core only, no Mathlib)
Namespace	`ChryseneFormalism.Article80`
Sections	S2–S10
Theorems	61
Proved by `decide`	61
Depending on no axioms	**61**
Depending on `propext`	0
Depending on `Quot.sound` or `funext`	0
`sorry`	0
`native_decide`	0

What This Certifies

The Lean 4.12.0 kernel independently verifies 61 claims from Article 80. Each is stated over `Nat` and discharged by `decide`: the goal is a decidable proposition, the kernel evaluates a verified Boolean decision procedure, and the proof is accepted only when that evaluation yields `true`. No external oracle; no `native_decide`; no `sorry`; no Mathlib import.

The file assumes nothing at all. An `#print axioms` audit over every theorem reports *"does not depend on any axioms"* in all 61 cases. Not `propext`, not `Quot.sound`, not `funext`, not `Classical.choice`. The customary three-axiom footprint quoted in Lean dossiers would overstate what is assumed here, so it is not quoted.

The proof covers: the scaled geometry of Article 78 Definition 5.2; the completeness predicate and its factorisation $\iota(n_x)\iota(n_y)\iota(n_z)$; the emptiness of Article 78's two ICL configurations; the minima of Theorem 5.1 by exhaustive search over every box in range; the unrestricted minima and the three counts that distinguish the cross from a lattice; the exact containment criterion and the genome thresholds, including the headline 222 base pairs; the Type B route and the Type A window; the carbon ledger against Article 77's witness; the incidence counts of the heritable unit; and the supply formula.

Encoding

Lengths as integers. Every span is scaled to hundredths of a nanometre, so the π-stacking rise 0.34 nm is 34, the chrysene body width 0.70 nm is 70, the lateral pitch 3.60 nm is 360, and the bilayer half-thickness 4.00 nm is 400. Nothing is rounded: these are the article's constants, exactly.

The completeness predicate. A site realises all six contractions exactly when its six axial neighbours are occupied, so in an $n_x \times n_y \times n_z$ box the count is $\iota(n_x)\,\iota(n_y)\,\iota(n_z)$ with $\iota(n) = \max(0, n-2)$. Lean's `Nat` subtraction truncates at zero, so ι is simply `n - 2` and the truncation is the desired behaviour rather than a hazard — `iota_truncates` records it.

The minima, by exhaustion. `minCompleteOK` and `minHeritableOK` range over every box with each edge below twelve (fourteen for the heritable case) and assert that any box with a complete site has at least 27 nodes, and any heritable box at least 54. The range is not a restriction: a complete site needs every edge at least three, so no smaller box qualifies, and enlarging an edge only enlarges the node count. The kernel evaluates the whole search.

The threshold, without forming a root. This is the one claim whose statement contains a square root, through the circumradius $r_{\mathrm{circ}} = \frac{1}{2}\sqrt{S} + d$ of the wrapped unit. Scaling to hundredths of a nanometre and clearing denominators with $c^2 = L_p h / 2E^* = 0.85$ turns the containment condition into

$$34000\,L \;\geq\; S + 3200\sqrt{S} + 2560000,$$

in which the only non-integer is $\sqrt{S}$. For the heritable unit $S = 1\,305\,800$ and $1142^2 < S < 1143^2$. Replacing $\sqrt{S}$ by 1143 gives a *sufficient* condition, which $L = 222$ satisfies; replacing it by 1142 gives a *necessary* one, which $L = 221$ fails. A bracket that settles both directions settles the threshold, so the root is never formed and nothing is lost. The same device certifies 219, 145, 637 and 389.

What is not encoded. The Lean file proves arithmetic and finite combinatorics. It says nothing about whether a covalent lateral bridge is chemically required — the load-bearing hypothesis of the article — nor about the continuous functional-analytic claims, which no `Nat` proof can reach.

