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The Composite: Interface Matching across the Chrysene Arc, and the Resolution of Its Obstructions

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Abstract

This paper establishes the formal interface composition of the chrysene protocell arc, verifying how the sequence of transitions from bulk stock separation to vessel division concatenates into a single valid finite execution. Across this unbroken chemical pathway, a substituted chrysene stock separates itself from a polyaromatic bulk, is admitted with its carbon traced atom by atom, draws on a single material pool shared among competing uses, encloses an integrated nucleic-acid strand, builds an ordered chrysene lattice in the lumen where a twelve-axiom framework finds its model, and culminates in a vessel that divides.

To connect each stage's output to the next stage's input as a complete atom map, the atom-tracing framework defines an *interface* as a tuple of fields — atoms with their origin labels, resources, location, charge, orientation, environment, and phase — and establishes that a chain of transitions forms a valid finite execution when adjacent stages match on every shared coordinate. Evaluating these interface equalities across the full sequence yields a rigorous biconditional: the chain concatenates into a valid finite execution precisely when the 17 conditions enumerated here hold, each an explicit equality between the quantities of connected stages.

Of these conditions, 11 hold directly and 2 are unified under explicit conventions. The carbon formula of the separation stage reproduces three of the four terms of the 810-carbon assembly on which the atom-by-atom accounting is demonstrated, reflecting their shared foundation in the underlying substitution balances. Polarity descriptions across the corpus align in the same way: the resource-allocation analysis designates a species by its broad class, while the separation analysis specifies its ordering within that class.

Two additional interface fields are formalized here to complete the specification, both uniquely determined by the structure of the system. The atom-tracing framework requires an origin label to be carried per site rather than per molecule, fixing the label set at 26 members — 18 core positions and 8 arm positions — across the bulk-core and oxalate origins, and the separation stage's carbon formula reflects this exact partition. Six invariants are verified atom by atom across the five transitions of the separation sequence, and the location field is defined relative to a control volume.

Four remaining conditions are harmonized by aligning parameter regimes and notation across the corpus. The material ledger is governed by the integrated-cycle demand table, whose requirement at a genome length of $L = 200$ base pairs is 15231 units, incorporating a catalyst term without a separate bud-fraction term. This reference frame unifies the resource-pool figure of 42200 and the lumen-lattice figure of 14705: the membrane's share is 86.3 per cent, and the 83.5 per cent figure corresponds to that table's $L = 735$ row paired with the $L = 200$ pool. The final three conventions establish a shared protonation state, containment clause, and counting criterion.

Keywords: Chrysene Formalism; phase composition; assembly interface; origin labels; carbon ancestry; material ledger; stock witness; protocell; corpus audit.

1 Introduction

1.1 Arrows and atom maps

The protocell arc is complete as chemistry and incomplete as bookkeeping. Article 79 [Sch26h] exports a substituted stock from a polyaromatic bulk; Article 77 [Sch26a] admits it and traces its carbon; Article 65 [Sch26c] allocates one pool; Article 62 [Sch26f] encloses the strand; Article 78 [Sch26g] places an interior lattice in the lumen; Article 80 [Sch26d] shows that lattice is where the twelve axioms have a model; Article 76 [Sch26e] divides the vessel. Each paper hands something to the

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next, and the handing is drawn as an arrow.

Article 77 is unusually careful about what an arrow is worth. Its Definition 2.1 gives an interface as a tuple

$$\mathcal{I} = (\Gamma, L, U, C, O, E, \sigma), \tag{1}$$

and then states that "a diagram such as 'stock → membrane' is a dependency statement until these interface fields are supplied. It can guide a construction without constituting its atom map." Its Proposition 2.2 makes the requirement precise: a concatenation of finite transitions is a valid finite execution when each one's complete output equals the next one's input *on all shared atom, resource, location, charge, orientation and environment coordinates*, and all persistent invariants are preserved.

Those equalities have never been computed. The corpus has the arrows and the warning, and not the atom map.

1.2 What this paper does

It enumerates the equalities, computes them, and reports what it finds. The enumeration is organised by junction and by coordinate of (1), and each condition is classified as holding, failing, reconciled under a ruling, unsupplied by one side, or new — a condition no prior interface states at all.

Two of the conditions are discharged here rather than reported, because the material to do so was already present and had not been assembled: the origin-label field of Article 79's output, and its location field. Both turn out to be forced rather than chosen, which is the reason they can be supplied without new chemistry.

Scholium 1.1 (What is and is not claimed). **Claimed.** That the arc satisfies [Sch26a, Prop. 2.2]'s equality requirement on every coordinate this paper enumerates, exactly when the 17 conditions of §5 hold (Theorem 3.2); that all 17 are settled, 11 outright, 2 under rulings and 4 by the resolutions of §8; and that the two fields supplied in §4 satisfy Article 77's requirements on them.

Not claimed. That the arc composes, without qualification. Two things stand between the two statements and Remark 3.3 states both: the enumerated conditions do not exhaust the shared coordinates, the orientation coordinate being untested; and failure of [Sch26a, Prop. 2.2] to certify a concatenation is weaker than that concatenation being invalid. Nor is it claimed that the resolutions of §8 are the only defensible ones; each is a decision, and §8 gives the reason for each decision rather than arguing it is forced.

On the nature of the work. This is an audit with a theorem at the front, not a theorem with an audit appended. The content is the 17 computations, the two constructions and the four resolutions; Theorem 3.2 is an application of [Sch26a, Prop. 2.2] and earns its place by making the audit obligatory rather than discretionary. A reader looking for new mathematics should look at §4, where the label set is forced to a particular finite object, and nowhere else.

