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The Placed Source: Exogenous Preparation of the Substituted-Chrysene Assembly, and the Equality of Its Reachable Set with the Endogenous One

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Abstract

This paper compares endogenous self-separation and exogenous site-specific placement of substituted-chrysene stocks, evaluating how transitioning from a bulk-partitioned source to purified, site-directed assembly affects both the set of reachable configurations and the information cost of structural selection. In the endogenous pathway, substitution in a polyaromatic bulk lowers a unit's transfer free energy so that departed units self-assemble into columns; in the placed pathway, the same three substitution families and arm words are delivered as purified material to prescribed sites. Comparing the two regimes establishes that both sources generate the exact same set of reachable structures, while exogenous placement provides direct control over alignment selection.

To establish the equivalence of the reachable sets, a configuration is defined as a finite occupied subset of $\mathbb{Z}^3$ carrying a substitution family and an arm word at each node, together with a binary registry label — Aligned or Unaligned — at each stacking contact. Because the set of such configurations on a column is finite and explicitly enumerable, comparing the two sources reduces to comparing the support of the endogenous distribution with the image of the placement map. Because the endogenous exit distribution is strictly positive at every admitted substitution degree and permits the full arm alphabet, every placed configuration lies within the endogenous support ($R_{\text{endo}} = R_{\text{exo}}$). Consequently, the reachable end materials coincide under any product map, and for the six-site stack composition their common image is bounded by 20 frame classes.

Where the two sources differ is in structural selection, governed by the symmetry properties of the separation filter. A unit's contact pattern is its set of locant–arm pairs, and inversion through the C_{2h} reference frame exchanges positions $3 \leftrightarrow 9$ and $6 \leftrightarrow 12$. Any contact containing at least one unit whose pattern is invariant under this inversion is indifferent to its binary alignment state. Because the disubstituted family $D = \{6, 12\}$ and tetrasubstituted family $T = \{3, 6, 9, 12\}$ are inversion-invariant and carry arm words constant across inversion orbits, every contact in the delivered stock is alignment-indifferent except the I–I contact, which exhibits a registration number of 3 when aligned and 2 when unaligned. Because the separation filter reads the alignment field solely through the initiator — where I–I contacts occur at a frequency of 2.5×10^{-7} — an endogenous contact energy without an additional orientation term induces a uniform distribution over the alignment field. Prescribing the alignment state of a stack of length ℓ therefore carries an exact information cost of $\ell - 1$ bits: 5 bits for a codon-length stack (corresponding to five binary agreements) and 53 bits at the heritable lattice minimum. By comparison, the kinetic species-selection cost at the reported demand ratios is 20.0 to 23.0 bits for a six-site alternating column, making the registry contribution about one-fifth of the specification cost.

This symmetry analysis also highlights the dual structural role of the trisubstituted initiator. As the sole admitted family that breaks inversion symmetry, the initiator is the unique unit capable of fixing an orientation frame. In the coding-stack architecture, stacks are capped with this trisubstituted unit, and polymerisation terminates there due to the absence of a sidechain at the 9-position — the exact image of the 3-position under C_{2h} inversion. Thus, the chemical feature that terminates chain extension is the same geometric feature that establishes stack orientation. Relative to this asymmetric cap, the two alignments of a symmetric contact become physically distinct states, anchoring the information content of the registry field.

Keywords: Chrysene Formalism; exogenous placement; directed placement; structural alphabet; frame group; registry field; flip blindness; reachability; selection cost; configuration space; interior chrysene lattice.

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1 Introduction

1.1 Two sources for one stock

Article 79 [Sch26g] constructs a source. Chrysene in a polyaromatic bulk is brominated, armed through an oxalate and reduced; each polar arm lowers the unit's transfer free energy out of the bulk, so the step that prepares the reagent is the step that removes it; the departed units stack into columns whose continuous arm registrations exclude a guest that cannot present the pattern. The stock so obtained feeds the protocell arc: Article 77's admission [Sch26a], Article 65's pool [Sch26c], Article 78's interior lattice [Sch26e], Article 80's completeness condition [Sch26d].

This paper asks what happens if the stock arrives instead as purified material, placed. Not manufactured in place and not separated from anything: delivered at specification and set, unit by unit, at sites chosen by an agent outside the system. The three admitted substitution families are the same, the arms are the same, the assemblies are the same stacks and lattices, and the end materials are to be the same. Only the source is different.

The question is not rhetorical, and it is not about plausibility. It is the question of what a source *is* in this architecture: which of the architecture's properties are properties of the material and which are properties of how the material arrived. Article 79 answers one half of that by construction; this paper answers the other half by comparison, and the comparison turns out to separate two things that the corpus has so far carried together.

1.2 The antecedent

Both routes are named in the filing, though not set against each other there, and the distinction this paper draws is its own. The filing's background treats each as a prior-art deficiency and objects to each differently: of de novo synthesis "processed through directed placement of monomers" it says that "there is limitation as to the number of bases synthesized"; of schemes in which "molecules randomly self-organize" it says that "these random events provide unknown variability and ambiguity in DNA and cellular production" [Sch25a, Background]. The second objection names the quantity this paper computes, and that is what is taken from the passage. What is not taken—and what an earlier form of this paper asserted—is that the filing identifies variability at fixed material as the difference between the two routes. It does not; the comparison at fixed material is this paper's, and the filing's two routes do not even operate on the same material, the first placing nucleotide monomers and the second having molecules self-organise into a strand that then "randomly couples with a phospholipid cellular membrane".

1.3 What the corpus leaves open here

Three places bear directly on the comparison and none makes it.

The endogenous source. Article 79 proves that its separation is selective in *species*: four filters act on the background, of which registry is the one that protects the disubstituted stream [Sch26g, Sch. "Four filters"]. It does not ask whether any filter acts on *orientation*. Section 7 shows that none of the four does, and that this is forced by the geometry the third filter uses.

The encoding. Article 57 [Sch26f] imports from the filing the assignment of the four orientation pairs to the four letters, counts the frame classes of a six-site stack at 20, and records that a product is a function of the registry field modulo end-to-end reversal [Sch26f, Prop. 4.7]. It takes the field as given. Where the field comes from is not its question, and it is this paper's.

The lattice. Articles 78 and 80 fix the geometry of the interior lattice—its node counts, its linker arms, its completeness minima of 27 and 54 nodes, the genome length that admits them—and treat the lattice as taken up from the environment [Sch26e, §7]. Whether a placed lattice differs from a self-assembled one in anything but provenance is not asked there.

1.4 What this paper addresses

(i) What a configuration of the assembly is, and in what exact sense the set of them is finite (Sections 4–5).
(ii) Whether the separation chemistry of Article 79 can select the orientation of a unit at all (Section 7).
(iii) Whether every placed configuration is endogenously reachable, and whether the end materials coincide (Section 8).
(iv) What the exogenous route therefore buys, measured in bits (Section 9).
(v) Whether a placed assembly is admissible, and what it costs the ledger (Section 10).

1.5 What is established here, by kind

Recovered. Article 40's structural alphabet $\Sigma_{\mathfrak{C}}$ and its additive law (Defs. 35–36, Prop. 12, Rem. 20), its registry field on the z-edges and the transport of that field (eqs. (108)–(113)), and the cipher flip with its translation covariance (Def. 37, Prop. 13) [Sch26b]; Article 57's frame group, capacity theorem with its withheld injectivity, cluster composition and identification clause, letter assignment and production operator (Defs. 4.1, 4.2, 5.2, Thm. 4.5, Props. 4.3, 4.4, 4.7) [Sch26f]; Article 79's exit distribution, orbit lemma, arm-word count, registry hypothesis and delivered stock (the exit distribution, the delivered one-reagent stock, the orbit lemma, the arm-word lemma and H_{rgy}, each cited below by title) [Sch26g]; Article 77's admission predicate and constitutional stock type (§3, Def. 3.1) and its reading of the filing's orientation dictionary (§2.4, eq. (1)) [Sch26a]; Article 80's site-class identities, completeness minima, saturated-bridging hypothesis with its arm-terminal count, its ruling on what models the registry field, and its statement that a lattice node is

tetrasubstituted (Prop. 3.4, Thm. 5.1, Hyp. 3.3, Rems. 2.3, 9.1, Prop. 3.6) [Sch26d]; Article 78's lattice composition, geometry, linker arm, entry mechanism and supply formula (§1.2, Defs. 5.2, 6.1, Hyps. 5.1, 7.1, eq. (1)) [Sch26e]; Article 65's species inventory, its two-sidedness property and its documentary bundle (Def. 5.1, Table 1, Hyp. 4.1) [Sch26c].

Proved here. Finiteness and the exact configuration count (Thm. 5.1, Cor. 5.2); the closed form for the registry classes at every length (Thm. 6.5); the frame group in codon coordinates (Prop. 6.11); the agreement word as the quotient by the flip, and a second derivation of the class count (Lem. 8.10); the two base-two coordinates of a register (Prop. 6.8); flip blindness and its consequence for Article 79's filter (Thm. 7.3, Cor. 7.5, Thm. 7.10); uniqueness of the frame carrier (Thm. 7.13); surjectivity of placement and equality of the reachable sets (Prop. 8.2, Thm. 8.4, Cor. 8.8); coincidence of the end materials, as images and not as counts (Cor. 8.12, Thm. 8.15, Cor. 8.17); the registry term of the selection cost (Thm. 9.2) and the species term (Prop. 9.5); admissibility of a placed assembly (Thm. 10.1); invariance of the ledger (Prop. 10.4).

Found. The separation of Article 79 is blind in orientation at every site of the stock it delivers, and blind for a reason internal to its own third filter (Sch. 7.12). The exogenous route therefore buys selection and not reach, and the part of the selection it buys that no kinetics can touch is the registry term, a fifth to a quarter of the cost of a coded hexamer (Rem. 9.6). Exactly one admitted family can carry an absolute frame, and it is the one the filing uses for that purpose (Thm. 7.13).

Computed. Blocks X1–X10 of Appendix B.

Supposed. Appendix A. H_{place} carries the exogenous route and is a stipulation of the same kind as Article 79's H_{ret}; H_{nf} carries the reachability theorem and is the weakest orientation premise that will do; H_{ν} is a refinement of Article 79's H_{rgy}, registered because the paper's central corollary is proved under it as well as under the corpus's own weaker reading.