Theorem Index

Section	Name	Statement (informal)
S2	`lat_two`, `lat_three`	lateral spans 430, 790
	`stk_two` … `stk_six`	stacking spans 104, 138, 172, 240
S3	`iota_truncates`	$\iota(0..3) = 0, 0, 0, 1$
	`minimal_B_incomplete`	2×2×2: 8 nodes, none complete, none z-engaged
	`whole_A_incomplete`	2×2×4: 16 nodes, 8 z-engaged, none complete
	`box_27_complete`	3×3×3: exactly one complete site
	`box_54_complete`	3×3×6: four, and one per half
S4	`min_complete_is_27`	**Thm 5.1**: exhaustive, no box below 27
	`min_complete_attained`	3×3×3 attains it
	`min_heritable_is_54`	exhaustive, no heritable box below 54
	`min_heritable_attained`	3×3×6 attains it
	`raises_A78_minima`	$54/16 = 27/8$
S5	`unrestricted_minima`	7 and 14 over arbitrary subsets
	`cross_is_a_star`	six of seven nodes are leaves
	`box_carries_translations`	overlap 2/7 against 18/27
	`box_cleaves_a_plane`	nine contacts severed, $= 3 \times 3$
	`cross_cleaves_a_bond`	one contact severed
S6	`sumSq_54` … `sumSq_16`	S for the four configurations
	`root_54_bracket` …	integer brackets for the four roots
	`threshold_54_holds_at_222`	**222 bp** suffices
	`threshold_54_fails_at_221`	and 221 does not
	`threshold_27_holds_at_219`	219 for a single complete unit
	`threshold_8_holds_at_145`	145 for Article 78's own
	`threshold_excess`	$222 - 200 = 22$
	`threshold_inside_old_bracket`	$206 \leq 222 \leq 316$
S7	`typeB_holds_at_637`	Type B needs 637
	`typeA_only_window`	222 to 636 is Type A only
	`two_eight_node_at_389`	two 8-node units need 389
	`one_unit_at_reference`	$145 \leq 200 < 389$
S8	`unit_is_C26`	a node is C_{26}
	`witness_partition`	$40 + 162 + 168 + 440 = 810$
	`A78_fits_witness`	$416 \leq 810$
	`A80_exceeds_witness`	$810 < 1404$
	`witness_excess_factor`	between 1.7 and 1.8
	`enlarged_witness`	$810 + 1404 = 2214$
S9	`heritable_bridges`	72 lateral bridges
	`heritable_stacks`	45 stack relations
	`heritable_arms`	216 arm terminals
	`heritable_free_arms`	72 free after bridging

Section	Name	Statement (informal)
	`arm_thirds`	one third free, two thirds bridged
S10	`supply_A78`	Article 78 eq. (1) gives 8
	`supply_A80`	restated, gives 27
	`supply_typeC`	16 against 54 for a Type C vessel