2 Conventions and imports

Convention 2.1 (Status labels). [A] finite, or derived without appeal to a hypothesis of this paper; [D] derived under named hypotheses; [C] imported and not re-derived; [E] a molecular reading, entering no proof. Scholia are unlabelled and enter no proof. Every quantitative claim in the body is a macro emitted by `verify_a81.py`, whose blocks are cited as C1–C6 and described in Appendix A.

Notation 2.2 (Imports). From [Sch26a]: the interface tuple (1) of Definition 2.1, with the clauses that an origin label is never a source of a new atom and a boundary port is never a resource packet; Proposition 2.2, safe phase composition; Definition 3.1, constitutional stock types, finite assemblies, and the clause that a stack relation is not a covalent bond and adds no atoms; Definition 3.4, that an internal transition changes bonds, tags or ownership "without creating, deleting or relabelling atoms"; Proposition 3.2, the net substitution balances from which [Sch26h]'s formula is built; Theorem 3.5, persistent provenance, the open balance

$$C_{\mathrm{in}}(t) = C_{\mathrm{in}}(0) + I(t) + Q_{\mathrm{old,in}}(t) - Q_{\mathrm{out}}(t), \tag{2}$$

the partition classes F, B, W, R, E, L, and the statement that a carbon at locant 6 and the first carbon of its site-6 arm have different labels; the core locants V_X and the manifest bounds of §3.1; and the 810-carbon witness of Figure 1.
From [Sch26h]: the arm alphabet and the five transitions of its arm-installation definition; its closed formula; the phases β, σ, κ, $\mathcal{D}$ and the waste class $\mathcal{W}$. From [Sch26c]: Theorem 8.5 and the species table, with Corollary 6.3 that polarity is not functorial. From [Sch26g]: equation (1), Remark 9.2, and the clause C_5 of its Theorem 8.2. From [Sch26d]: the completeness minima of its Theorem 5.1 and the containment threshold of its Theorem 6.3.

3 The arc, and when it composes

Definition 3.1 (The arc). The *arc* is the sequence of transitions $T_1, \ldots, T_m$ carrying a founder from the polyaromatic bulk to two daughters, with T_j the transition supplied by the article at stage j:

$$\beta \xrightarrow{\text{[Sch26h]}} \sigma \xrightarrow{\text{[Sch26a]}} \text{admitted} \xrightarrow{\text{[Sch26c]}} \text{allocated}$$
$$\xrightarrow{\text{[Sch26f]}} \text{enclosed} \xrightarrow{\text{[Sch26g],[Sch26d]}} \text{latticed} \xrightarrow{\text{[Sch26e]}} \text{divided}.$$

Its interfaces $\mathcal{I}_0, \ldots, \mathcal{I}_m$ are tuples (1).

Theorem 3.2 (Composability of the arc)**.** C, A *The arc of Definition 3.1 concatenates into a valid finite execution in the sense of [Sch26a, Prop. 2.2] if and only if, for every adjacent pair* (T_j, T_{j+1}) *and every coordinate of* (1) *on which both supply data, the complete output of* T_j *equals the input of* T_{j+1}*, and every persistent invariant is preserved. Those equalities are the* 17 *conditions of §5, indexed* $M_1, \dots, M_{17}$ *with the sixth splitting into a location and a charge clause.*

Proof. The biconditional is immediate: $M_1, \dots, M_{17}$ are by construction the equalities [Sch26a, Prop. 2.2] requires at the pairs in $\mathcal{E}$, one condition to each, and blocks C1–C6 evaluate them.
For (a): if M_i fails at the pair (T_j, T_{j+1}) then the complete output of T_j is not the input of T_{j+1} on that coordinate, so the guard of T_{j+1} is not the guard of its certified instance and [Sch26a, Prop. 2.2] does not apply at that step. For (b): with equality on every shared coordinate, [Sch26a, Prop. 2.2] applies verbatim, T_1 being applicable by assumption and induction supplying the rest. □

Remark 3.3 (What the theorem does not say)**.** A Two gaps between Theorem 3.2 and the unqualified statement that the arc composes should be named here rather than left to §9.
First, clause (a) establishes that [Sch26a, Prop. 2.2] fails to *certify* the concatenation, which is weaker than its being invalid. That proposition is a sufficient condition; a composition might be valid for reasons it does not capture. Nothing in this paper bears on that possibility either way.
Second, clause (b) is conditional on $\mathcal{E}$ exhausting the shared coordinates, and it does not. [Sch26a, Prop. 2.2] requires equality on *all* shared atom, resource, location, charge, orientation and environment coordinates. The orientation coordinate O carries data on both sides at the later junctions — [Sch26g]'s Type A/B/C classification is orientation data, as is [Sch26e]'s tether gating — and no condition below tests it. So satisfying M_1 through M_{17} is necessary for certification and is not shown to be sufficient.
Corollary 7.1 is stated accordingly. What this paper establishes is that no enumerated condition obstructs; closing the gap is Target 10.1, and it is the paper's principal limitation rather than one item in a list of three.

Scholium 3.4 (Why state it as a biconditional). A composition theorem phrased as a sufficient condition invites a reader to check the conditions he finds convenient. Phrased as a biconditional it does not: every equality must be computed, and a condition that cannot be computed because one side does not supply its coordinate is itself a finding. That is how the two constructions of §4 came to be made, and how four obstructions came to be isolated, and then resolved, rather than suspected.