Not claimed. Proposition 3.1, eleven clauses.

2 Status, nomenclature, scope

Convention 2.1 (Status labels)**.** [A][A] finite or axiom-free; [D] derived under hypotheses named and registered in Appendix A; [C] imported, source named; [E] molecular reading, entering no proof. Scholia are unlabelled and enter no proof. Every quantitative claim in the body is a macro emitted by `verify_placed.py`; the emitting block is cited as X1–X10 and the blocks are described in Appendix B. A small number of macros carry a fixed symbol rather than a computed number—a point group, a family name, a closed form in m or ℓ—and the generated file lists them under a separate heading so that the distinction can be checked rather than taken on trust.

Convention 2.2 (Patent citation and documentary marks)**.** [A]Adopted from Article 65 [Sch26c, Convs. 2.2–2.3] and used as Article 79 uses it: the filing [Sch25a] is cited through Article 65's documentary bundle H_{mat}, as amended by Article 79's clause (i) and by clause (j) of Ruling 13.7. A sentence quoting the filing is [E]; a chemical premise carries the mark A (attested by a clause of the bundle), S (stated by the author) or B (to be discovered). No theorem rests on a B entry.

Convention 2.3 (Labels and their premises)**.** [A]A label records how a statement is established and not what it is free of. Theorem 5.1 is [A] as an identity about functions on a finite set; that its bound is attained where the corpus's saturated bridging holds is a separate matter, recorded at Table 1. Theorem 9.2 is [A] as arithmetic on a uniform law; which law obtains is Theorem 7.10, which is [D]. Theorem 8.7 is [A] as an inequality about a product, and the formula it operates on is imported. Where an in-line label and the register appear to disagree, the register governs.

Convention 2.4 (Seniority)**.** [A]Where this paper and an earlier article speak to the same object, Section 13 states which governs. Nothing earlier is amended silently. This paper is parallel to Article 79 and supersedes nothing in it.

Notation 2.5. $\Lambda = \{3,6,9,12\}$ the primary locants of Axiom A_1 [Sch26b]; $D = \{6,12\}$, $I = \{3,6,12\}$, $T = \{3,6,9,12\}$ the three admitted families (Article 65's notation); $\mathcal{A}_{\mathrm{arm}} = \{h, o, e, e_P\}$ the arm alphabet of Article 79; ι the inversion of the C_{2h} reference of Axiom A_2, acting on the locants of A_3 by $3 \leftrightarrow 9$, $6 \leftrightarrow 12$; $\Sigma_{\mathfrak{C}} = \mathbb{Z}_2^{\mathrm{dc}} \times \mathbb{Z}_2^{z}$ the structural alphabet of Axiom A_8, with elements $s_{\epsilon,\eta}$; $\mathrm{Fr} = \langle \mathrm{flip}, \mathrm{rev} \rangle$ the frame group; Ω a finite occupied subset of $\mathbb{Z}^3$ and $B(n_x, n_y, n_z)$ an occupied box; $\jmath(n) = \max(0, n-2)$; ℓ a column length; (p_n) Article 79's exit distribution; $\rho^* = p_2/p_4$.

3 Non-implications

Proposition 3.1 (Non-implications)**.** *[A]None of the following is asserted.*

(i) No new end material. *That placement reaches an end material the endogenous source cannot reach.*

Theorem 8.15 says the two product sets are equal, and Corollary 8.17 states the consequence. A reader who expects an exogenous route to enlarge the architecture's output should read those two statements first.

(ii) No placement mechanism. H_{place} *(Hypothesis 4.19) is a stipulation. No sorter is designed, no apparatus is specified, no actuation chemistry is offered, and no energy, carbon or time is costed for the act of placing. What the exogenous agent is remains Target 14.3.*

(iii) No probability claim about either source. *Every comparison between the two routes in Section 8 is between an image and a support. The probabilities that appear are the admitted fractions built from Article 79's* (p_n)*, the orientation weight* θ *and the cipher weight* γ*, and they enter the reachability theorem only through strict positivity (Cor. 8.8). The bit figures of Section 9 are a separate and explicitly distributional statement.*

(iv) No argument about origins from the bit count. *Theorem 9.2 bounds the endogenous mass of one prescribed configuration. It is not a statement about how many configurations are tried, over what volume, or for how long, and it supports no inference about whether the endogenous route occurs. Scholium 9.7 says this at the point where the temptation is greatest.*

(v) No chronology. *That either source precedes the other, or that one is prebiotic and the other technological. The two are compared as sources of the same material and nothing more; Article 57's Section 3(iv) is read here as well.*

(vi) No absolute orientation read. *That any pathway reads the orientation of a unit as such. Article 57's Proposition 4.7 excludes it, and Corollary 8.12 is the consequence drawn here: a product is a function of the registry field modulo the frame group. What the placement system sets is a field; what a reading recovers is at most its class. How many distinct products the classes give is not claimed (Rem. 8.16).*

(vii) No field structure. *That* $\Sigma_{\mathfrak{C}}$ *carries a multiplication making it* $GF(4)$*. Article 40's Remark 20 declines the inference and it is declined here [Sch26b, Rem. 20]; Proposition 6.8 uses the additive group only.*

(viii) No new axiom. *Nothing is added to the twelve of Article 40. The structural alphabet, the cipher flip and the local tensor space are imported as written.*

(ix) No completeness without extent. *That a placed lattice realises Article 40's contractions at a site merely by being placed. Article 80's minima of* 27 *and* 54 *nodes and its genome bracket are geometric and are untouched by provenance (Rem. 10.3).*

(x) No selectivity claim transferred. *That the placement system is pure, or that it delivers only admitted species. Its purity is a stipulation of* H_{place} *and an impurity in the placed stream behaves exactly as Article 79's* c_d^{σ} *does, by the same column bound.*

(xi) No claim about which occupied sets arise. *That the endogenous route produces the occupied set* Ω*, or any particular one.* Ω *is a datum of the comparison and is handed to both routes alike (Def. 8.1);* R_{endo} *and* R_{exo} *are both relative to it. Since the difference between self-assembly and placement is in the first instance a difference in which occupied sets arise, this is the largest restriction on what the comparison settles, and it is stated here rather than left to be inferred.*

(xii) No defect found in Article 79. *Flip blindness (Thm. 7.3) is a property of Article 79's filter and not an error in it. The filter does what that paper proves it does; what it additionally does not do was never claimed there.*

Scholium 3.2. Clauses (i) and (iv) are the two that decide how the paper should be read. The first refuses the expectation that an exogenous route is a more powerful route: it is not, and the equality of Theorem 8.4 is the reason. The second refuses the use of a bit count as a probability argument about the origin of life. The paper computes how much information a prescribed assembly requires and says which source supplies it. It does not compute how often that requirement is met, and the difference between those two questions is the difference between this paper and an argument from improbability, which it is not.

4 Objects

4.1 The structural alphabet

Definition 4.1 (Structural alphabet)**.** [C]Axiom A_8 equips each local cell with the alphabet

$$\Sigma_{\mathfrak{C}} \;=\; \mathbb{Z}_2^{\mathrm{dc}} \times \mathbb{Z}_2^{z}, \qquad |\Sigma_{\mathfrak{C}}| = 4,$$

whose elements are written $s_{\epsilon,\eta} = (\epsilon, \eta)$ with ϵ the cipher component and η the discrete z-registry state, under componentwise addition modulo two [Sch26b, Defs. 35–36, eqs. (230)–(235)]. The cipher flip F_{dc} acts on the first factor and its localised copies $\Delta_{\mathbf{k}}$ satisfy $u_{\mathbf{m}}\Delta_{\mathbf{k}}u_{\mathbf{m}}^{*} = \Delta_{\mathbf{k}+\mathbf{m}}$ [Sch26b, Def. 37, Prop. 13].

Definition 4.2 (The alignment field)**.** [C]Axiom A_8 assigns to each z-stacking relation $\mathbf{k} \sim_z \mathbf{k} + e_z$ a registry label $\eta_{\mathbf{k}} \in R_z := \mathbb{Z}_2$ and writes $\eta : E_z \to \mathbb{Z}_2$ for the resulting *registry field on the oriented z-edges*, with translation transport at its equations (112)–(113) [Sch26b, eqs. (108)–(113)]. The

orientation datum of this paper is that field and nothing else: it is carried by *contacts*, not by sites. The *flip* at a contact is $\eta_{\mathbf{k}} \mapsto \eta_{\mathbf{k}} + 1$; its effect on the pair is to relabel the locants of one of the two units by ι, the involution of Axiom A_2's C_{2h} reference acting as $3 \leftrightarrow 9, 6 \leftrightarrow 12$ [Sch26b, eqs. (52)–(53)]. Article 40 adds that "the registry label is not identified with molecular chirality" [Sch26b, eq. (109)], which this paper also does not do.

Remark 4.3 (The filing's name for the same datum)**.** [E]The filing carries the field under the name Aligned/Unaligned and defines it as a relation between the two members of a contact: "an Aligned orientation refers to when the top catalyst molecule π-stacks with the bottom substrate molecule . . . in the same relative orientation where the central core is in the orientation configuration (e.g., in this case, both are in the Forward direction)", against Figure 4(f), "which depicts the 'Unaligned' orientation" [Sch25a, Fig. 4 description]; and it builds its four states from two bits exactly as this paper does—"at least four different orientations between the heterodimers (e.g., Aligned, Forward; Aligned, Backward; Unaligned, Forward; and Unaligned, Backward)". Its Figure 5(c) codes the summary in that vocabulary, "where CA represents catalyst aligned; CU represents catalyst unaligned". Clause (j) of Ruling 13.7 carries all three. The alignment bit is therefore not a construction of this paper: it is the filing's own second coordinate and Article 40's own registry field, and Lemma 8.10 is the statement that the two are the same object.

Remark 4.4 (Why the datum cannot sit at a site)**.** [D]An earlier form of this paper put η at each site and called it that site's orientation. That will not do, and the reason is worth stating because it is also the reason the paper needs Hypothesis 4.5.

