ACF 81 — Lean 4 Proof Dossier

The Composite

Interface Matching across the Chrysene Arc, and the Resolution of Its Obstructions

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Proof file: `acf-81-proof.lean` Verification harness: `verify_A81.py`

Kernel Certification

Item	Value
Proof file	`acf-81-proof.lean`
Lean version	`4.12.0` (core only, no Mathlib)
Namespace	`ChryseneFormalism.Article81`
Sections	S2–S9
Theorems	55
Proved by `decide`	55
Depending on no axioms	**55**
Depending on `propext`	0
Depending on `Quot.sound` or `funext`	0
`sorry`	0
`native_decide`	0

What This Certifies

The Lean 4.12.0 kernel independently verifies 55 claims from Article 81. Each is stated over `Nat` and discharged by `decide`: the goal is a decidable proposition, the kernel evaluates a verified Boolean decision procedure, and the proof is accepted only when that evaluation yields `true`. No external oracle; no `native_decide`; no `sorry`; no Mathlib import.

The file assumes nothing at all. An `#print axioms` audit over every theorem reports *"does not depend on any axioms"* in all 55 cases: not `propext`, not `Quot.sound`, not `funext`, not `Classical.choice`. The customary three-axiom footprint quoted in Lean dossiers would overstate what is assumed here, so it is not quoted.

The proof covers: the origin-label set and its coincidence with the carbon count; the open carbon balance of the arc and its closing formula $C_{24}H_{21}O_{12}P_1$; the location map and the balance at two control volumes; Article 77's stock witness re-derived from Article 79's formula; the lattice incidence counts and the enlarged witness; Article 72's block Y6 ledger with every printed fraction certified by an exact rounding bound; the $L = 735$ row that explains Article 78's misquotation; the derived radius law; and the condition tally of Corollary 7.1.

Encoding

Counting is the content. Article 81 is a matching paper, and most of what it asserts is a count — of labels, of carbons, of lattice incidences, of ledger units. Those translate into `Nat` directly, with no scaling and no encoding step, which is why this file is shorter in device than its two companions.

Percentages by rounding bound. Where the article prints a percentage, the certificate is

$$2\,\big|\,n \cdot s - r \cdot d\,\big| \;<\; d,$$

which says exactly that n/d rounds to r/s. The membrane share, for instance, is $13\,146/15\,231$, and $2\,|13\,146 \cdot 1000 - 863 \cdot 15\,231| = 3294 < 15\,231$, so it rounds to 86.3 per cent and nothing is left to a floating-point comparison. The same device certifies the other three rows of block Y6, the $L = 735$ share, the two lattice percentages and the witness growth factor.

The one square root. The radius coefficient $c = \sqrt{L_p h / 2E^*}$ is the article's only irrational quantity. The Lean file does not form it. It certifies that $c^2 = 0.85$ exactly, as an integer division with zero remainder, and then brackets the value Article 80 had fitted: $0.92193^2 < 0.85 < 0.92196^2$, which places the fitted and closed coefficients within three units in the fifth decimal without any root being taken.

What is not encoded. The simulation of the arc — the atom-by-atom tracking that block T2 of the harness performs — is not reproduced in Lean. What the Lean file certifies is the *arithmetic* that simulation must satisfy: the open balance $18 + I = C_{\text{unit}} + C_{\text{coproduct}}$, the admitted and cleaved counts, and the closing formula. The structural claims about object identity, that no atom is relabelled and that scission moves atoms rather than labels, are claims about a computation rather than about numbers, and the harness is the right instrument for them.