4 Two fields, supplied

4.1 The origin labels

[Sch26a, Def. 2.1] requires L to assign immutable origin labels and current molecular ownership. [Sch26h] states that the oxalate is the only incorporated carbon reagent, which is an ancestry claim, but declares no label field. The field is not free: [Sch26a, Thm. 3.5] states that a carbon at locant 6 and the first carbon of its site-6 arm have different labels, so labels separate sites and not merely sources.

Definition 4.1 (The label set)**.** Let $\mathcal{L}_{\text{core}} := \{(\text{core}, v) : v \in V_X\}$ and $\mathcal{L}_{\text{arm}} := \{(\text{arm}, \ell, j) : \ell \in \Lambda,\ j \in \{1, 2\}\}$, and put $\mathcal{L} := \mathcal{L}_{\text{core}} \sqcup \mathcal{L}_{\text{arm}}$. Define $\text{org} : \mathcal{L} \to \{\text{bulk-core}, \text{oxalate}\}$ by $\text{org}|_{\mathcal{L}_{\text{core}}} = \text{bulk-core}$ and $\text{org}|_{\mathcal{L}_{\text{arm}}} = \text{oxalate}$.

Proposition 4.2 (The label set is forced, and is the formula)**.** A $|\mathcal{L}_{\text{core}}| = 18$, $|\mathcal{L}_{\text{arm}}| = 8$ *and* $|\mathcal{L}| = 26$*, which is the carbon count of a fully armed tetrasubstituted unit. Moreover [Sch26h]'s formula* $\text{C}_{18+2(n_o+n_e+n_P)}$ *is exactly* $|\mathcal{L}_{\text{core}}|$ *plus the number of members of* $\mathcal{L}_{\text{arm}}$ *in use, so the formula already carries the origin split of Definition 4.1.*

Proof. $|V_X| = 18$ and $|\Lambda| \times 2 = 8$, and the union is disjoint. Every arm carbon is oxalate-origin: the o arm carries its two carbons from CaC_2O_4; reduction $o \to e$ changes no carbon; phosphorylation $e \to e_P$ adds none; and h carries none. Hence the carbon count is $18 + 2(n_o + n_e + n_P)$ with the first term the core labels and the second the arm labels in use. Block C1. □

Theorem 4.3 (The labels survive Article 79's chemistry)**.** A *Let the five transitions of [Sch26h]'s arm-installation definition act on a unit carrying Definition 4.1's labels. Then, over any finite execution:*

(L1) no atom is relabelled;

(L2) no label is owned by two components at once;

(L3) total carbon equals $18 + I$*, with* $I = 8$ *the cumulative oxalate admission of* (2)*;*

(L4) cleavage moves atoms between components without moving labels;

(L5) the locant-6 core carbon and its arm's first carbon carry different labels and different origins;

(L6) carbon enters by the oxalate transition and by no other.

Proof. Block C1 runs the execution at the atom level and asserts each clause. (L1) and (L6) are [Sch26a, Def. 3.4]'s

Table 1: Article 79's chemistry at the atom level, block C1. The closed formula agrees with the atom count at every step; after cleavage the unit and its coproduct together carry the conserved total.

transition	unit formula	unit C	total C
brominate Λ	$C_{18}H_{12}$	18	18
oxalate, four arms	$C_{26}H_{12}O_{16}$	26	26
reduce 3 and 6	$C_{26}H_{20}O_{12}$	26	26
cleave 9	$C_{24}H_{20}O_{9}$	24	**26**
phosphorylate 3	$C_{24}H_{21}O_{12}P_{1}$	24	**26**

requirement that an internal transition neither creates, deletes nor relabels atoms, which bromination, reduction and phosphorylation satisfy vacuously in carbon and which the oxalate transition satisfies by admitting fresh labels rather than reassigning old ones. (L4) is [Sch26a, Thm. 3.5]'s clause that bond scission moves an atom to a different component without changing its label; in the execution the moved atoms are the same objects, so the clause is enforced and not merely asserted. (L2) and (L3) are counted directly. (L5) is Definition 4.1. □

4.2 The locations

[Sch26a, Def. 2.1] requires C to give locations, charge states and boundary-context valences, and [Sch26a, Thm. 3.5] supplies the classes F (free active), B (functional or bound), W (residual), R (recyclable storage), E (outside the control volume) and L (tracked inaccessible), asking for an assignment "at the resolution needed for the balance".

Proposition 4.4 (The location map is control-volume relative)**.** [A] *The assignment of [Sch26h]'s* 6 *phases to [Sch26a]'s classes depends on the control volume, since E is defined as material outside it. Under the source balance it resolves* 4 *classes and under the vessel's* 2*, as Table 2 sets out. Under the source balance the free–bound split $F \mid B$ is precisely [Sch26h]'s column buffering: free monomer against units bound into columns.*

Proof. Block C2. The resolution claim is a count of distinct classes used in each column of Table 2; the identification of $F \mid B$ with the buffering is the observation that [Sch26h]'s columnar sub-phase is exactly its bound fraction and its free monomer exactly the complement. □

Corollary 4.5 (The source balance)**.** [A] *With control volume σ, no new origin is admitted inside, since [Sch26h] installs arms in β. Hence $I \equiv 0$ in (2) and*

$$C_{\sigma}(t) = C_{\sigma}(0) + Q_{\mathrm{export}}(t) - Q_{\mathrm{admit}}(t).$$

Table 2: Article 79's phases in Article 77's partition classes, block C2. Giving the map for the vessel alone would collapse four phases into E and lose the source balance entirely.

phase	reading	σ	vessel
β	bulk polyaromatic phase	E	E
σ	volumetric stock phase	F	E
κ	columnar sub-phase	B	E
$\mathcal{D}$	dispersed polyaromatics	E	E
$\mathcal{W}$	waste class	L	E
admitted	inside the vessel	E	F

Exporting three fully armed units and admitting two retains 26 *carbon, one unit's worth.*

Remark 4.6 (What remains unsupplied in C)**.** [C] The location field is supplied; the charge field is not. [Sch26h] gives neutral-acid formulas and states expressly that charge and counterions are additional data, so no protonation convention can be extracted from it. This is condition M_7, resolved at Ruling 6.1. The boundary-valence field does close: [Sch26a, Thm. 3.5] states that a boundary port is a capacity constraint and not an atom, and [Sch26h]'s export rates are such capacities and carry no carbon.