A site datum would have to be the state of one unit in some frame. For a unit whose substituted pattern is carried to itself by ι—which by Theorem 7.3 is exactly the blind case, and by Article 79's one-reagent stock is the case of D and T—the motion that would exchange the two states is a symmetry of that unit, so the two states are one state described twice, and there is no datum. Nor does turning such a unit over supply one: reflection in the molecular plane of an idealised planar core fixes every atom. Absolute orientation is therefore not available at a blind site, which is also what Article 57 concludes from the other direction—"absolute orientation is excluded" [Sch26f, Prop. 4.7]—and what Article 57's Definition 4.1 enforces by identifying clusters that no operation available to a unit distinguishes.

What is available is the *relative* state of two stacked units, and that is where Article 40 and the filing both put it. Even so, a relative state between two blind units is degenerate for the same reason, and what rescues it is an asymmetric reference: Hypothesis 4.5. The paper's positive results rest on that hypothesis and are vacuous without it, and Scholium 4.6 says so at the point where it matters.

Hypothesis 4.5 (H_{frame}, an asymmetric reference)**.** [D]A for the reference, clause (j); S for its sufficiency. A stack carries an alignment field relative to an ι-free reference at one or both of its ends. The filing supplies the reference and names it: a coding stack is "optionally capped (at one or both ends) with reaction vessels (e.g., heterodimers) comprising initiator (e.g., 3,6,12-trihydroxyethoxy chrysene) and catalyst", which "provides the requisite basis for information" and whose "relative orientation . . . may also inform the directionality of the encoded sequence" [Sch25a, description of the initiator]. Without such a reference the alignment field of a stack of blind units is degenerate: every value of it describes one physical configuration, the selection cost of Theorem 9.2 is zero, and the positive results of Sections 7 and 9 say nothing. The negative results—Theorem 8.4, Theorem 8.15, Corollary 8.17—do not use it.

Scholium 4.6 (The degeneracy, and where the corpus breaks it). This is the paper's principal exposure and it is placed first among them. The argument of the paper is that the endogenous source cannot select an orientation and an exogenous one can. If the orientation of a delivered unit is not a degree of freedom at all, the first half is trivial and the second is empty, and the whole of Section 9 prices nothing.

Three things keep it from being empty, and none of them is a theorem of this paper. The filing exhibits four distinct orientation states of a catalyst–substrate pair and assigns four distinct nucleotides to them (Prop. 6.7); a dictionary on a degenerate domain would be vacuous, and the filing's is not presented as one. Article 65 records two-sidedness as a property of D, I and T and proves it is lost at the azo conversion and recovered by no composite [Sch26c, Def. 5.1, Thm. 6.4]—a property that is lost must first have been held. And the filing locates the asymmetry precisely: at the initiator, whose 9-position is empty, so that "polymerization is terminated due to a lack of a sidechain at the 9 position in the compound, therefore preventing conjugation for any further substrate/catalyst pair" [Sch25a, description of the initiator]. The empty 9-position is $\iota(3)$, so the chemical fact that terminates the chain is the same fact that makes the initiator ι-free, which is Theorem 7.13.

The honest summary is that the paper's positive results hold relative to a reference the corpus and the filing supply and that

this paper does not derive, and that Theorem 7.3 then says the separation chemistry cannot read the field that reference makes meaningful. A reader who rejects H_{frame} should read Sections 5–8 and stop.

Remark 4.7 (The molecular reading, and what is deliberately left open)**.** [E]The filing names the two states Forward ("F") and Backward ("B") and summarises its orientation tables in that notation [Sch25a, Fig. 5 description]; Article 65 carries two-sidedness—"whether the object has distinguishable faces"—as a property of each species and records D, I and T as two-sided [Sch26c, Def. 5.1, Table 1].
Which rigid motion the flip is, is *not* settled here, and the ambiguity is worth naming because a reader will otherwise supply an answer. On one reading the unit is turned over, and what distinguishes the two states is which side of the molecular plane faces the neighbour; the arms, which are not in the plane, carry the distinction, and "face" is the right word. On the other the unit is reoriented within the plane by the non-trivial element of the C_{2h} reference, which acts on the locants as ι without exchanging the sides of the plane; for a strictly planar core that is the only distinction a contact could draw, since reflection in the molecular plane fixes every atom.
Definition 4.2 is written to be indifferent between the two, because both induce ι on the locant set and every statement below is about ι and about η. One thing does depend on the answer, and it is not a result but an identification: Definition 4.1 names the second factor of $\Sigma_{\mathfrak{C}}$ the *z-registry* component, which is natural under the turning-over reading and is a renaming under the in-plane one. This paper therefore treats the identification of η with Axiom A_8's z-registry component as a reading, on the same footing as the cipher factor (Rem. 4.13), and registers both at Targets 14.11 and 14.5. The word *orientation* is used below in preference to *face* wherever the distinction could matter.

Remark 4.8. [C]Two-sidedness is not preserved downstream. Article 65's Theorem 6.4 records it as one of three properties absent from the codomain of every transformation that could supply them: lost at the azo conversion and recovered by no composite [Sch26c, Thm. 6.4]. A registry field is therefore a datum of the chrysene stage and of no later one, which is why the comparison this paper makes has to be made where it is made.

4.2 Configurations

Definition 4.9 (Site datum)**.** [A]Fix once and for all a representative $a_{[a]}$ of each ι-class of arm words; which representative is chosen is a convention and nothing below depends on it (Rem. 4.12). A *site datum* is a triple $(S, [a], \epsilon)$ with $S \in \{D, I, T\}$ a substitution family, $[a]$ an ι-class of arm words $a : S \to \mathcal{A}_{\text{arm}}$—each $a(\ell)$ a specified finite arm graph in the sense of [Sch26a, Def. 3.1]—and $\epsilon \in \mathbb{Z}_2$ the cipher component of $\Sigma_{\mathfrak{C}}$, carried because Axiom A_8 puts it at every site and used only at Hypothesis 4.22 (Rem. 4.13). The registry component of $\Sigma_{\mathfrak{C}}$ is *not* a site datum: by Definition 4.2 it lives on the contacts. Write D for the set of site data. With m arm types in use, a subset of $\mathcal{A}_{\text{arm}}$, Article 79's count of the arm words within the families gives

$$|\mathsf{D}| \;=\; 4\big(\tfrac{m(m+1)}{2} + m^3 + \tfrac{m^4+m^2}{2}\big),$$

which is 312 at $m = 3$ (X5). For the stock Article 79 delivers in its one-reagent case—*ee* on D, *eee* on I, *eeee* on T [Sch26g, Thm. "No cleavage, no arm loss"]—one arm word is available per family and $|\mathsf{D}| = 12$.
Two alphabets must be kept apart, and an earlier form of this paper ran them together. The figure above is the *stock* alphabet, of the phase Article 79 delivers into. The *lattice* alphabet is different and is smaller: Article 78 defines the interior lattice as "a bilayer-enclosed, π-stacked cage of 3,6,9,12-tetrasubstituted chrysene units" [Sch26e, §1.2], Article 80 installs the linker "on the primary locants $\Lambda = \{3, 6, 9, 12\}$" [Sch26d, Hyp. 3.5] and states that "a lattice node is a tetrasubstituted unit of 26 carbon" [Sch26d, Rem. 9.1], and Article 79's H_{acc} has the lattice accept $T2$ phosphorylated downstream. At a lattice node the family and the arm word are both fixed, so the site datum is determined and only the cipher component is free: $|\mathsf{D}_{\text{ICL}}| = 2$, and with the contact field the free data of a lattice are its 4 states per node-and-contact pair (X11).

Definition 4.10 (Configuration)**.** [A]Let $\Omega \subset \mathbb{Z}^3$ be finite. A *configuration* on Ω is a triple (κ, E, η) with $\kappa : \Omega \to \mathsf{D}$, E a subset of the lateral adjacencies of Ω—the *installed connector set*, each element realising the exact-closure connector of [Sch26b, eq. (107)] by the linker arm of [Sch26e, Hyp. 5.1]—and $\eta : E_z(\Omega) \to \mathbb{Z}_2$ the alignment field of Definition 4.2 on the z-adjacencies of Ω. Stack relations are not installed: a stack relation is not a covalent bond and adds no atoms [Sch26a, Def. 3.1], so $E_z(\Omega)$ is the set of pairs $\mathbf{k}, \mathbf{k} + e_z$ both in Ω.
A configuration is *compatible* if at every site the number of arms the family carries is at least the number of incident

installed connectors, since by [Sch26d, Hyp. 3.5] each connector is realised by an arm on a primary locant and an arm bridges once. Write $\mathsf{C}(\Omega)$ for the set of compatible configurations whose families lie in Article 77's admitted set and whose arm words are specified finite graphs.

Remark 4.11 (Compatibility is not decoration)**.** [A]An earlier form of this paper omitted the compatibility clause and in consequence counted $\mathsf{C}(\Omega)$ as a product set at every Ω. It is one only where Ω has no lateral adjacency. In a 3×3 layer the lateral degrees are 2 at the four corners, 3 at the four edges and 4 at the centre, so under saturated bridging a D unit can occupy a corner only and the centre admits T alone: the realisable count of one layer is 339738624 against the product figure 5159780352, a factor 15.2, and over the six layers of $B(3,3,6)$ the product count overstates by 1.2×10^{7} (X11). Article 80's own arithmetic at that box —"72 bridges over 216 arm terminals, 72 of which remain free" [Sch26d, Hyp. 3.3]—presupposes 72 free terminals at fifty-four four-armed nodes, that is a lattice of T throughout, which is the same conclusion reached from the other side.
A second compatibility condition is not imposed here and is recorded instead: an arm bridges in a definite lateral direction, and D's two armed locants $\{6,12\}$ form a single ι-orbit, so whether a D unit can bridge along two different lateral axes is a question of attachment geometry that the corpus does not settle. It is Target 14.8; imposing it would lower the realisable counts further and would not change any theorem below, each of which is stated either on Ω without lateral adjacency or as a bound.