Proof Listing: `acf-80-proof.lean`

`acf-80-proof.lean` — complete source

```
/-
  acf-80-proof.lean
  Annals of the Chrysene Formalism, Vol. 1, Issue 5.
  Article 80 -- "The Completeness Condition: Realisation of the Twelve-Axiom
  Framework at a Site of the Interior Chrysene Lattice, and the Genome
  Length It Requires"

  Lean 4 formal proof file.  Core only: no Mathlib, no `sorry`, no
  `native_decide`.  Every theorem is discharged by `decide`: the goal is a
  decidable proposition over Nat and the kernel evaluates a verified
  Boolean decision procedure.

  ENCODING.  Lengths are scaled to hundredths of a nanometre so that every
  span is an integer: the pi-stacking rise 0.34 nm is 34, the chrysene
  body width 0.70 nm is 70, the lateral pitch 3.60 nm is 360, and the
  bilayer half-thickness 4.00 nm is 400.

  The genome threshold is the one claim whose statement contains a square
  root, through the circumradius of the wrapped unit.  It is certified
  here WITHOUT leaving Nat, by bracketing that root between consecutive
  integers and showing that the threshold holds at L even under the
  pessimistic bracket and fails at L-1 even under the optimistic one.  A
  bracket that settles both directions settles the threshold, so nothing
  is lost by never forming the root.

  Build:  lean acf-80-proof.lean
-/

namespace ChryseneFormalism.Article80

/-! ## S2.  Scaled constants (hundredths of a nanometre) -/

def width  : Nat := 70      -- chrysene body width, 0.70 nm   (A78 Def 5.2)
def pitch  : Nat := 360     -- lateral chrysene pitch, 3.60 nm
def rise   : Nat := 34      -- pi-stacking rise, 0.34 nm
def dBL    : Nat := 400     -- bilayer half-thickness, 4.00 nm (A78 Prop 5.3)

/-- Lateral span of an n-wide block. -/
def lat (n : Nat) : Nat := (n - 1) * pitch + width
/-- Stacking span of an n-tall column. -/
def stk (n : Nat) : Nat := (n - 1) * rise + width

theorem lat_two   : lat 2 = 430 := by decide
theorem lat_three : lat 3 = 790 := by decide
theorem stk_two   : stk 2 = 104 := by decide
theorem stk_three : stk 3 = 138 := by decide
theorem stk_four  : stk 4 = 172 := by decide
theorem stk_six   : stk 6 = 240 := by decide

/-! ## S3.  The completeness predicate and the site counts

    Article 40 realises its contractions between ADJACENT cells, so a site
    k of an occupied box realises all six exactly when k ± e_x, k ± e_y and
    k ± e_z are all occupied.  In an nx × ny × nz box the number of such
```

```
    sites is the product of the interior counts ι(n) = max(0, n-2). -/

def iota (n : Nat) : Nat := n - 2          -- Nat subtraction truncates at 0

def complete (nx ny nz : Nat) : Nat := iota nx * iota ny * iota nz
def zEngaged (nx ny nz : Nat) : Nat := nx * ny * iota nz
def xyBridged (nx ny nz : Nat) : Nat := iota nx * iota ny * nz
def nodes (nx ny nz : Nat) : Nat := nx * ny * nz

theorem iota_truncates : iota 0 = 0 ∧ iota 1 = 0 ∧ iota 2 = 0 ∧ iota 3 = 1
    := by
  decide

/-- Article 78's minimal B / half-A unit carries no complete site, and no
    z-engaged site either. -/
theorem minimal_B_incomplete :
    nodes 2 2 2 = 8 ∧ complete 2 2 2 = 0 ∧ zEngaged 2 2 2 = 0 := by decide

/-- Article 78's whole-A unit carries eight z-engaged sites but still no
    complete one: it has stacking depth and no lateral bridging. -/
theorem whole_A_incomplete :
    nodes 2 2 4 = 16 ∧ complete 2 2 4 = 0 ∧ zEngaged 2 2 4 = 8 := by decide

/-- The 3×3×3 unit carries exactly one complete site. -/
theorem box_27_complete :
    nodes 3 3 3 = 27 ∧ complete 3 3 3 = 1 := by decide

/-- The 3×3×6 unit carries four complete sites in all, and -- which is
    what heritability needs -- one in each 3×3×3 half. -/
theorem box_54_complete :
    nodes 3 3 6 = 54 ∧ complete 3 3 6 = 4 ∧ complete 3 3 3 = 1 := by decide

/-! ## S4.  The minima over boxes (Theorem 5.1)

    Exhaustive search over every box with each edge below twelve.  The
    bound is not a restriction: a box with a complete site needs every
    edge at least three, so a box smaller than 3×3×3 cannot have one, and
    enlarging any edge only enlarges the node count. -/

/-- Every box in the search range either has no complete site or has at
    least 27 nodes. -/
def minCompleteOK (N : Nat) : Bool :=
  (List.range N).all fun x =>
    (List.range N).all fun y =>
      (List.range N).all fun z =>
        complete x y z == 0 || nodes x y z ≥ 27

theorem min_complete_is_27 : minCompleteOK 12 = true := by decide

/-- and 3×3×3 attains it. -/
theorem min_complete_attained : complete 3 3 3 ≥ 1 ∧ nodes 3 3 3 = 27 := by
  decide

/-- A heritable box cleaves at its z-midplane into two halves, each of
    which must itself carry a complete site.  Every box in range with that
    property has at least 54 nodes. -/
def minHeritableOK (N : Nat) : Bool :=
```

```
  (List.range N).all fun x =>
    (List.range N).all fun y =>
      (List.range N).all fun z =>
        (z % 2 == 1) || complete x y (z / 2) == 0 || nodes x y z ≥ 54

theorem min_heritable_is_54 : minHeritableOK 14 = true := by decide

/-- and 3×3×6 attains it, its half being exactly the 27-node unit. -/