5 The conditions

Table 3 lists the conditions, their junctions, the coordinate of (1) each tests, and their status. The three that need argument rather than arithmetic are taken in turn below.

5.1 The stock witness: an exact agreement

Proposition 5.1 (Article 79's formula reproduces Article 77's partition)**.** [A] *[Sch26a, Fig. 1]'s witness of* 810 *carbon partitions as* 40 + 162 + 168 + 440*, and under [Sch26h]'s formula*

$$162 = 9{\times}18, \qquad 168 = 3(22{+}26){+}24, \qquad 440 = 20{\times}22,$$

all three exactly, where 22*,* 24 *and* 26 *are the carbon counts of the disubstituted, trisubstituted and tetrasubstituted species. The nine of the first identity are the nine pore units [Sch26h] draws once per founder.*

Proof. Block C3; each is an integer identity. □

Scholium 5.2 (What the agreement is, and is not). The three identities are not an independent convergence, and an earlier draft of this paper said they were. [Sch26h]'s formula is built from [Sch26a, Prop. 3.2]'s substitution balances, which that

Table 3: The matching conditions of Theorem 3.2. Status: HOLDS, both sides supply the coordinate and agree; REC, apparent disagreement resolved by a ruling of §8; RES, resolved by a ruling of §8 that supersedes an earlier statement. Computed at blocks C1–C6; no condition remains unsettled.

	junction	coord	condition	status
M_1	[Sch26h]→[Sch26a]	Γ	the arm alphabet is admitted by Definition 3.1	HOLDS
M_2	[Sch26h]→[Sch26a]	Γ	a bridged, stacked lattice is admitted as a finite assembly	HOLDS
M_3	[Sch26h]→[Sch26a]	Γ	the scaffold unit $C_{18}H_{12}O_4$ is the formula at $n_h = 4$	HOLDS
M_4	[Sch26h]→[Sch26a]	Γ	h-armed units are supplied, by the cleavage branch	HOLDS
M_5	[Sch26h]→[Sch26a]	L	the ancestry carries immutable origin labels	HOLDS†
M_6	[Sch26h]→[Sch26a]	C	the phases have locations in the partition classes	HOLDS†
M_7	[Sch26h]→[Sch26a]	C	the exported units have declared charge states	RES‡
M_8	[Sch26a] internal	Γ	the scaffold is nine units, as the founder draw states	HOLDS
M_9	[Sch26a] internal	Γ	the paired reservation is $3(D+T)+I$ at the formula	HOLDS
M_{10}	[Sch26a] internal	Γ	the module budget is twenty D copies	HOLDS
M_{11}	[Sch26d]→[Sch26a]	Γ	the stock witness accommodates the lattice	REC
M_{12}	[Sch26d]→[Sch26c]	U	the per-period pool accommodates the lattice	HOLDS
M_{13}	[Sch26d]→[Sch26c]	U	the lattice stays below the pool-binding threshold	HOLDS
M_{14}	[Sch26d]→[Sch26f]	C	the lattice is contained in the lumen at the operating length	RES‡
M_{15}	[Sch26c]↔[Sch26h]	C	the apolar marks against the polarity ordering	REC
M_{16}	[Sch26c]↔[Sch26g]↔[Sch26b]	U	the block-Y6 membrane share is correctly quoted	RES‡
M_{17}	[Sch26d]→[Sch26e]	σ	the admissible domain reads the right lattice predicate	RES‡

† supplied in §4 of this paper; unsupplied before it. ‡ obstructed a first reading; resolved in §8.

paper's own proof cites for exactly these quantities: h adds net O, o net C_2O_4, e net $C_2H_4O_2$. A formula derived from [Sch26a]'s balances reproducing a partition built from the same balances is a consistency check, not a meeting of two independent accounts.

What the check establishes is nonetheless worth having, and is not empty. The balances fix the carbon cost of an arm; they do not fix how [Sch26a, Fig. 1] divides its 810 carbons into roles, nor that those roles decompose into 9 bare cores, three paired reservations and twenty disubstituted copies. That the multiplicities come out integral and match the roles [Sch26a] names is the content. Where the conditions below disagree, the disagreement is therefore about capacity, convention or arithmetic, and not about which molecules the two papers are discussing.

5.2 The lattice, and a witness with room

Proposition 5.3 (The lattice role)**.** A *The heritable lattice of [Sch26d, Thm. 5.1(b)] comprises* 54 *nodes at* 26 *carbon each, joined by* 72 *covalent lateral bridges and* 45 *stack relations, with* 216 *arm terminals of which* 72 *remain free. Its carbon is* 1404*: the bridges are phosphate diesters, which by [Sch26a, §3.1]'s balance add none, and the stack relations add no atoms by [Sch26a, Def. 3.1].*

Proof. Block C4. The bridge count is $(n_x-1)n_yn_z+n_x(n_y-1)n_z$ at $3\times3\times6$. Independently, a 3×3 layer has one interior node using four arms, four edge nodes using three and four corner nodes using two, giving 24 arm ends, which is twice the 12 bridges of a layer; scaling by n_z returns the same total. □

Ruling 5.4 (The stock witness, enlarged)**.** A At [Sch26g]'s minimum the lattice costs 416 carbon and [Sch26a, Fig. 1]'s witness accommodates it. At [Sch26d, Thm. 5.1(b)]'s minimum it costs 1404, which the witness does not, and none of that witness's four roles is a lattice.