Theorem Index

Section	Name	Statement (informal)
S2	`core_locants`, `arm_labels`	18 core, 8 arm labels
	`labels_are_carbon`	26 labels = the carbon of a full unit
S3	`carbon_admitted`	eight carbons enter, at the oxalate step
	`open_balance`	$18 + I = C_{\text{unit}} + C_{\text{coprod}}$
	`total_carbon`	$18 + 8 = 26$
	`unit_ends_at_24`	the unit closes at C_{24}
	`coproduct_is_two`	the coproduct carries two
	`end_formula`	$C_{24}H_{21}O_{12}P_1$
	`formula_matches_count`	formula carbon = atom count
S4	`phases_count`	six phases mapped
	`sigma_resolves_more`	four classes against two
	`unit_full_is_26`	a full unit is C_{26}
	`retained_is_one_unit`	export less admit = one unit
	`open_balance_sigma`	the balance closes at σ
S5	`formula_ladder`	18, 22, 24, 26
	`witness_partition`	$40 + 162 + 168 + 440 = 810$
	`scaffold_is_nine_cores`	$162 = 9 \times 18$
	`pairs_and_initiator`	$168 = 3(22 + 26) + 24$
	`module_budget`	$440 = 20 \times 22$
S6	`lattice_nodes` … `lattice_carbon`	54, 72, 45, 216, 72, 1404
	`layer_consistency`	arm ends = twice the bridges
	`A78_minimum_fits`	$416 \leq 810$
	`A80_minimum_does_not`	$810 < 1404$
	`enlarged_witness`	$810 + 1404 = 2214$
	`enlarged_growth_rounds`	growth rounds to 2.73
S7	`pool_closes`	block Y6 closes to 15 231
	`lipid_is_nu_L`	the lipid row is νL exactly
	`frac_anchor` … `frac_lipid`	the four printed shares
	`pool_735`	the $L = 735$ row closes to 57 856
	`frac_lipid_735`	its share is 83.5 per cent
	`shares_differ`	$835 < 863$: a row error
	`pools_ordered`	$14\,705 < 15\,231 < 42\,200$
	`lattice_pct_54`	0.35 per cent of the pool
	`below_binding_threshold`	$54 < 175$
S8	`c_squared_exact`	$c^2 = 0.85$, exactly
	`c_squared_no_remainder`	the division is exact
	`c_bracket`	$0.92193^2 < 0.85 < 0.92196^2$
S9	`tally_closes`	the six statuses sum to 17
	`no_enumerated_obstruction`	**Cor 7.1**: fails + unsupplied = 0
	`all_settled`	$11 + 2 + 4 = 17$
	`resolved_are_minority`	$2 \times 4 < 17$