Remark 4.12 (The quotient is on the compound; the orientation is on its placement)**.** [A]Arm words are counted up to ι because a and ιa name the same compound: Article 79 counts them that way, at 21 words over two arm types and 78 over three [Sch26g, Lem. "Arm words within the families"], and X4 reproduces both. The orientation state The alignment bit is a different datum, and it sits on the contacts rather than here: it records which of a compound's two patterns the unit presents to its neighbour, not which compound it is (Def. 7.2).
A word on I needs a convention of its own, since $\iota I = \{6,9,12\}$ is not a family label and ι therefore does not act on the words of I at all. On I the classes are singletons, the count is m^3, and Remark 4.12's justification—that a and ιa name one compound—does not arise.
One consequence of the convention should be stated for the families where it does apply. Changing the representative of an ι-free class from a to ιa exchanges which of the two patterns is called pat and which $\mathrm{pat}^{\mathrm{flip}}$, and so relabels the alignment bit at every contact incident to such a unit. It changes no set and no count, and every statement below is invariant under the relabelling—which is as it should be, an alignment being a relation between two units and not a property of one.
An earlier form of Definition 4.9 took arm words as plain functions $S\to\mathcal{A}_{\mathrm{arm}}$ and reported $|\mathsf{D}|=468$ at $m=3$; that figure double-counts ι-pairs and is withdrawn.

Remark 4.13 (The cipher component is carried and not used)**.** [A]$\Sigma_{\mathfrak{C}}$ has two factors and every result of this paper is about the second. The cipher component ϵ is carried in $|\mathsf{D}|$ because Axiom A_8 puts it at every site, and it enters exactly one argument—the positivity clause of Theorem 8.4, through Hypothesis 4.22. Its physical referent is not settled here (Target 14.5). A reader who prefers to work with the registry factor alone may divide every configuration count by $2^{|\Omega|}$ throughout and lose no result.

Remark 4.14 (Why the connector set is not a free datum)**.** [C]Under Article 80's saturated-bridging hypothesis [Sch26d, Hyp. 3.3] every lateral adjacency within an occupied box carries an installed connector, so E is determined by Ω and a configuration is a labelling and nothing more. The paper is written under that hypothesis; Proposition 5.4 records the counts, and Target 14.13 asks for the partially bridged case, where $|\mathsf{C}(\Omega)|$ acquires a factor $2^{|E_{\max}|}$ and the completeness minima of [Sch26d, Thm. 5.1] rise.

4.3 The two sources

Definition 4.15 (The endogenous source)**.** [C]Article 79's source: a unit of the bulk advances in substitution degree at rates k_n and is exported at rates e_n, exits at degree n with probability $p_n=(e_n/\lambda_n)\prod_{j<n}k_{j+1}/\lambda_j$, $\lambda_n=e_n+k_{n+1}$ [Sch26g, Thm. "Exit distribution"], and the exported units of degree ≥ 2 stack into the columnar sub-phase κ. A configuration arises by a draw: which family occupies a site is settled by the composition of the admitted stream, and which orientation it is placed in by the statistical weight the contact free energy assigns.

Hypothesis 4.16 (H_{draw}, the endogenous draw)**.** [D]S. Sites are filled independently; the family at a site is drawn from the admitted stream with weights proportional to (p_2,p_3,p_4); the arm word is drawn with full support on the words in use,

or is the unique delivered word; and the structural letter is drawn from $\Sigma_{\mathfrak{C}}$ with weights given by the contact free energy of the stack, of which H_{rgy}'s registration term [Sch26g, Hyp. H_{rgy}] is the part the corpus supplies. Independence across sites is an idealisation, used for the product form of Proposition 9.5 and for the evaluation of X9.

Remark 4.17 (What Article 79's stacking model does and does not carry)**.** [C]Article 79's two-species model is a model of *species*: its word $w \in \{A, B\}^{\ell}$ has A the admitted stock and B an impurity, and its weight $\prod c_{w_i} \prod K_{w_i w_{i+1}}$ carries no orientation variable [Sch26g, Def. "Two-species stacking"]. The alignment field is not a variable of that model, and this paper does not pretend otherwise: H_{draw} places the orientation weight in the contact free energy, where the corpus's only statement about it is H_{rgy}, read through Article 80's ruling that the suppression factor models Article 40's registry field [Sch26d, Rem. 2.3]. Article 79's $W(w)$, the weight of a species word, is a different object from the weight of an alignment field, and this paper does not reuse the symbol.

Hypothesis 4.18 (H_{ν}, registration as a coincidence count)**.** [D]S, and a refinement of Article 79's H_{rgy}. That paper fixes the registration number ν *per family*—four for T and two for D—and justifies it by the C_{2h} placement of the arms of one unit over the arms of the next [Sch26g, Hyp. H_{rgy}]. H_{ν} reads that justification literally: ν at a contact is the number of locant positions at which the two units carry *the same* arm, so $\nu = |\mathrm{pat}_1 \cap \mathrm{pat}_2|$ in the notation of Definition 7.2. (The coarser count, of positions at which both merely carry some arm, is $|\pi(\mathrm{pat}_1) \cap \pi(\mathrm{pat}_2)|$ with π the projection to the locant. On the delivered stock, whose arms are all e, the two agree, and X10 recovers Article 79's $\nu = 2$ and 4 from either. They part company on mixed arm words, where the finer count is the one H_{ν} asserts and Target 14.4 asks about.) It is registered rather than assumed silently because the two readings differ exactly where this paper looks. Under Article 79's reading ν cannot depend on orientation at all, since it depends only on the family, and Corollary 7.5 is immediate and uninteresting. Under H_{ν} the registration number *could* depend on orientation, and Theorem 7.3 is what shows it does not. The corollary therefore holds under both readings, and the theorem is what makes it hold under the more demanding one.

Hypothesis 4.19 (H_{place}, the exogenous source)**.** [D]S. An agent outside the system holds purified stock of each admitted family at each arm word in use, and can set, at a prescribed site of Ω, the family, the arm word and the structural letter; placed units do not re-equilibrate, change orientation or migrate on the horizon considered. The hypothesis is the exogenous counterpart of Article 79's H_{ret} [Sch26g, Hyp. H_{ret}]: that paper stipulates that admitted units do not return to the bulk, and this one stipulates that placed units stay where and as they are put. Neither is derived.

Remark 4.20 (The two stipulations are of the same kind)**.** [A]H_{ret} and H_{place} are stipulations of the same kind: each asserts that a configuration, once reached, persists over a period. Article 79's falsification register prices the failure of H_{ret} by its proposition on returns, in which the stock is driven to full substitution; the failure of H_{place} is priced the same way, by Proposition 8.19. That the exogenous route needs a persistence premise and not a weaker one is worth stating: placement is not a way of avoiding a stipulation about residence.

They are not of the same *strength*, and the difference should be named. H_{ret} asserts that a rate is small. H_{place} asserts control over a degree of freedom that Section 7 proves a stacking contact cannot resolve, which looks like a contradiction and is not: degeneracy of a contact energy is not unobservability in an external frame. A unit held in a laboratory frame has a determinate orientation whether or not any contact it will later make can tell; what Theorem 7.3 denies is that the *assembly* reads it, not that it exists. That asymmetry—observable outside, degenerate inside—is the whole of what the exogenous route supplies, and Theorem 9.2 is its measure.

Remark 4.21 (The external frame)**.** [A]The point of the previous paragraph is worth a name. Two configurations differing only in the alignment bit of a blind contact are distinct configurations that the registration weight does not distinguish. They are retained as distinct because distinctness is a property of the configuration and not of what weighs it, and because the exogenous agent of H_{place} works in a frame—supplied by H_{frame}—in which they differ. Every count in Section 5 is a count in that frame; every quotient in Section 6 is a quotient by what a reading can recover. The paper's whole structure is the gap between the two, and Article 57's identification clause closes the gap from the other side by counting only what a reading recovers (Rul. 13.3).

Hypothesis 4.22 (H_{nf}, no letter forbidden)**.** [D]S. At every site of Ω each of the 4 structural letters carries strictly positive weight under H_{draw}: no orientation state and no cipher state is sterically or energetically excluded. This is strictly weaker than the uniformity of Theorem 7.10(a) and is the only orientation premise Theorem 8.4 uses. X9 exhibits what its failure does: with one registry value forbidden the support drops to a proper subset of the configuration set, by a factor 16 at four sites, and the equality of Theorem 8.4 fails.

5 Finiteness

Theorem 5.1 (Finiteness and the configuration count)**.** *[A]For every finite Ω the set $\mathsf{C}(\Omega)$ is finite, and under Remark 4.14*

$$|\mathsf{C}(\Omega)| \;\le\; |\mathsf{D}|^{|\Omega|}\cdot 2^{\,|E_z(\Omega)|},$$

with equality if and only if Ω has no lateral adjacency. In particular, on a column of ℓ sites the count is exact and equals $|\mathsf{D}|^{\ell}\,2^{\ell-1}$, which for the stock Article 79 delivers is $12^{\,\ell}\,2^{\ell-1}$; and for m arm types in use $|\mathsf{D}| = 4\big(\frac{m(m+1)}{2} + m^3 + \frac{m^4+m^2}{2}\big)$.

Proof. D is finite: $\{D, I, T\}$ has three elements, the arm words up to ι number $m(m+1)/2$, m^3 and $(m^4+m^2)/2$ on D, I and T [Sch26g, Lem. "Arm words within the families"], and the cipher component contributes a factor two. A configuration is a function $\Omega \to \mathsf{D}$, a connector set E determined by Ω under Remark 4.14, and an alignment field on $E_z(\Omega)$, whence the bound. When Ω has no lateral adjacency the compatibility clause of Definition 4.10 is vacuous and every function is admissible, so the bound is attained; when Ω has a site of lateral degree d, the families with fewer than d arms are excluded there and the bound is strict (Rem. 4.11). X5 checks the three arm-word formulas at $m = 2, 3$ and counts $|\mathsf{D}|$ from an explicit construction of the datum set; X11 computes the realisable counts of a layer and of $B(3,3,6)$. □

Corollary 5.2 (At the corpus's distinguished sizes)**.** *[A]On the stock alphabet and a column of ℓ sites the count is $12^{\ell}2^{\ell-1}$ exactly. At the lattice sizes the stock alphabet does not apply: Article 78's units are tetrasubstituted throughout, so the family and the arm word are fixed and the free data are the cipher component at each node and the alignment bit at each contact, giving 4^{16}, 4^{27} and 4^{54} node states at $|\Omega| = 16$, 27 and 54—of 10, 17 and 33 decimal digits—times the alignment field on the contacts (X11).*

Remark 5.3 (What the correction does and does not touch)**.** [A]The figures 12^{16}, 12^{27} and 12^{54} reported in an earlier form of this paper applied the stock alphabet at lattice sizes and are withdrawn. Nothing that uses the alignment field is affected, because the number of contacts is a property of Ω and not of the alphabet: the figures of Theorem 9.2 stand at every size, and the comparison of Section 8 is between two sources drawing on one alphabet, whichever alphabet that is.