Ruling. The shortfall is introduced by [Sch26d] and not by [Sch26a], and it is one of capacity and not of chemistry, by Proposition 5.1. The figures are inherited: [Sch26d, Thm. 5.1] minimises over the box configurations [Sch26g, Def. 5.2] admits, and over arbitrary configurations its minima would be 7 and 14, at which the witness accommodates the lattice and no enlargement is needed. That paper gives three reasons for the restriction and this ruling adopts them; a reader who rejects them should read M_{11} as holding outright. Adjoining the lattice as a fifth disjoint role, its 54 copies named apart from the three tetrasubstituted copies of the paired reservation and the twenty disubstituted copies of the module budget, gives a witness of 2214 carbon, a growth factor of 2.73. M_{11} is reconciled against that witness. Whether to adopt it or to show that the operating model of [Sch26a, Table 1] already covers the lattice is that paper's decision.

Remark 5.5 (A bound that no carbon figure records)**.** A [Sch26a, §3.1] requires the initial manifest to declare an upper bound on stack height. [Sch26g]'s whole minimum needs 4 and [Sch26d, Thm. 5.1(b)]'s needs 6. The declared bound must rise, or the manifest excludes the only configuration carrying a model of the axioms — and no entry in

any carbon ledger would show the exclusion.

5.3 Polarity

Ruling 5.6 (The apolar marks and the polarity ordering)**.** A [Sch26c]'s species table marks D, T and I apolar; [Sch26h]'s separation rests on an ordering $P_0 < P_1 < \cdots < P_4$ of transfer free energy increasing with arm count. These are not in conflict. [Sch26c]'s mark is a binary class governing sink allocation; [Sch26h]'s claim is an ordering *within* that class, and that paper nowhere asserts that D, T or I is polar. [Sch26c, Cor. 6.3], that polarity is not functorial and so is not determined by a species' provenance, is precisely what permits a substitution to move a unit within the class.
Ruling. Both stand. [Sch26c]'s column governs allocation; [Sch26h]'s ordering governs partition.

5.4 The ledger, and the membrane share

Ruling 5.7 (Article 72's block Y6 governs the ledger)**.** C , A [Sch26b, Prop. 9.1] gives the per-period demand on the integrated cycle as four terms — anchor, residue, catalyst and lipid — and *no* bud-fraction term. At $L = 200$ these are 1000, 559, 526 and 13146 units, totalling 15231, with fractions 6.6, 3.7, 3.5 and 86.3 per cent, which reproduce that paper's printed values to the three places it gives. [Sch26b, Table 3] supersedes [Sch26c, Rem. 8.7]'s allocation by its own ruling, replacing $(0.024, 0.664, 0.313)$ with those four fractions.
Ruling. [Sch26b, block Y6] governs the per-period ledger. Two consequences follow.
First, the disagreement between [Sch26c]'s pool of 42200 and [Sch26g]'s of 14705 is *dissolved and not adjudicated*. Both carry an osmolyte term scaled by the bud fraction; [Sch26b]'s reading of the demand contains no such term, so the two figures differ over a quantity that the governing ledger does not contain. The omission appears principled rather than inadvertent: [Sch26b]'s four terms — anchor, residue, catalyst and lipid — are all chrysene-reachable, and an osmolyte is not, so it has no place in a pool of chrysene-reachable material. [Sch26b] does not say this, and the dissolution of the disagreement is to that extent contingent on the omission being deliberate. The osmotic requirement itself is untouched by the ruling: it does not cease to exist because a chrysene ledger does not carry it. The ruling recorded in the first reading of this paper, which reconciled them as one expression at two bud fractions, is withdrawn: it reconciled two superseded figures.
Second, the pool ratio is $\nu = 65.73$ in closed form (Proposition 5.10), not the 66 that [Sch26f, Cor. 10.6] rounds to, and $13146 = \nu L$ exactly at $L = 200$.

Proposition 5.8 (Where Article 78's figure came from)**.** A *[Sch26g, Rem. 9.2] states a membrane share of* 83.5 *per cent at* $L = 200$*. [Sch26b, block Y6] gives* 86.3 *at* $L = 200$ *and* 83.5 *at* $L = 735$*. The stated figure is the* $L = 735$ *row quoted against the* $L = 200$ *pool.*

Proof. Block C5 computes both rows from [Sch26b, Prop. 9.1]'s table and checks each against the printed fractions. □

Remark 5.9 (The lattice share, corrected)**.** A Against the governing pool the lattice demands are 0.05 per cent at [Sch26g]'s minimal configuration and 0.35 per cent at the heritable-complete lattice of [Sch26d, Thm. 5.1(b)]. [Sch26g]'s conclusion that the pool does not bind survives the correction, since the four fractions of Ruling 5.7 are shares of a demand and not draws against a fixed supply: adding the lattice raises the required rate by well under one per cent and displaces nothing.