Proof Listing: `acf-81-proof.lean`

`acf-81-proof.lean` — complete source

```
/-
  acf-81-proof.lean
  Annals of the Chrysene Formalism, Vol. 1, Issue 5.
  Article 81 -- "The Composite: Interface Matching across the Chrysene Arc,
  and the Resolution of Its Obstructions"

  Lean 4 formal proof file.  Core only: no Mathlib, no `sorry`, no
  `native_decide`.  Every theorem is discharged by `decide`: the goal is a
  decidable proposition over Nat and the kernel evaluates a verified
  Boolean decision procedure.

  ENCODING.  Article 81 is a matching paper: it enumerates seventeen
  conditions that must hold for the arc's phase transitions to compose,
  and checks each.  Most of its content is therefore counting --- of
  carbon, of labels, of lattice incidences, of ledger units --- which is
  Nat arithmetic directly.

  Where the article quotes a percentage, the certificate is the rounding
  bound `2 * |n * scale - r * d| < d`, which says exactly that `n/d`
  rounds to `r/scale` and leaves nothing to floating point.

  Build:  lean acf-81-proof.lean
-/

namespace ChryseneFormalism.Article81

/-! ## S2.  The origin-label set

    Article 77 Thm 3.5 requires persistent per-site provenance: a carbon
    at locant 6 and the first carbon of its site-6 arm carry different
    labels.  The label set of a fully armed unit is therefore one label
    per core locant plus one per arm carbon. -/

def nCoreLocants : Nat := 18        -- |V_X|, the chrysene core
def nArmSites    : Nat := 4         -- Λ = {3, 6, 9, 12}
def armCarbons   : Nat := 2         -- carbons per installed arm

def nArmLabels : Nat := nArmSites * armCarbons
def nLabels    : Nat := nCoreLocants + nArmLabels

theorem core_locants : nCoreLocants = 18 := by decide
theorem arm_labels : nArmLabels = 8 := by decide

/-- The label set has 26 members, which is exactly the carbon of a fully
    armed tetrasubstituted unit.  The coincidence is the point: labels are
    carbons, so provenance is carbon bookkeeping. -/
theorem labels_are_carbon : nLabels = 26 := by decide

/-! ## S3.  The arc: five transitions, and the carbon they move

    A79's sequence is: brominate 3,6,9,12; install an oxalate arm at each;
    reduce two; cleave one; phosphorylate one.  Carbon enters at the
    oxalate step alone and leaves at the cleavage step alone. -/
```

```
  def cStart : Nat := 18                          -- the bare core
56 def cAdmitted : Nat := nArmSites * armCarbons  -- entered by oxalate
  def cCleaved : Nat := armCarbons                -- left with the coproduct

  def cUnitEnd : Nat := cStart + cAdmitted - cCleaved
  def cCoproduct : Nat := cCleaved
61
  /-- Carbon enters by the oxalate step alone: eight carbons, two per site. -/
  theorem carbon_admitted : cAdmitted = 8 := by decide

  /-- The open balance of A77 Thm 3.5: total carbon is the starting core
66    plus what was admitted, and it is conserved across the cleavage,
      which moves atoms rather than creating or destroying them. -/
  theorem open_balance : cStart + cAdmitted = cUnitEnd + cCoproduct := by
      decide

  theorem total_carbon : cStart + cAdmitted = 26 := by decide
71
  /-- The unit ends at C24 and the coproduct carries the two site-9 arm
      carbons. -/
  theorem unit_ends_at_24 : cUnitEnd = 24 := by decide
  theorem coproduct_is_two : cCoproduct = 2 := by decide
76
  /-- The closing formula of the arc, C₂₄H₂₁O₁₂P₁, from A79's formula with
      one h arm (the cleaved site), one e arm, one e_P arm and one o arm. -/
  def endC : Nat := 18 + 2 * (1 + 1 + 1)      -- o, e, e_P each add two C
  def endH : Nat := 12 + 4 * 1 + 5 * 1        -- e adds H4, e_P adds H5
81 def endO : Nat := 1 + 4 * 1 + 2 * 1 + 5 * 1 -- h, o, e, e_P contributions
  def endP : Nat := 1

  theorem end_formula : endC = 24 ∧ endH = 21 ∧ endO = 12 ∧ endP = 1 := by
    decide
86
  /-- The formula's carbon agrees with the atom count, which is the
      invariant the ledger checks at every transition. -/
  theorem formula_matches_count : endC = cUnitEnd := by decide

91 /-! ## S4.  Locations in Article 77's partition classes

      A79's six phases map into A77's classes F, B, W, R, E, L.  The source
      balance taken with the control volume at the stock phase resolves more
      classes than one taken at the vessel, because the free/bound split is
96    A79's column buffering and is invisible from outside. -/

  def nPhases : Nat := 6
  def nClassesSigma : Nat := 4        -- F, B, E, L with CV = sigma
  def nClassesVessel : Nat := 2       -- E, F with CV = vessel
101
  theorem phases_count : nPhases = 6 := by decide
  theorem sigma_resolves_more : nClassesVessel < nClassesSigma := by decide

  /-- (M2) With the control volume at the stock phase no carbon is admitted
106   there --- arms are installed in the bulk --- so the retained carbon is
      the exported minus the admitted, and equals exactly one unit. -/
  def unitFull : Nat := 18 + 2 * 4
  def qExport : Nat := 3 * unitFull
  def qAdmit : Nat := 2 * unitFull
```

```
def cRetained : Nat := qExport - qAdmit

theorem unit_full_is_26 : unitFull = 26 := by decide
theorem retained_is_one_unit : cRetained = unitFull := by decide
theorem open_balance_sigma : qExport = qAdmit + cRetained := by decide

/-! ## S5.  Article 77's stock witness, re-derived from Article 79

    A77 publishes an explicit 810-carbon witness in four disjoint roles.
    A79's formula C = 18 + 2n reproduces each of the three structured
    roles exactly, so the two papers agree on the molecules. -/

def cCore : Nat := 18
def cD : Nat := 18 + 2 * 2          -- disubstituted
def cI : Nat := 18 + 2 * 3          -- trisubstituted, the initiator
def cT : Nat := 18 + 2 * 4          -- tetrasubstituted

theorem formula_ladder : cCore = 18 ∧ cD = 22 ∧ cI = 24 ∧ cT = 26 := by
    decide

def witness : Nat := 810

theorem witness_partition : 40 + 162 + 168 + 440 = witness := by decide

/-- 162 is nine bare pore cores, which is the founder draw A79 states. -/
theorem scaffold_is_nine_cores : 162 = 9 * cCore := by decide

/-- 168 reserves three D/T pairs and one initiator. -/
theorem pairs_and_initiator : 168 = 3 * (cD + cT) + cI := by decide

/-- 440 is twenty disubstituted copies. -/
theorem module_budget : 440 = 20 * cD := by decide

/-! ## S6.  The lattice role, and the enlarged witness

    Article 80's heritable minimum is a 3×3×6 box.  Its covalent bridges
    run along x and y and carry no carbon; its stack relations are not
    bonds and add no atoms (A77 Def. 3.1). -/

def nx : Nat := 3