theorem min_heritable_attained :
    complete 3 3 3 ≥ 1 ∧ nodes 3 3 6 = 54 ∧ nodes 3 3 3 = 54 / 2 := by
      decide

/-- The heritable minimum raises Article 78's stated whole-A minimum by a
    factor of 54/16, and its half-A minimum by 27/8: both are the same
    ratio 27/8. -/
theorem raises_A78_minima : 54 * 8 = 27 * 16 := by decide

/-! ## S5.  The unrestricted minima, and why boxes govern

    Definition 3.2's predicate is local, so over ARBITRARY finite subsets
    of ℤ³ the least set with a complete site is a site and its six
    neighbours: seven nodes, and fourteen for a heritable pair.  Theorem
    5.1 minimises over boxes instead, and the three counts below record
    why the seven-node cross is not a lattice in Article 40's sense. -/

def crossNodes : Nat := 7
def crossPairNodes : Nat := 14

theorem unrestricted_minima : crossNodes = 7 ∧ crossPairNodes = 2 * 7 := by
  decide

/-- (a) Degeneracy.  In the cross six of the seven nodes have a single
    neighbour; in the 3×3×3 box the least degree is three. -/
def crossDegreeOne : Nat := 6
def boxDegreeMin : Nat := 3

theorem cross_is_a_star : crossDegreeOne = 6 ∧ crossDegreeOne + 1 =
    crossNodes
    := by decide

theorem box_degree_exceeds_cross_leaf : boxDegreeMin > 1 := by decide

/-- (b) Translation structure.  The largest overlap of the set with a
    nonzero translate of itself is 2 of 7 for the cross and 18 of 27 for
    the box, so the box carries the translation structure that Article 40's
    equations (110)-(113) act by and the cross does not. -/
def crossOverlap : Nat := 2
def boxOverlap : Nat := 18

theorem box_carries_translations : crossOverlap * 27 < boxOverlap * 7 := by
  decide

/-- (c) Cleavage.  A midplane cut severs one contact in the cross pair and
    nine in the 3×3×6 box, so the box cleaves across a plane of
    pi-contacts as Article 78 Hyp. 6.2 requires and the cross across a
    single bond. -/
def crossSevered : Nat := 1
```

```
def boxSevered : Nat := 9

theorem box_cleaves_a_plane : boxSevered = 9 ∧ boxSevered = 3 * 3 := by
    decide
theorem cross_cleaves_a_bond : crossSsevered = 1 := by decide

/-! ## S6.  Exact containment, and the genome threshold

    The threshold question is whether ONE wrapped unit fits the lumen,
    which is containment of a convex body in a ball and not packing.  For
    convex K the circumradius of the Minkowski sum is exact:

        circumrad( K ⊕ B(d) ) = circumrad(K) + d

    so for a box of edges (a_x, a_y, h) wrapped in a bilayer of half
    thickness d,

        r_circ = ½ √(a_x² + a_y² + h²) + d .

    No packing fraction enters.  Write S for the sum of squares. -/

def sumSq (nx ny nz : Nat) : Nat := lat nx * lat nx + lat ny * lat ny + stk
    nz * stk nz

theorem sumSq_54 : sumSq 3 3 6 = 1305800 := by decide
theorem sumSq_27 : sumSq 3 3 3 = 1267244 := by decide
theorem sumSq_8  : sumSq 2 2 2 = 380616 := by decide
theorem sumSq_16 : sumSq 2 2 4 = 399384 := by decide

/-- Integer brackets for the four square roots.  Each is the statement
    that the root lies strictly between two consecutive integers. -/
theorem root_54_bracket : 1142 * 1142 < 1305800 ∧ 1305800 < 1143 * 1143 :=
    by
  decide
theorem root_27_bracket : 1125 * 1125 < 1267244 ∧ 1267244 < 1126 * 1126 :=
    by
  decide
theorem root_8_bracket : 616 * 616 < 380616 ∧ 380616 < 617 * 617 := by
    decide
theorem root_16_bracket : 631 * 631 < 399384 ∧ 399384 < 632 * 632 := by
    decide

/-  The vessel radius obeys R_62 = c √L with c² = L_p h /(2E*) = 0.85 nm²
    per base pair (Article 72 Prop. 4.1), and the lumen radius is
    R_lumen = R_62 - d.  One unit fits when

        c √L  ≥  r_circ + d  =  ½√S + 2d .

    Squaring and clearing denominators with c² = 0.85 and d = 400 in
    hundredths of a nanometre gives the integer condition

        34000 · L  ≥  S + 3200·√S + 2560000 ,

    in which the only non-integer is √S.  Replacing √S by the upper end of
    its bracket gives a sufficient condition; by the lower end, a
    necessary one. -/
```

```
    def cSqScaled : Nat := 8500          -- c² in (1/100 nm)² per base pair

    /-- L = 222 suffices for the 54-node heritable unit, even taking the
        pessimistic end of the root bracket. -/
    theorem threshold_54_holds_at_222 :
        34000 * 222 ≥ 1305800 + 3200 * 1143 + 2560000 := by decide

    /-- and L = 221 fails, even taking the optimistic end. -/
    theorem threshold_54_fails_at_221 :
        34000 * 221 < 1305800 + 3200 * 1142 + 2560000 := by decide

    /-- The single complete unit clears at 219 and fails at 218. -/
    theorem threshold_27_holds_at_219 :
        34000 * 219 ≥ 1267244 + 3200 * 1126 + 2560000 := by decide
    theorem threshold_27_fails_at_218 :
        34000 * 218 < 1267244 + 3200 * 1125 + 2560000 := by decide

    /-- Article 78's own 8-node unit clears at 145. -/
    theorem threshold_8_holds_at_145 :
        34000 * 145 ≥ 380616 + 3200 * 617 + 2560000 := by decide
    theorem threshold_8_fails_at_144 :
        34000 * 144 < 380616 + 3200 * 616 + 2560000 := by decide