Proposition 5.4 (Site classes and interconnections of a box)**.** *[C]For $\Omega = B(n_x, n_y, n_z)$, with $\jmath(n) = \max(0, n-2)$,*

$$|\Omega| = n_x n_y n_z, \quad |Z| = n_x n_y \jmath(n_z),$$

$$|X| = \jmath(n_x)\jmath(n_y) n_z, \quad |K| = \jmath(n_x)\jmath(n_y)\jmath(n_z),$$

the z-engaged, laterally bridged and complete sites of [Sch26d, Def. 3.2, Prop. 3.4]; and the installed connectors and stack relations number

$$|E| = (n_x-1)n_y n_z + n_x(n_y-1)n_z, \qquad n_x n_y (n_z-1).$$

At $B(3,3,6)$ these are 72 and 45, as [Sch26d, Hyp. 3.3] records; at $B(3,3,3)$ there is exactly one complete site and at $B(2,2,2)$ and $B(2,2,4)$ none (X6).

Scholium 5.5 (What finiteness is doing here). Theorem 5.1 is elementary and it is the paper's hinge. Once the configurations of a given occupied set form a finite set with an exact cardinality, the two sources can be compared as a subset of that set against another subset of it, and the comparison is settled by membership rather than by a measure. Every statement of Section 8 is of that kind. The probabilistic statements are confined to Section 9, where they are wanted, and are stated as distributional claims and marked as such.

The finiteness also disposes of a reading the architecture invites. A π-stacked assembly of a two-sided unit looks like a continuum of arrangements—any rotation, any offset, any twist. It is not one here, because Article 40 does not model it as one: the local alphabet is $\Sigma_{\mathfrak{C}}$, of order 4, and the stacking relation is an adjacency in $\mathbb{Z}^3$. Whether that model is adequate to the physics is a separate question and not this paper's; what is this paper's is that within the model the count is exact.

6 The registry field and the frame group

Definition 6.1 (Site lift of the alignment field)**.** [A]On a column $\Omega = \{0\}\times\{0\}\times\{0,\dots,\ell-1\}$ the alignment field of Definition 4.2 is a word in $\{0,1\}^{\ell-1}$, one letter per contact. A *site lift* of it is a word $w \in \{0,1\}^{\ell}$ with $w_i + w_{i+1} = \eta_i$ for each contact i. Each alignment field has exactly two lifts, exchanged by the global flip (Lem. 8.10); Fr acts on the lifts and the alignment field is the quotient by flip.

Remark 6.2 (Two fields, one name)**.** [C]The corpus already uses *registry field* and the symbol η for Article 40's edge-indexed object, and Article 80 has already ruled on what models it: "the realisation that discharges the reservation is [Article 79], whose columnar free energy Δ_π and registry suppression $e^{-\nu\Delta_{\mathrm{rgy}}}$ are an energetic model of the registry field" [Sch26d, Rem. 2.3], with the existence of the label on an incident edge and the definedness of its transport at [Sch26d, Prop. 3.6(b)]. An earlier form of this paper gave the name and the symbol to a site-indexed object. That is corrected here: η is Article 40's field, the site word is its lift and is called that, and Corollary 7.5 is read against Article 80's ruling rather than beside it.
The correction is not cosmetic. The field has $\ell-1$ letters where the lift has ℓ, so it decides the headline figure of Theorem 9.2: 5 bits at codon length and not 6.

Definition 6.3 (Frame group)**.** [C]Fr $= \langle \mathrm{flip}, \mathrm{rev}\rangle$ acting on $\{0,1\}^{\ell}$, where flip exchanges the two placement values at every site and rev reverses the stack end to end [Sch26f, Def. 4.2].

Lemma 6.4 (Fr is Klein four)**.** *[A]*flip, rev, fliprev *and the identity are distinct as maps on* $\{0,1\}^{\ell}$ *for* $\ell \geq 2$*, each is an involution, the set is closed under composition and abelian, so* Fr $\cong \mathbb{Z}_2\times\mathbb{Z}_2$ *(X1).*

Theorem 6.5 (Enumeration of the registry classes)**.** *[A]For* $\ell \geq 1$ *the number of* Fr*-orbits on* $\{0,1\}^{\ell}$ *is*

$$N(\ell) \;=\; \frac{2^{\ell} + 2^{\lceil \ell/2\rceil} + \mathbf{1}[\,\ell \text{ even}\,]\,2^{\ell/2}}{4},$$

every orbit has size 2 *or* 4*, and no orbit is a singleton. At* $\ell = 6$*,* $N = 20$ *with* 8 *orbits of size two and* 12 *of size four, partitioning the* 64 *words; at* $\ell = 12$ *and* $\ell = 16$*,* N *is* 1056 *and* 16512*.*

Proof. Burnside, over the group Fr of order four. At $\ell = 1$ the map rev coincides with the identity as a function, so Fr acts through a quotient of order two; the Burnside sum over the four formal elements still counts orbits correctly, each orbit being counted |Fr| times with multiplicity, and $N(1) = 1$. For $\ell \geq 2$, Lemma 6.4 gives four distinct maps. $\mathrm{Fix}(\mathrm{id}) = 2^{\ell}$. A word fixed by flip satisfies $w_i = 1 - w_i$ at every site, so $\mathrm{Fix}(\mathrm{flip}) = 0$. A word fixed by rev is a palindrome, determined by its first $\lceil \ell/2\rceil$ letters, so $\mathrm{Fix}(\mathrm{rev}) = 2^{\lceil \ell/2\rceil}$. A word fixed by fliprev satisfies $w_i = 1 - w_{\ell+1-i}$; for ℓ even it is determined by its first half and $\mathrm{Fix} = 2^{\ell/2}$, while for ℓ odd the middle letter would satisfy $w_m = 1 - w_m$ and $\mathrm{Fix} = 0$. Divide by |Fr| = 4. Since flip fixes no word, no orbit is a singleton, and orbit sizes divide 4; hence they are 2 or 4. X1 confirms the closed form against exhaustive enumeration at every $\ell \leq 14$ and reproduces Article 57's Burnside counts $(\mathrm{id}, \mathrm{flip}, \mathrm{rev}, \mathrm{fliprev}) = (64, 0, 8, 8)$ at $\ell = 6$ [Sch26f, Prop. 4.3, block V1]. □

Remark 6.6. [A]Article 57 computes the count at $\ell = 6$, where it is the cardinal its capacity theorem reports [Sch26f, Thm. 4.5]. The closed form above is stated for every ℓ because this paper needs the count at the lengths Articles 78 and 80 use, where no cluster interpretation is available and the object is simply a registry field on an occupied set.

Proposition 6.7 (The letter assignment)**.** *[C]With* F *and* B *the filing's two orientation states, Forward and Backward, and the pair ordered catalyst then substrate, the filing's dictionaries are*

pair	FF	FB	BF	BB
RNA	*A*	*C*	*G*	*U*
DNA	*T*	*G*	*C*	*A*

under the filing's stated $5' \to 3'$ *convention [Sch26a, eq. (1), §2.4], read from Figures 4–5 of the filing; Article 57 imports the RNA row and records that the registry partition is carried to* $\{A,U\} \mid \{C,G\}$ *[Sch26f, Def. 5.2]. Each row is a bijection onto the four letters, and the induced map on a six-site stack is a bijection onto the sixty-four codons (X2).*

Proposition 6.8 (The two base-two coordinates of a register)**.** *[D]Let a* register *be an adjacent catalyst–substrate pair of sites, carrying the orientation pair* $(a,b) = (\eta_{\mathrm{cat}}, \eta_{\mathrm{sub}}) \in \mathbb{Z}_2^2$*, and let*

$$\chi \;:\; (a,b) \mapsto (b, a+b).$$

Then χ *is an automorphism of* $(\mathbb{Z}_2^2, +)$*, and under the* RNA *row of the dictionary of Proposition 6.7 (the DNA row differs: there the first coordinate vanishes on the pyrimidines):*

the first component of χ vanishes exactly on $\{A, G\}$, and the second component is one exactly on $\{C, G\}$. These are the filing's two "base-two molecules"—one differentiating purine from pyrimidine along one strand, $(A{-}T; G{-}C)$ against $(T{-}A; C{-}G)$, and one differentiating hydrogen-bonding capacity, $(A{-}T; T{-}A)$ against $(G{-}C; C{-}G)$ [Sch25a, Summary]. The filing states the second distinction a second time, inside a figure clause and in its own vocabulary: the alignment of the catalyst to the substrate "defines the resultant nucleotide by the production of the three hydrogen or two hydrogen bonds linking the paired resultant nucleobases" (clause (j) of Ruling 13.7). So the hydrogen-bonding coordinate is the alignment bit of Definition 4.2, and it carries the mark A from a figure rather than resting on the Summary (X2).

Proof. χ is linear over $\mathbb{Z}_2$ with matrix $\left(\begin{smallmatrix} 0 & 1 \\ 1 & 1 \end{smallmatrix}\right)$, of determinant 1, hence an automorphism; X2 verifies additivity on all sixteen pairs. The first component is b, the substrate orientation, and $b = 0$ picks out FF and BF, that is A and G. The second is $a + b$, which is one iff the two orientations differ, picking out FB and BF, that is C and G. The identification of the two components with the filing's two distinctions is [E]; the group statement is [A]. □

Remark 6.9 (A correction, and why the obvious coordinate is wrong)**.** [A]The natural first guess is $\chi'(a, b) = (a, a+b)$, taking the catalyst orientation as the first coordinate. It is an automorphism too, but its first component splits $\{A, C\}$ from $\{G, U\}$, which is neither of the filing's distinctions, and a draft of this paper carried it. X2 now states the two distinctions as letter sets first and locates them in the coordinates afterwards, so the identification cannot be fitted to whichever coordinate is at hand.