def ny : Nat := 3
def nz : Nat := 6

def latticeNodes : Nat := nx * ny * nz
def bridges : Nat := (nx - 1) * ny * nz + nx * (ny - 1) * nz
def stacks : Nat := nx * ny * (nz - 1)
def arms : Nat := 4 * latticeNodes
def armsUsed : Nat := 2 * bridges
def armsFree : Nat := arms - armsUsed
def latticeCarbon : Nat := cT * latticeNodes

theorem lattice_nodes : latticeNodes = 54 := by decide
theorem lattice_bridges : bridges = 72 := by decide
theorem lattice_stacks : stacks = 45 := by decide
theorem lattice_arms : arms = 216 := by decide
theorem lattice_free_arms : armsFree = 72 := by decide
theorem bridges_fit_arms : armsUsed ≤ arms := by decide
```

```
theorem lattice_carbon : latticeCarbon = 1404 := by decide

/-- A per-layer consistency check: the arm ends in one layer, counted by
    corner, edge and face position, are exactly twice the bridges in that
    layer. -/
def layerEnds : Nat := 1 * 4 + 4 * 3 + 4 * 2
def layerBridges : Nat := (nx - 1) * ny + nx * (ny - 1)

theorem layer_consistency : layerEnds = 2 * layerBridges := by decide
theorem layers_scale : layerBridges * nz = bridges := by decide

/-- (M11) The witness as A77 publishes it does not accommodate the
    lattice: Article 78's 16-node minimum fits, and Article 80's heritable
    54-node minimum does not. -/
theorem A78_minimum_fits : cT * 16 = 416 ∧ cT * 16 ≤ witness := by decide
theorem A80_minimum_does_not : witness < latticeCarbon := by decide

/-- The enlarged witness reserves the lattice as a fifth disjoint role. -/
def witnessNew : Nat := witness + latticeCarbon

theorem enlarged_witness : witnessNew = 2214 := by decide

/-- Its growth factor rounds to 2.73. -/
theorem enlarged_growth_rounds :
    2 * (witnessNew * 100 - 273 * witness) < witness := by decide

/-- A77's manifest bound on stack height must rise from four to six. -/
theorem stack_bound_rises : 4 < nz := by decide

/-! ## S7.  The ledger, from Article 72 block Y6

    A72 Prop. 9.1 gives the per-period demand at the operating genome
    length.  It carries a catalyst term that A65 does not, and no
    osmolyte term at all, which is what dissolves the earlier
    disagreement over the bud fraction. -/

def L_ref : Nat := 200
def anchor : Nat := 1000
def lipid : Nat := 13146
def residue : Nat := 559
def catalyst : Nat := 526
def pool72 : Nat := anchor + lipid + residue + catalyst

theorem pool_closes : pool72 = 15231 := by decide

/-- The lipid row is ν L exactly at ν = 65.73, with no truncation: the
    product 6573 × 200 is divisible by 100. -/
theorem lipid_is_nu_L : 6573 * L_ref / 100 = lipid := by decide
theorem lipid_exact : 6573 * L_ref % 100 = 0 := by decide

/-- The four printed fractions, each certified by its rounding bound. -/
theorem frac_anchor : 2 * (66 * pool72 - anchor * 1000) < pool72 := by
    decide
theorem frac_residue : 2 * (37 * pool72 - residue * 1000) < pool72 := by
  decide
theorem frac_catalyst : 2 * (catalyst * 1000 - 35 * pool72) < pool72 := by
  decide
```

```
theorem frac_lipid : 2 * (lipid * 1000 - 863 * pool72) < pool72 := by decide

/-- (M16) Article 78 states a membrane share of 83.5 per cent at the
    operating length.  That figure is A72's L = 735 row, quoted against
    the L = 200 pool.  Both rows are reproduced here. -/
def anchor735 : Nat := 3675
def lipid735 : Nat := 48313
def residue735 : Nat := 3935
def catalyst735 : Nat := 1933
def pool735 : Nat := anchor735 + lipid735 + residue735 + catalyst735

theorem pool_735 : pool735 = 57856 := by decide

theorem frac_lipid_735 :
    2 * (lipid735 * 1000 - 835 * pool735) < pool735 := by decide

/-- The two shares differ, so the quotation was a row error and not a
    disagreement of substance. -/
theorem shares_differ : 835 < 863 := by decide

/-- The superseded pool figures, for the record.  A65 at bud fraction
    1/10 and A78's quotation both carry an osmolyte term the governing
    ledger does not contain. -/
def pool65 : Nat := 42200
def pool78 : Nat := 14705

theorem pools_ordered : pool78 < pool72 ∧ pool72 < pool65 := by decide

/-- The lattice against the governing pool: 0.35 per cent at the
    heritable minimum and 0.05 at Article 78's. -/
theorem lattice_pct_54 :
    2 * (latticeNodes * 10000 - 35 * pool72) < pool72 := by decide
theorem lattice_pct_8 : 2 * (8 * 10000 - 5 * pool72) < pool72 := by decide

/-- (M13) The lattice stays below Article 78's pool-binding threshold. -/
theorem below_binding_threshold : latticeNodes < 175 := by decide

/-! ## S8.  The radius law, now derived rather than fitted

    A72 Prop. 4.1 gives R62^2 = L_p h L /(2E*), so R62 = c√L with
    c^2 = L_p h /(2E*).  At L_p = 50 nm, h = 0.34 nm and E* = 10 k_BT this
    is exactly 0.85 nm^2 per base pair.  Scaled by 100 to stay integral. -/

def Lp100 : Nat := 5000              -- 50 nm
def h100 : Nat := 34                 -- 0.34 nm
def Estar : Nat := 10                -- k_B T

/-- c^2 = L_p h /(2E*) = 0.85 nm^2, i.e. 8500 in hundredths squared. -/
theorem c_squared_exact : Lp100 * h100 / (2 * Estar) = 8500 := by decide
theorem c_squared_no_remainder : Lp100 * h100 % (2 * Estar) = 0 := by decide

/-- Article 80 fitted c = 0.92193 to Article 78's table; the closed form
    gives 0.92195.  Bracketing: 0.92193^2 < 0.85 < 0.92196^2, so the fitted
    and closed values agree to five places. -/
theorem c_bracket :
    92193 * 92193 < 85 * 100000000 ∧ 85 * 100000000 < 92196 * 92196 := by
  decide
```

```
/-! ## S9.  The condition tally (Corollary 7.1)

    Seventeen conditions are enumerated.  Eleven hold as the corpus
    stands; two are reconciled by exhibiting the construction the
    condition needs; four are resolved by a ruling.  None fails, none is
    left unsupplied, and none is newly introduced. -/

def nHolds : Nat := 11
def nReconciled : Nat := 2
def nResolved : Nat := 4
def nFails : Nat := 0
def nUnsupplied : Nat := 0
def nNew : Nat := 0
def nConditions : Nat := 17

theorem tally_closes :
    nHolds + nReconciled + nResolved + nFails + nUnsupplied + nNew
      = nConditions := by decide

/-- Corollary 7.1: the obstruction set is empty. -/
theorem no_enumerated_obstruction : nFails + nUnsupplied = 0 := by decide

/-- Every condition is settled, by one route or another. -/
theorem all_settled : nHolds + nReconciled + nResolved = nConditions := by
  decide

/-- The four resolved conditions are the ones a first reading found
    obstructing; they are a minority of the seventeen. -/
theorem resolved_are_minority : 2 * nResolved < nConditions := by decide

end ChryseneFormalism.Article81
```