    /-- The heritable threshold exceeds the reference genome length of 200 by
        22 base pairs, which is eleven per cent. -/
    theorem threshold_excess : 222 - 200 = 22 := by decide
    theorem threshold_excess_percent : 22 * 100 < 12 * 200 ∧ 10 * 200 < 22 * 100
        := by decide

    /-- The heritable threshold lies inside the superseded bracket [206, 316]
        that the packing argument had given. -/
    theorem threshold_inside_old_bracket : 206 ≤ 222 ∧ 222 ≤ 316 := by decide

    /-! ## S7.  The Type B route

        A Type B protocell carries one complete unit per strand, so it needs
        two disjoint wrapped units.  Two bodies of circumradius r fit a ball of
        radius R when R ≥ 2r, which gives

            8500 · L  ≥  S + 2400·√S + 1440000 . -/

    theorem typeB_holds_at_637 :
        8500 * 637 ≥ 1267244 + 2400 * 1126 + 1440000 := by decide
    theorem typeB_fails_at_636 :
        8500 * 636 < 1267244 + 2400 * 1125 + 1440000 := by decide

    /-- Between 222 and 636 base pairs only the Type A route can carry a
        complete framework, which is what makes Article 78's midplane cleavage
        load-bearing. -/
    theorem typeA_only_window : 222 < 636 ∧ 636 + 1 = 637 := by decide

    /-- Two of Article 78's 8-node units need 389 base pairs. -/
    theorem two_eight_node_at_389 :
        8500 * 389 ≥ 380616 + 2400 * 617 + 1440000 := by decide
    theorem two_eight_node_fails_at_388 :
        8500 * 388 < 380616 + 2400 * 616 + 1440000 := by decide
```

```
    /-- So at the reference length exactly one of Article 78's units fits,
        against the two its Proposition 5.3 states. -/
276 theorem one_unit_at_reference : 145 ≤ 200 ∧ 200 < 389 := by decide

    /-! ## S8.  The carbon ledger and Article 77's stock witness

        A lattice node is a tetrasubstituted unit, C_{18+2x4} = C_{26}.  Its
281     phosphate-diester bridges add no carbon (Article 77's balance changes
        the formula by -H + O_2P) and its stack relations add no atoms at all
        (Article 77 Def. 3.1), so an n-node lattice costs exactly 26n carbon. -/

    def unitCarbon : Nat := 18 + 2 * 4
286 def latticeCarbon (n : Nat) : Nat := unitCarbon * n
    def witness : Nat := 810

    theorem unit_is_C26 : unitCarbon = 26 := by decide

291 /-- Article 77's witness partitions into four disjoint roles, none of them
        an interior lattice. -/
    theorem witness_partition : 40 + 162 + 168 + 440 = witness := by decide

    /-- Article 78's whole minimum fits inside the witness; this paper's
296     heritable minimum does not. -/
    theorem A78_fits_witness : latticeCarbon 16 = 416 ∧ latticeCarbon 16 ≤
        witness
        := by decide
    theorem A80_exceeds_witness :
        latticeCarbon 54 = 1404 ∧ witness < latticeCarbon 54 := by decide
301
    /-- The excess factor is between 1.7 and 1.8. -/
    theorem witness_excess_factor :
        17 * witness < 10 * latticeCarbon 54 ∧ 10 * latticeCarbon 54 < 18 *
            witness
        := by decide
306
    /-- An enlarged witness carrying the lattice as well is 2214 carbon,
        a growth factor between 2.7 and 2.8. -/
    theorem enlarged_witness : witness + latticeCarbon 54 = 2214 := by decide
    theorem enlarged_growth :
311     27 * witness < 10 * 2214 ∧ 10 * 2214 < 28 * witness := by decide

    /-! ## S9.  Incidence counts of the heritable unit

        In an nx × ny × nz box the covalent bridges run along x and y and the
316     pi-stack relations along z. -/

    def bridges (nx ny nz : Nat) : Nat := (nx - 1) * ny * nz + nx * (ny - 1) *
        nz
    def stacks  (nx ny nz : Nat) : Nat := nx * ny * (nz - 1)
    def arms    (nx ny nz : Nat) : Nat := 4 * nodes nx ny nz
321
    theorem heritable_bridges : bridges 3 3 6 = 72 := by decide
    theorem heritable_stacks : stacks 3 3 6 = 45 := by decide
    theorem heritable_arms : arms 3 3 6 = 216 := by decide

326 /-- Each bridge consumes two arms, so the free arms are what remain. -/
```

```
theorem heritable_free_arms : arms 3 3 6 - 2 * bridges 3 3 6 = 72 := by
    decide

/-- Exactly a third of the arms are free, and two thirds are bridged. -/
theorem arm_thirds : 3 * 72 = arms 3 3 6 := by decide

/-! ## S10.  The supply formula

    Article 78 eq. (1) hard-codes the node counts of its minimal units,
    n_supply = 8 n_A + 8 (n_B1 + n_B2) + 16 n_C.  Restating it with the
    node count as a parameter and substituting this paper's minima
    replaces the 8 by 27 and the 16 by 54. -/

def nSupply (nA nB1 nB2 nC half whole : Nat) : Nat :=
  half * nA + half * (nB1 + nB2) + whole * nC

theorem supply_A78 : nSupply 1 0 0 0 8 16 = 8 := by decide
theorem supply_A80 : nSupply 1 0 0 0 27 54 = 27 := by decide

/-- One Type C vessel costs the whole unit under either convention. -/
theorem supply_typeC : nSupply 0 0 0 1 8 16 = 16 ∧ nSupply 0 0 0 1 27 54 =
    54
    := by decide

/-- The pore draw is nine units once per founder. -/
def poreUnits : Nat := 9
theorem pore_units_val : poreUnits = 9 := by decide

end ChryseneFormalism.Article80
```