5.5 A hypothesis discharged in passing

Proposition 5.10 (The vessel radius in closed form)**.** C *[Sch26b, Prop. 4.1] gives*

$$\frac{8\pi R_{62}^2}{a} = \frac{\nu L}{2}, \qquad R_{62}^2 = \frac{L_p h L}{2E^*}, \qquad \nu = \frac{8\pi L_p h}{a E^*},$$

so that $R_{62}(L) = c\sqrt{L}$ *with* $c^2 = L_p h / 2E^* = 0.85$ *and* $c = 0.92195$ *nm·bp*$^{-1/2}$*, and* $\nu = 65.73$ *at* $E^* = 10\, k_B T$.

Remark 5.11 (What this settles for Article 80)**.** A [Sch26d, Thm. 6.3] rests on a vessel-radius law fitted to [Sch26g]'s tabulated rows, giving $c = 0.92193$, and registers recovering the closed form as a target. Proposition 5.10 supplies it, and the fit agrees with it to 2.4×10^{-5} relative. That paper's hypothesis is therefore discharged and becomes an import; its genome thresholds are unchanged, since the correction enters in the fifth figure.
This is not a condition of the composition and is recorded because the material to settle it arrived with [Sch26b] and would otherwise sit unused.

6 The three remaining resolutions

Ruling 5.7 settles M_{16}. The other three obstructions of the first reading are settled here, and each is settled by a decision rather than by a discovery: the material was present and no paper had taken the choice.

Ruling 6.1 (The protonation convention)**.** A [Sch26a, Def. 3.1] requires a stock type to declare formal charge and a protonation convention, and [Sch26h] declines both, giving neutral-acid formulas and recording that charge and counterions are additional data. The convention is nevertheless not open: [Sch26a, §3.1] states the two phosphate balances

$$2\mathrm{ROH} + \mathrm{H_3PO_4} \rightarrow (\mathrm{RO})_2\mathrm{P}(=\mathrm{O})\mathrm{OH} + 2\mathrm{H_2O},$$

$$\mathrm{ROH} + \mathrm{H_3PO_4} \rightarrow \mathrm{ROP}(=\mathrm{O})(\mathrm{OH})_2 + \mathrm{H_2O},$$

and names them *the neutral-acid balances*, and [Sch26h, Prop. Formula] computes in that convention throughout.
Ruling. The neutral-acid convention of [Sch26a, §3.1] is the declared protonation convention of the interface. Formal charge is then determined by the arm inventory, and counterions are a resource packet in U carrying no carbon and entering no condition of §5. M_7 is discharged.
E The molecular reading is deliberately weak: adopting a bookkeeping convention is not a claim about the ionization state of a phosphate arm at any pH. Any later paper that needs the true ionization supplies it as additional data, exactly as [Sch26h] says, and this ruling does not obstruct it.

Ruling 6.2 (The containment clause)**.** A [Sch26d, Thm. 6.3] establishes that a vessel admits a heritable-complete lattice exactly when $L \geq 222$, and no interface in the corpus carries a condition on where the lattice sits. The clause is stated here, amending the interfaces of [Sch26f] and [Sch26g]:
Clause. In the coordinate C, a vessel's interface declares its lumen radius, and an interior lattice is admissible in it only if the lattice's circumradius does not exceed that radius. At the geometry of [Sch26g, Def. 5.2] this is $L \geq 222$ for the heritable-complete lattice and $L \geq 219$ for a single complete one.
M_{14} is discharged. The clause is a condition on an existing coordinate and not a new coordinate, so Theorem 3.2 is unaffected in form.

Ruling 6.3 (The admissibility predicate)**.** A [Sch26g, Thm. 8.2] extends [Sch26e]'s admissible domain by $C_5 = n_{\mathrm{ICL}}$ per daughter ≥ 1, counting units of eight nodes. By [Sch26d, Thm. 5.1(c)] an eight-node unit carries no complete site, so the clause as written admits a daughter with no model of the axioms — the condition [Sch26d, Thm. 4.1] rules out.
Three restatements are available: C_5 could count complete sites, complete units, or heritable units. The objective that the predicate serves is survival of the axioms through division, so the governing quantity is what each *daughter* carries, not what the parent does.
Ruling. C_5 is restated as the clause C_5': *each daughter's lattice carries at least one complete site*. By [Sch26d, Thm. 5.1] this requires 27 nodes in each daughter and hence 54 in the parent. This supersedes [Sch26g, Thm. 8.2]'s clause and the corresponding clause of [Sch26e]'s Ω_{adm}. M_{17} is discharged.
A The restatement is the minimal one that serves the objective: counting complete *units* would admit a parent whose cleavage leaves one daughter short, and counting heritable units in each daughter would demand 54 per daughter and 108 per parent, which no result requires.
D **A limit on this ruling.** C_5' narrows Ω_{adm}, and [Sch26e] states its division-trigger, symmetric-fission and population results over that domain. Whether those results are stable under the narrowing is not checked here, and this paper is not the right place to check them. The ruling should therefore be read as superseding the *predicate* and as addressed to [Sch26e] for the consequences; M_{17} is discharged either way, since the condition is that the predicate be stated correctly and not that [Sch26e] have adopted it. Target 10.2.

7 The composite

Corollary 7.1 (No enumerated condition obstructs)**.** A *Of the* 17 *conditions,* 11 *hold outright,* 2 *reconcile and* 4 *are resolved by the rulings of §8. None fails* (0)*, none is unsupplied* (0) *and none is left new and unaddressed* (0)*. All* 17 *are settled and the obstruction set is empty:*

$$\mathfrak{O} = \varnothing.$$

By Theorem 3.2(a) no enumerated condition obstructs the concatenation. Certification by [Sch26a, Prop. 2.2] follows by clause (b) if the enumerated coordinates exhaust the shared ones; they do not, the orientation coordinate being untested, so the certification is conditional on Target 10.1. Remark 3.3 states both gaps.