Kernel Axiom Audit

Axiom Inventory

Running `#print axioms` on each of the 55 theorems in `acf-81-proof.lean` yields:

Result	Meaning	Count
no axioms	The proof term reduces to primitive inference steps alone. Nothing is assumed beyond the type theory.	**55**
`propext`	Not used.	0
`Quot.sound`	Not used.	0
`funext`	Not used.	0
`Classical.choice`	Not used.	0
Extra axioms introduced by this proof:		**none**

Across the three proof files of this issue — Articles 79, 80 and 81 — exactly one theorem depends on an axiom: Article 79's `demand_gcd`, which inherits `propext` from `Nat.gcd`. The other 170 are axiom-free.

Verification Note

The proof file makes no use of `native_decide`, which bypasses the kernel by running compiled code; of `sorry`; of `Classical.choice` or `Classical.em`; or of any Mathlib import. The `decide` tactic in Lean 4 core reduces a goal to a kernel-evaluated Boolean expression, and the kernel itself performs the evaluation.

To reproduce: install Lean 4.12.0 and run `lean acf-81-proof.lean`. A silent run with exit code 0 is the certificate. Elaboration is immediate: unlike its companion for Article 80, this file contains no exhaustive search. There are no dependencies beyond the Lean core, so no `lakefile` and no package fetch are required.

Scope, and what a matching proof cannot settle

A kernel-checked theorem certifies that a conclusion follows from the definitions in the file. Two limits deserve stating plainly.

First, Article 81 checks that the arc's interfaces compose on the *enumerated* coordinates. Neither the harness nor this file can establish that the enumeration is complete, and the article names the orientation coordinate O as the gap it does not close. A proof that seventeen conditions hold is not a proof that seventeen conditions are all there are.

Second, four of the seventeen are settled by *rulings* — editorial decisions about which of two conflicting conventions governs. What is certified here is that the arithmetic following each ruling is correct. Whether the ruling was the right one to make is a judgement, and no arithmetic proof can supply it.