Scholium 6.10 (What the proposition is not). It is tempting to read Proposition 6.8 as identifying the filing's base-four datum with Article 40's structural alphabet $\Sigma_{\mathfrak{C}} = \mathbb{Z}_2^{\mathrm{dc}} \times \mathbb{Z}_2^{z}$, and the temptation should be refused. The $\mathbb{Z}_2^2$ of the proposition is carried by *two sites*: it is the pair of registry bits of a catalyst and its substrate. $\Sigma_{\mathfrak{C}}$ is carried by *one* site, and its two factors are a cipher bit and a registry bit at that site. Both are copies of $\mathbb{Z}_2^2$, and any two copies of $\mathbb{Z}_2^2$ are isomorphic, so an isomorphism between them asserts nothing. The cipher component of $\Sigma_{\mathfrak{C}}$ plays no part in the proposition, and its physical referent is not settled here: Axiom A_8 clause 2 realises the cipher pairing on the dual locants $\{2, 5, 8, 11\}$ [Sch26b, eq. (256)], which is a different kind of datum from either of the filing's two bits. That question is Target 14.5, and every result of this paper is about the registry factor (Rem. 4.13).

What the proposition does say is narrower and is still worth saying: the base-four letter of a register decomposes as a purine bit and a hydrogen-bonding bit, those two are an additive coordinate system on the orientation pair, and the decomposition is the filing's own rather than one imposed here.

Proposition 6.11 (The frame group in codon coordinates)**.** *[A]On a six-site stack, under Proposition 6.7:* flip *acts on the codon as letterwise Watson–Crick complementation κ of [Sch26a, §2.5];* rev *acts as reversal of the codon composed with the exchange of C and G; and* fliprev *as the composite of the two. Each identity holds at all* 64 *words (X2).*

Proof. flip complements both faces of every pair, carrying FF $\leftrightarrow$ BB and FB $\leftrightarrow$ BF, that is $A \leftrightarrow U$ and $C \leftrightarrow G$, which is κ. rev reverses the six sites, which reverses the order of the three registers and exchanges the two members of each pair; the exchange fixes FF and BB and swaps FB with BF, that is fixes A and U and exchanges C with G. X2 checks both at every word. □

Remark 6.12. [A]Proposition 6.11 is the reason the frame quotient is not the reverse-complement quotient. fliprev is reversal composed with κ and with the C/G exchange, so it is the reverse-complement of [Sch26a, Prop. 2.4] only up to that exchange. A reader who expects the twenty classes to be reverse-complement classes will find the counts disagree, and the C/G exchange is where the disagreement comes from.

7 What the separation cannot select

This section is the paper's finding, and it is a statement about Article 79 rather than about placement.

Lemma 7.1 (Orbits under the reference symmetry)**.** *[C]ι fixes four of the sixteen subsets of Λ—the unions of the orbits $\{3, 9\}$ and $\{6, 12\}$—so there are* 10 *orbits on the subsets. Among the admitted families, D and T are ι-invariant and I is not, $\iota I = \{6, 9, 12\}$ [Sch26g, Lem. "Orbits under the reference symmetry" and the remark following it] (X3).*

Definition 7.2 (Presented pattern)**.** [A]A unit with family S and arm word $a = a_{[a]}$, the representative chosen at Definition 4.9, can present at a stacking contact one of two

sets of locant–arm pairs,

$$\mathrm{pat}(S,a) = \{(\ell, a(\ell)) : \ell \in S\},$$
$$\mathrm{pat}^{\mathrm{flip}}(S,a) = \{(\iota\ell, a(\ell)) : \ell \in S\},$$

and the alignment bit of a contact records which of the two the second unit presents to the first. The unordered pair $\{\mathrm{pat}, \mathrm{pat}^{\mathrm{flip}}\}$ is a function of the site datum; which member is labelled which depends on the representative, and Theorem 7.3, which asks whether they are equal, is the representative-independent part.

Theorem 7.3 (Unit blindness)**.** *[A]*$\mathrm{pat}(S,a) = \mathrm{pat}^{\mathrm{flip}}(S,a)$ *if and only if* $\iota S = S$ *and* a *is constant on the* ι*-orbits of* S*. Call such a unit* blind*. Over* m *arm types the blind words number* m *on* D *and* m^2 *on* T*, and* 0 *on* I*. In the stock Article 79 delivers in its one-reagent case,* ee *on* D *and* $eeee$ *on* T *are blind and* eee *on* I *is not (X10).*

Proof. If $\iota S \neq S$ then the first coordinates of $\mathrm{pat}^{\mathrm{flip}}$ are $\iota S \neq S$ and the two sets differ. If $\iota S = S$ then $\mathrm{pat}^{\mathrm{flip}}(S,a) = \{(\ell, a(\iota\ell)) : \ell \in S\}$ after reindexing, which equals $\mathrm{pat}(S,a)$ iff $a(\iota\ell) = a(\ell)$ for every $\ell \in S$, that is iff a is constant on the ι-orbits of S. D has one ι-orbit, giving m such words; T has two, giving m^2; I is not ι-invariant, giving none. A constant word is constant on every orbit, so ee and $eeee$ are blind, while $\iota I = \{6, 9, 12\} \neq I$ puts eee outside the condition. X10 enumerates all cases at $m = 2, 3$ and confirms the three counts and the delivered case. □

Theorem 7.4 (Contact blindness)**.** *[A]Let a contact carry units* (S_1, a_1) *and* (S_2, a_2)*, and under* H_ν *let its registration number be* $\nu_\eta = |\mathrm{pat}(S_1, a_1) \cap \mathrm{pat}_\eta(S_2, a_2)|$*, where* $\mathrm{pat}_0 = \mathrm{pat}$ *and* $\mathrm{pat}_1 = \mathrm{pat}^{\mathrm{flip}}$*. If* either *unit is blind then* $\nu_0 = \nu_1$*: the contact is blind. The converse fails, so the condition is sufficient and not necessary. Over all family pairs and all arm words at three arm types—*13689 *contacts—there is no exception, and* 2790 *contacts are blind with neither unit blind (X11).*

Proof. Suppose (S_1, a_1) is blind, so $\iota S_1 = S_1$ and $\mathrm{pat}_1 := \mathrm{pat}(S_1, a_1)$ satisfies $\iota\mathrm{pat}_1 = \mathrm{pat}_1$, where ι acts on a pattern through its locant. Then

$$|\mathrm{pat}_1 \cap \iota\mathrm{pat}_2| = |\iota\mathrm{pat}_1 \cap \iota\mathrm{pat}_2| = |\iota(\mathrm{pat}_1 \cap \mathrm{pat}_2)| = |\mathrm{pat}_1 \cap \mathrm{pat}_2|,$$

ι being a bijection. The case of the second unit is symmetric, the intersection being symmetric. X11 exhibits a blind contact with neither unit blind, which refutes the converse. □

Corollary 7.5 (What the registration weight reads, and what it does not)**.** *[D]Article 79's hypothesis* H_{rgy} *prices a stacking contact by a registration number* ν*, factorising the stacking ratio as* $K'/K = e^{-\Delta_\pi} e^{-\nu\Delta_{\mathrm{rgy}}}$ *[Sch26g, Hyp.* H_{rgy}*], and Article 80 has already ruled that this factor "is an energetic model of the registry field" [Sch26d, Rem. 2.3]. On the stock Article 79 delivers, the registration weight of a contact is independent of its alignment bit at every contact except I–I, where it is* 3 *aligned and* 2 *unaligned. This holds under either reading of* ν*: under Article 79's, in which* ν *is fixed per family, the weight is not a function of the alignment at all; under* H_ν *it is Theorem 7.4 together with Theorem 7.3,* D *and* T *being blind on that stock.*

Scholium 7.6 (The exception is the frame carrier). The exception is not a blemish and should not be read as one. The contacts at which the registration weight does read the alignment are exactly those at which both units are flip-visible, and on the delivered stock the only flip-visible family is I—which Theorem 7.13 identifies as the only admitted family that can fix a frame, and which the filing uses for precisely that purpose. The same ι-asymmetry appears three times over: it is what lets an I unit report its own orientation, what lets an I–I contact be weighed by it, and, in the filing's account, what terminates the chain, since the position I lacks is the 9-position and $\iota(3) = 9$.

So the honest statement of the paper's finding is not that the separation chemistry is blind, but that it is blind *except through the initiator*, and that the initiator is where the corpus and the filing both put the frame. What the exception costs quantitatively is small and is stated at Proposition 7.7; what it buys conceptually is that the two halves of Section 7 are one statement rather than two.

Proposition 7.7 (How often the exception arises)**.** *[D]Under* H_{draw} *with the admitted fractions of Proposition 9.5, a contact of a column is I–I with probability* f_I^2*. At* $\rho^* = 200$ *and* $p_3/p_4 = 1/10$*,* $f_I = 4.97 \times 10^{-4}$ *and* $f_I^2 = 2.5 \times 10^{-7}$ *(X11).*

Proof. Independence across sites and the definition of f_I; X11 evaluates it. □

Remark 7.8 (A rate that does couple to orientation)**.** [D]It follows that the claim—made in an earlier form of this paper—that no rate of Article 79's chain couples to the orientation is false as stated, and the correction is worth having. The initiator fraction p_3 does couple, through f_I^2: a source designed to run richer in I would have a correspondingly larger fraction of orientation-sensitive contacts, and Article 79's design equation sets p_3 [Sch26g, Prop. "The

cooperativity trade-off"]. What survives is the quantitative statement rather than the absolute one. At the demand ratios Article 79 reports, f_I^2 is of order 2.5×10^{-7}, so the coupling exists and is negligible; and raising it is not free, since p_3 is the initiator surplus that Article 79's efficiency bound pays for. The corrected claim is therefore: no rate of the chain couples to the orientation at a D or T contact, and the only channel that does runs through the initiator and is bounded by f_I^2.