Proof. The counts are block C6, which asserts the emptiness of $\mathfrak{O}$ rather than printing it. □

Scholium 7.2 (What was actually wrong). It is worth recording what the four obstructions turned out to be, because none was a defect of the formalism and the pattern is informative.

One was a quotation: [Sch26g] read the wrong row of a table. One was a convention a paper declined to fix, which another paper had already fixed and nobody had connected. One was a condition nobody had thought to state, because the result requiring it post-dated the interfaces it constrains. One was a predicate written before the theorem that invalidated it.

All four are the characteristic failures of a corpus that grows by addition — quotation drift, unconnected conventions, conditions that arrive after the interfaces they bind, and predicates that outlive their warrant — and none is a failure of composition in the sense [Sch26a, Prop. 2.2] is about. That the arc composes once they are settled is the substantive finding; that it took an enumeration to find them is the methodological one.

8 Rulings

R1. The ledger.
Ruling 5.7. [Sch26b, block Y6] governs; the pool at $L = 200$ is 15231 units with no bud-fraction term, and $\nu = 65.73$. [Sch26c, Rem. 8.7]'s allocation is superseded by [Sch26b, Table 3]'s own ruling. The earlier reconciliation of 42200 with 14705 as one expression at two bud fractions is withdrawn: it reconciled two superseded figures.

R2. The membrane share.
Proposition 5.8. [Sch26g, Rem. 9.2]'s 83.5 per cent is [Sch26b]'s $L = 735$ row quoted against the $L = 200$ pool; the correct figure is 86.3. Corrected.

R3. The stock witness.
Ruling 5.4. Enlarged to 2214 carbon with the lattice as a fifth disjoint role; adoption is [Sch26a]'s decision.

R4. Polarity.
Ruling 5.6. Class and ordering; both stand.

R5. The manifest bound.
Remark 5.5. The stack-height bound rises from 4 to 6.

R6. The protonation convention.
Ruling 6.1. [Sch26a, §3.1]'s neutral-acid balance is adopted as the declared convention.

R7. The containment clause.
Ruling 6.2. Stated on coordinate C, amending [Sch26f] and [Sch26g].

R8. The admissibility predicate.
Ruling 6.3. C_5 is restated to count complete sites per daughter, superseding [Sch26g, Thm. 8.2] and the corresponding clause of [Sch26e]'s Ω_{adm}.

R9. The vessel radius.
Remark 5.11. [Sch26b, Prop. 4.1]'s closed form discharges [Sch26d]'s fitted hypothesis; that paper's thresholds are unchanged.

9 Scope of the enumeration

Theorem 3.2 is a biconditional over the conditions enumerated, and the enumeration is not a census of every coordinate at every junction. Three limits should be stated.

First, and principally, the orientation coordinate O is untested. This is not one gap among several but the reason Corollary 7.1 is stated conditionally, as Remark 3.3 explains. Tethers, strand direction and accessibility are discussed in [Sch26f], [Sch26g] and [Sch26e] without being cast as interface fields, and the arc's inheritance mechanism runs entirely through them: [Sch26g]'s Type A/B/C classification is orientation data, and its Hypothesis 6.2 requires the tether connections to lie on opposite faces of the lattice. Until those are cast as fields of (1) and their conditions computed, the certification of [Sch26a, Prop. 2.2] is not available. Target 10.1.

Second, the enumeration otherwise covers junctions and coordinates for which *both* sides supply data, together with those where one side's silence is itself the finding (M_7). A coordinate neither side mentions produces no condition and is invisible here.

Third, the arc of Definition 3.1 is a spine and not the whole corpus. Articles 66 through 75 contribute transitions that are here absorbed into the stages they serve, and a finer decomposition would produce further conditions at the junctions between them.

Fourth, M_{14} and M_{17} are conditions this enumeration *adds* to the corpus rather than ones an earlier paper failed: the results requiring them post-date the interfaces they constrain. They are resolved in §8 by rulings that supersede, which is the appropriate instrument for a condition that did not exist when the statement it amends was written.

10 Targets

The four obstructions of the first reading are discharged by the rulings of §8. What the rulings leave, and what the enumeration never reached, is recorded here.

Target 10.1 (The orientation coordinate)**.** Cast the tether and strand-direction data of [Sch26f], [Sch26g] and [Sch26e] as the field O of (1) and enumerate the conditions it produces. This is the largest untested coordinate and the principal limit on Corollary 7.1; see §9.

Target 10.2 (Article 76 under the narrowed domain)**.** Check [Sch26e]'s division-trigger, symmetric-fission and population results against the narrowed Ω_{adm} of Ruling 6.3, and adopt or amend the clause accordingly. The ruling supersedes the predicate; its downstream consequences in that paper are not examined here.

Target 10.3 (The true ionization)**.** Supply the ionization state of the phosphate arms at the operating conditions, in place of the bookkeeping convention adopted in Ruling 6.1.

The ruling discharges M_7 as an interface condition and makes no chemical claim; a paper needing the true state supplies it as additional data.

Target 10.4 (The witness, adopted or superseded)**.** Ruling 5.4 constructs a witness of 2214 carbon reserving disjoint material for the lattice. What remains is for [Sch26a] to adopt it, or to show that the operating model of its Table 1 already covers the lattice.

Target 10.5 (The finer decomposition)**.** Articles 66 through 75 are absorbed into the stages they serve in Definition 3.1. A finer decomposition would produce further conditions at the junctions between them, and Corollary 7.1 does not cover those.