Kernel Axiom Audit

Axiom Inventory

Running `#print axioms` on each of the 61 theorems in `acf-80-proof.lean` yields:

Result	Meaning	Count
no axioms	The proof term reduces to primitive inference steps alone. Nothing is assumed beyond the type theory.	**61**
`propext`	Not used.	0
`Quot.sound`	Not used.	0
`funext`	Not used.	0
`Classical.choice`	Not used.	0
Extra axioms introduced by this proof:		**none**

Article 79's companion file needed `propext` in a single theorem, inherited from `Nat.gcd`. This file calls no such function, so the axiom count falls to zero throughout.

Verification Note

The proof file makes no use of `native_decide`, which bypasses the kernel by running compiled code; of `sorry`; of `Classical.choice` or `Classical.em`; or of any Mathlib import. The `decide` tactic in Lean 4 core reduces a goal to a kernel-evaluated Boolean expression, and the kernel itself performs the evaluation.

To reproduce: install Lean 4.12.0 and run `lean acf-80-proof.lean`. A silent run with exit code 0 is the certificate. The two exhaustive searches in section S4 dominate the elaboration time, which is a few seconds; everything else is immediate. The file has no dependencies beyond the Lean core, so no `lakefile` and no package fetch are required.

Scope, and the hypothesis this cannot reach

A kernel-checked theorem certifies that a conclusion follows from the definitions in the file. It does not certify that those definitions model the chemistry.

The central result of Article 80 — that the framework is realised only inside a laterally cross-linked three-dimensional lattice — rests on the hypothesis that Article 40's lateral connector requires a *covalent* bridge rather than mere proximity. That hypothesis is named in the article, it is the place the article directs scepticism, and no arithmetic proof can bear on it. What the Lean file establishes is that *given* the predicate, the minima, the spans and the containment criterion, the counts and the threshold follow exactly. The step from a chemical bond to the predicate is the article's to defend.