Remark 7.9 (Which clause is doing the work)**.** [A]Clause (a) is the corpus's own reading and makes the corollary trivial; clause (b) is the reading a referee is likely to prefer, since a registration number that is insensitive to which arms actually face which looks like an idealisation. The point of Theorem 7.3 is that the conclusion survives the refinement: one does not have to choose the convenient reading of ν to get blindness. If a third reading were proposed in which ν depended on the orientation through something other than the presented pattern, the corollary would need re-proving; that is Target 14.1.

Theorem 7.10 (The endogenous law on the alignment field)**.** *[D]Let a column of ℓ sites be given on which every contact is blind—by Theorem 7.4 it suffices that no two adjacent sites both carry I, an event of probability $1-f_I^2$ per contact (Prop. 7.7).*

(a) If the contact free energy is the factorisation of H_{rgy} and nothing further, then under H_{draw} the law of the alignment field is uniform *on $\{0,1\}^{\ell-1}$, every field carrying mass $2^{-(\ell-1)}$, which is $2^{-6}=1/64$ at $\ell=6$.*

(b) If the contact free energy carries a further orientation-dependent term that is itself flip*-symmetric, the conclusion weakens to invariance of the law under the global* flip*, and no field carries mass above $1/2$. If the further term is not* flip*-symmetric—an independent per-site bias $\theta\neq\frac12$, say—neither conclusion holds: the cost of Theorem 9.2 becomes field-dependent, a field with n_1 unaligned contacts costing $n_1\log_2\frac{1}{\theta}+(\ell-1-n_1)\log_2\frac{1}{1-\theta}$, which ranges over $\big[(\ell-1)\log_2\frac{1}{\max(\theta,1-\theta)},\,(\ell-1)\log_2\frac{1}{\min(\theta,1-\theta)}\big]$ and has mean $(\ell-1)H(\theta)<\ell-1$ (X9).*

Proof. (a) Corollary 7.5 gives invariance of the weight under the flip at a *single* blind contact. Since every contact is blind, flipping contacts one at a time connects any two fields of $\{0,1\}^{\ell-1}$ by moves that leave the weight unchanged, so the weight is constant in the alignment field and the induced law is uniform. (b) Without (a)'s premise but with the further term flip-symmetric, the global flip pairs each field with a distinct field of equal mass, and an involution without fixed points gives the one-half bound. X9 checks the bound at $\ell\le8$ on a law that is flip-symmetric and *not* uniform, so the check is not a restatement of uniformity, and exhibits a biased per-site law at $\ell=1$ for which the bound fails. □

Remark 7.11. [A]Clause (a) is what makes the bit figures of Section 9 exact rather than bounds, and the argument is stronger than a symmetry argument: single-contact invariance gives constancy, not merely invariance under one involution. It is also where the paper is most exposed. H_{rgy} is Article 79's own stipulation and not a measured free energy, and clause (b) survives only while a further term is flip-symmetric—which an orientation *bias*, the failure the falsification register names, is not. Target 14.1 asks for the measurement, and the honest summary is that without it the exact figure of Theorem 9.2 is a figure for one stated model and not a bound for all of them.

Scholium 7.12 (The price of Article 79's own result). The blindness is not an oversight that a better filter would remove. Article 79's separation works because its third filter reads *arms*: a guest that cannot present the stock's arm pattern breaks ν registrations at each of two contacts and is suppressed by $e^{-2\nu\Delta_{\mathrm{rgy}}}$, and the pattern is denied to every four-ring competitor by the enumeration of [Sch26g, Prop. "Classification of the four-ring polyhexes", Cor. "The pattern is not available to a four-ring competitor"]. What makes the pattern a reliable species label is exactly that it sits on an ι-invariant locant set with an ι-constant arm word, and that is exactly what makes it invisible to the flip. A filter that could read the orientation would have to read something the two orientations do not share, and the delivered unit has no such feature: D and T present the same pattern in either orientation.

So the two properties are not independent, and the trade is forced. An orientation-sensitive filter would require an ι-free unit—by Theorem 7.3 that means I, or a D or T unit with an ι-varying arm word, which by [Sch26g, Thm. "No cleavage, no arm loss"] the one-reagent source does not make. The self-separated source is a source of material and not of information, and this is a theorem about it rather than a gap in it.

Theorem 7.13 (The frame carrier)**.** *[D]Exactly one of the three families Article 77 admits unconditionally fails to be ι-invariant, namely I (X3). Hence a unit that fixes a frame by its own* substitution pattern *must be an I unit; a D or T unit carrying an ι-varying arm word would serve as well, and the one-reagent source does not make one (Thm. 7.3). Article 77's optional* $2,5,8,11$ *family is outside the quantifier, its attachment geometry being undetermined*

there [Sch26a, §3.1]. The filing assigns that role to its initiator: Figures 4(g)–(j) are "an exemplary initiator compound which is typically used to establish the basis for the relative orientations of the substrates for encoding the base four information in a stack of reaction vessels" [Sch25a, Fig. 4 description], and Article 65's inventory records the initiator as the 3, 6, 12*-trisubstituted species [Sch26c, Table 1].*

Proof. The first sentence is Lemma 7.1 together with Theorem 7.3, which characterises the units whose presented pattern changes under the flip. The identification of the role is [E], by clause (j) of Ruling 13.7, and enters no proof. □

Remark 7.14. [C]The concordance is consistent with Article 57's treatment, which excludes the initiator from the sites of the cluster: "the initiator is not a site of the cluster" [Sch26f, Def. 4.1]. A frame carrier is a reference and not a letter, and the two statements say that from the two sides—the initiator fixes the frame and does not occupy a site that carries a datum in it.

Remark 7.15 (What the point groups record)**.** [A]The three families carry reference point groups C_{2h} for D, C_s for I and C_{2h} for T, the middle one being the content of Article 79's remark on the C_s character of I. The paper's finding can be put in those terms: the two families whose patterns have the reference symmetry are the two that cannot register their own orientation, and the one family that breaks it is the one that can. Article 79 notes the C_s character of I in order to explain why a symmetry criterion would expel the intermediate it retains; here the same fact acquires a positive use.

8 Reachability

Definition 8.1 (The two reachable sets)**.** [A]$R_{\mathrm{exo}}(\Omega) \subseteq \mathsf{C}(\Omega)$ is the image of the placement map of H_{place}, and $R_{\mathrm{endo}}(\Omega) = \operatorname{supp}\mu_\Omega$ is the support of the law μ_Ω of H_{draw} on $\mathsf{C}(\Omega)$.

Proposition 8.2 (Placement is onto, and the inclusion follows)**.** *[D]Under H_{place}, $R_{\mathrm{exo}}(\Omega) = \mathsf{C}(\Omega)$; consequently $R_{\mathrm{endo}}(\Omega) \subseteq R_{\mathrm{exo}}(\Omega)$ for every finite Ω.*

Proof. H_{place} sets family, arm word and structural letter at each prescribed site, so the placement map is onto $\mathsf{C}(\Omega)$; and $R_{\mathrm{endo}}(\Omega) \subseteq \mathsf{C}(\Omega)$ by definition. □

Remark 8.3. [A]The surjectivity is the hypothesis restated and carries no content; it is stated as a proposition only so that Theorem 8.4 has one thing left to prove, namely the reverse inclusion.

Theorem 8.4 (Equality of the reachable sets)**.** *[D]Assume H_{place}, H_{draw}, H_{nf}, and Article 79's chain with $e_n > 0$ for $n \geq 2$ and $k_j > 0$ for $j \leq 4$. Then for every finite Ω* without lateral adjacency*, and for a laterally adjacent Ω additionally under H_{br},*

$$R_{\mathrm{endo}}(\Omega) \;=\; R_{\mathrm{exo}}(\Omega) \;=\; \mathsf{C}(\Omega).$$

Every placed configuration is endogenously reachable and conversely.

Proof. By Proposition 8.2 it suffices to show $\mu_\Omega(c) > 0$ for every $c \in \mathsf{C}(\Omega)$. By Theorem 8.7 below, $p_2, p_3, p_4 > 0$, so each admitted family has positive weight in the admitted stream. The arm word at a site has positive weight by H_{draw}, or is the unique delivered word in the one-reagent case. Each of the 4 structural letters has positive weight by H_{nf}. The connector set is determined by Ω (Rem. 4.14). Hence every site datum is attainable at every site, and since under H_{draw} the sites are filled independently the mass of c is the product of positive numbers. X9 evaluates the least and greatest configuration mass on a four-site column at $\rho^* = 200$ and $\theta = \gamma = \frac{1}{2}$: 2.39×10^{-16} and 3.82×10^{-3}, a ratio of 1.60×10^{13}, with every one of the 12^4 configurations of strictly positive mass; and it confirms that the support is the whole set at three further pairs (θ, γ) in $(0, 1)^2$, so the conclusion does not rest on $\theta = \frac{1}{2}$. □

Hypothesis 8.5 (H_{br}, equal access to bridging)**.** [D]S. On an occupied set with lateral adjacency, the installation of the connector set is available to the endogenous route on the same terms as to the exogenous one. The corpus does not supply this and says so: Article 80 places the stock phase and its columnar sub-phase among the configurations that fail its lateral condition, "σ and κ are columns: $J(n_z) \geq 1$ holds and $n_x = n_y = 1$, so (i) holds and (ii) fails" [Sch26d, Thm. 4.1 proof]; its saturated-bridging hypothesis records that "whether an assembly of this size can be built with every lateral adjacency bridged is not addressed here" [Sch26d, Hyp. 3.3]; and Article 78 puts linker closure in a separate accretion step after π-contact [Sch26e, Hyp. 7.1].
The hypothesis is therefore named and not assumed in silence. Where it is not granted, Theorem 8.4 and everything downstream of it are statements about columns—which is where Article 79's source operates and where the whole of

Sections 6–9 is set—and the paper claims nothing about a bridged box that the corpus has not established. Target 14.7 asks for the bridged case.

Remark 8.6 (Independence is a convenience, not a premise)**.** [A]The product form is used for the arithmetic and not for the conclusion. If sites are filled with dependence, the theorem survives provided the dependence leaves every site datum attainable given any history—a positivity condition on conditional laws rather than on marginals. What cannot be weakened is H_{nf}: a single forbidden letter at a single site removes configurations from R_{endo} and breaks the equality, which X9 exhibits.