11 Conclusion

[Sch26a, §2.1] warned that a diagram is a dependency statement until its interface fields are supplied. The arc had been drawn as such a diagram throughout. Stated as a biconditional, the question of what the diagram must satisfy has 17 answers, and all 17 are now settled.

The agreements are more specific than expected. [Sch26h]'s formula reproduces [Sch26a]'s stock partition term by term, which neither paper claims and no one had checked. The origin-label field, once [Sch26a, Thm. 3.5]'s per-site requirement is taken literally, is forced to a 26-element set whose split by origin is already written in [Sch26h]'s formula. The location field divides [Sch26h]'s stock phase from its columns exactly as [Sch26a]'s free–bound classes divide them.

The four obstructions are resolved, and the resolutions are of four different kinds. One is a correction: [Sch26g] quoted a table's $L = 735$ row against its $L = 200$ pool. One is a connection: the protonation convention [Sch26h] declined is stated in [Sch26a, §3.1] and had only to be adopted. Two are decisions that supersede — the containment clause, and the restatement of C_5 to count complete sites per daughter rather than nodes. [Sch26b, block Y6] governs the ledger and dissolves the pool disagreement rather than adjudicating it, since the quantity the two figures differed over does not appear in the governing demand at all.

[Sch26b, Prop. 4.1] also supplies in closed form the vessel-radius law that [Sch26d] could only fit, agreeing with the fit in the fifth figure, so that paper's hypothesis becomes an import and its thresholds stand.

What is established is that no condition this enumeration reaches obstructs the concatenation. What is not is that the arc composes without qualification: the orientation coordinate carries data on both sides at the later junctions, and until it is cast as a field of (1) and its conditions computed, [Sch26a, Prop. 2.2]'s certification is out of reach. That is one coordinate rather than a diffuse doubt, and naming it is the last thing this paper can do for the question it asks.

A Verification blocks

`verify_a81.py` uses the standard library only, with exact `Fraction` and integer arithmetic throughout; no published value passes through floating point. Every number in the body is a macro it emits, and the script asserts each rather than printing it, exiting non-zero on any failure.

C1 The label set.
Builds $\mathcal{L}$ and org, runs [Sch26h]'s five transitions over an explicit atom set, and asserts (L1)–(L6) of Theorem 4.3 together with the agreement of the closed formula with the atom count at every step.

C2 Locations.
Assigns the phases per control volume, counts the classes resolved, and checks Corollary 4.5.

C3 The stock witness.
Re-derives [Sch26a, Fig. 1]'s partition from [Sch26h]'s formula, asserting the three identities of Proposition 5.1.

C4 The lattice role.
Computes nodes, bridges, stack relations, arm terminals and carbon, with the per-layer cross-check, and forms the enlarged witness of Ruling 5.4.

C5 Pool and membrane.
Recovers both pool figures, decomposes [Sch26c]'s, and evaluates the four readings of Proposition 5.8.

C6 Conditions.
Tabulates the conditions, counts the statuses and asserts the membership of $\mathfrak{O}$.

References

[Sch26a] Charles D. Schaper. Carbon Traceable Protocell Construction From Substituted Chrysene Stocks. *Annals of the Chrysene Formalism*, 1(4):1567–1605, 2026. Article 77. DOI: https://doi.org/10.68366/ees7wd38.

[Sch26b] Charles D. Schaper. Cycle Integration: Copy Number, the Closure Vesicle, and the Band of the Genome. *Annals of the Chrysene Formalism*, 1(3):1472–1483, 2026. Article 72. DOI: https://doi.org/10.68366/7fdnjs23.

[Sch26c] Charles D. Schaper. The Chrysene-Reachable Material Ledger: Species, Properties, and the Allocation of a Single Pool. *Annals of the Chrysene Formalism*, 1(3):1371–1383, 2026. Article 65. DOI: https://doi.org/10.68366/dwqk8m15.

[Sch26d] Charles D. Schaper. The Completeness Condition: Realisation of the Twelve-Axiom Framework at a Site of the Interior Chrysene Lattice, and the Genome Length It Requires. *Annals of the Chrysene Formalism*, 1(4):1639–1651, 2026. Article 80. DOI: https://doi.org/10.68366/0a19r769.

[Sch26e] Charles D. Schaper. The Division of the Vessel: The Coded Chain, the Photochrome Brake, and the Genome that Selects Itself. *Annals of the Chrysene Formalism*, 1(4):1545–1566, 2026. Article 76. DOI: https://doi.org/10.68366/s7zef416.

[Sch26f] Charles D. Schaper. The Enclosure via the Encoded Chrysene Cluster: Integrated Synthesis of Nucleic Acids and Phospholipids, and the Orientational Encoding of the Membrane. *Annals of the Chrysene Formalism*, 1(3):1279–1320, 2026. Article 62. DOI: https://doi.org/10.68366/24m3wx88.

[Sch26g] Charles D. Schaper. The Interior Chrysene Lattice: Mathematical Necessity, Distributed Inheritance, and Catalytic Organization in the Protocell. *Annals of the Chrysene Formalism*, 1(4):1606–1615, 2026. Article 78. DOI: https://doi.org/10.68366/2kkrhm64.

[Sch26h] Charles D. Schaper. The Source: Substitution-Initiated Self-Separation of the Substituted-Chrysene Stock from a Polyaromatic Bulk, and the Supply It Delivers to the Protocell. *Annals of the Chrysene Formalism*, 1(4):1616–1638, 2026. Article 79. DOI: https://doi.org/10.68366/s8sfwq50.