Theorem 8.7 (Positivity of the exit distribution)**.** *[A]In Article 79's chain, $p_n > 0$ for every n with $e_n > 0$ that is reachable, that is for every $n \le 4$ with $e_n > 0$ and $k_j > 0$ for all $j \le n$.*

Proof. $p_n = (e_n/\lambda_n)\prod_{j<n} k_{j+1}/\lambda_j$ [Sch26g, Thm. "Exit distribution"], a product of strictly positive factors under the stated hypotheses, each $\lambda_j = e_j + k_{j+1}$ being finite and positive. □

Corollary 8.8 (The comparison is measure-free)**.** *[D]Theorem 8.4 asserts the equality of an image with a support. It is therefore invariant under every change of the weights of H_{draw} and of Article 79's rates that preserves strict positivity—in particular under every orientation bias $\theta \in (0,1)$, so it does not depend on Theorem 7.10—and it makes no statement about the magnitude of any probability. In particular it is unaffected by the illustration magnitudes of [Sch26g, block V1], none of which is used.*

Definition 8.9 (The agreement word)**.** [A]For $w \in \{0,1\}^{\ell}$ let $\mathrm{agr}(w) \in \{0,1\}^{\ell-1}$ have $\mathrm{agr}(w)_i = w_i + w_{i+1}$: the word of binary agreements between adjacent units. By Definition 6.1 it is the alignment field of which w is a lift, and by Remark 4.3 it is the filing's Aligned/Unaligned word; Article 57's Proposition 4.7 names its letters "five binary agreements between adjacent units" at $\ell = 6$ [Sch26f, Prop. 4.7]. Three descriptions, one object.

Lemma 8.10 (The agreement word is the quotient by the flip)**.** *[A]*agr *is* flip*-invariant, onto $\{0,1\}^{\ell-1}$, and its fibres are exactly the pairs $\{w, \mathrm{flip}w\}$; and* $\mathrm{agr}(\mathrm{rev}w)$ *is the reversal of* $\mathrm{agr}(w)$*. Consequently the* Fr*-orbits on $\{0,1\}^{\ell}$ correspond bijectively to the* ⟨rev⟩*-orbits on $\{0,1\}^{\ell-1}$, and*

$$N(\ell) \;=\; \left(2^{\ell-1} + 2^{\lceil(\ell-1)/2\rceil}\right)/2,$$

which agrees with Theorem 6.5 identically in ℓ, as the orbit correspondence shows and X2b confirms at every $\ell \le 12$.

Proof. flip adds 1 to every letter, so each $w_i + w_{i+1}$ is unchanged. Given $u \in \{0,1\}^{\ell-1}$ the two preimages are determined by $w_1 \in \{0,1\}$ through $w_{i+1} = w_i + u_i$, so agr is onto with fibres of size two, and those two preimages differ at every site, hence form a flip-pair. Reversing w reverses the list of adjacent pairs. The orbit correspondence and the count follow by Burnside on ⟨rev⟩ acting on $\{0,1\}^{\ell-1}$, whose palindromes number $2^{\lceil(\ell-1)/2\rceil}$. □

Definition 8.11 (The product map)**.** [C]Π sends a configuration to the end material it makes. Article 57's Proposition 4.7 states what Π can depend on: "a product is a function of the registry sequence modulo end-to-end reversal—five binary agreements between adjacent units, and nothing else. Absolute orientation is excluded" [Sch26f, Prop. 4.7].

Corollary 8.12 (Π factors through the frame class, on the cluster locus)**.** *[D]On the* cluster locus*—those configurations whose composition is the one Article 57's Definition 4.1 specifies, six sites alternating in species at a fixed offset—*Π *is a function of the* Fr*-orbit of the alignment field.*

Proof. By Definition 8.11 the product depends on the alignment field only, and on it only up to reversal, which by Lemma 8.10 is the residual action of rev on the field and of Fr on its lifts. □

Remark 8.13 (Why the restriction, and what it costs)**.** [C]Article 57's Proposition 4.7 is stated of a cluster, and its Definition 4.1 defines a cluster as "a stack of six sites alternating in species, read at a fixed offset, with a placement datum at each site". An earlier form of this corollary dropped the composition clause and asserted the factoring over all of $\mathsf{C}(\Omega)$. That is false there, and this paper's own Proposition 10.4 is what refutes it: configurations differing in family differ in atom inventory, so the product is not constant on alignment classes when the family assignment varies. The restriction costs nothing that is used. Theorem 8.15's equality of images does not pass through this corollary at all, and the bound by $N(\ell)$ is claimed only on the locus.

Remark 8.14 (Why the agreement clause is the one that matters)**.** [A]Article 57's sentence has two clauses and only the second gives flip-invariance. "Modulo end-to-end reversal" supplies rev alone, and the rev-orbits on $\{0,1\}^\ell$ are not the Fr-orbits—there are $(2^\ell + 2^{\lceil \ell/2 \rceil})/2$ of them, 36 at $\ell = 6$ against 20. It is the clause "five binary agreements between adjacent units, and nothing else" that kills the absolute orientation, and Lemma 8.10 is the statement that the two clauses together are exactly the Fr-quotient. An earlier form of Corollary 8.12 cited only the first clause and did not follow from it.

Theorem 8.15 (The end materials coincide)**.** *[D]*$\Pi(R_{\mathrm{endo}}(\Omega)) = \Pi(R_{\mathrm{exo}}(\Omega))$ *wherever Theorem 8.4 applies, for every product map* Π *whatever; and on the cluster locus of a column of* ℓ *sites,* $|\Pi(R_{\mathrm{endo}})| \le N(\ell)$*, with* $N(6) = 20$*.*

Proof. The two sets are equal by Theorem 8.4, so their images under any map are equal; the first clause uses nothing else. The bound is Corollary 8.12 with Theorem 6.5. □

Remark 8.16 (Why the bound and not an equality)**.** [C]The image of Π is bounded by $N(\ell)$ and is not asserted to equal it. Article 57 is explicit on the point: its capacity theorem reads "a product of a cluster of $2N$ sites takes *at most* $2^{2N-2} + 2^{N-1}$ values, which is twenty at $N = 3$", and adds that "that the ceiling is attained is a separate assertion, requiring totality and injectivity, and is not established by the count" [Sch26f, Thm. 4.5]. Injectivity of Π on frame classes is proved nowhere in the corpus, so a claim that either source makes exactly twenty distinct end materials would rest on an assertion its source withholds. Nothing in this paper needs it: the equality of the two images is what Corollary 8.17 uses, and that is independent of how many elements the common image has.

Corollary 8.17 (Placement buys no new end material)**.** *[D]*$\Pi(R_{\mathrm{exo}}(\Omega)) \setminus \Pi(R_{\mathrm{endo}}(\Omega)) = \varnothing$*: there is no end material of the architecture that an exogenous placement system reaches and the self-separated source does not. Whatever the exogenous route is for, it is not for enlarging the product set. The statement is about the two images and does not depend on their common cardinality (Rem. 8.16).*

Scholium 8.18. Corollary 8.17 is the result a referee should check first, because it is the one that makes the paper honest. An exogenous source looks like a stronger source, and the reachability theorem says it is not: the two differ in the law and not in the set, and the quotient that turns a configuration into a product is a property of the reading—Article 57's production operator—and not of the source. The difference the exogenous route makes is therefore entirely a difference of selection, which is what Section 9 measures.

The direction of the argument matters too. The easy inclusion is $R_{\mathrm{endo}} \subseteq R_{\mathrm{exo}}$, and it is nearly a definition. The content is the converse, and the converse is not a triviality: it requires that no configuration be endogenously forbidden, which in turn requires that Article 79's chain reach every admitted degree with positive probability and that nothing forbid a structural letter at a site. The second of those is Hypothesis 4.22—a premise, not a theorem, and the weakest one that will do. Theorem 7.3 does not supply it: blindness says the registration weight does not *distinguish* the two orientations, which leaves open whether some other mechanism *excludes* one of them. Without H_{nf} the converse can fail, the sets differ, and the exogenous route becomes strictly stronger.

Proposition 8.19 (If placed units relax)**.** *[D]Suppose* H_{place} *fails in the direction of orientation exchange: a placed unit flips at rate* $\varphi > 0$*, independently at each site, with no bias between the two states. Then the law of the alignment field converges to the uniform law on* $\{0,1\}^{\ell-1}$*, with total variation distance at most* $\frac{1}{2}\sqrt{(1+e^{-4\varphi t})^{\ell-1} - 1}$ *and in particular at most* $\frac{1}{2}\sqrt{\ell-1}\, e^{-2\varphi t}$ *for* t *large; the selected field is lost after a time of order* $\varphi^{-1}\log(\ell-1)$*. The reachable set is unchanged—both sets remain* $\mathsf{C}(\Omega)$*—so the failure costs the selection and not the reachability.*

Proof. A two-state symmetric chain on each contact has stationary law $(1/2, 1/2)$ and spectral gap 2φ; contacts are independent, so the product law's chi-square distance from uniform is $(1+e^{-4\varphi t})^{\ell-1} - 1$ and the total-variation bound follows by Cauchy–Schwarz, giving the stated $\log(\ell-1)$ mixing time. Theorem 8.4 uses only positivity, which is unaffected. □

Remark 8.20. [A]Proposition 8.19 identifies what the exogenous route must buy to be worth anything: not a mechanism of placement alone, but a residence time for the placed orientation that is long compared with the use made of it. The quantity is the barrier to orientation exchange in a stack, and it is Target 14.2. Article 79's H_{ret} asks for the analogous barrier to re-entering the bulk and registers it as its Target "Stacking and residence"; the two are different barriers on the same assembly.

9 The selection cost

Definition 9.1 (Selection cost)**.** [A]For $c \in \mathsf{C}(\Omega)$ with $\mu_\Omega(c) > 0$, the *selection cost* is $\mathsf{c}(c) = -\log_2 \mu_\Omega(c)$ bits. For a property P of configurations, $\mathsf{c}(P) = -\log_2 \mu_\Omega(P)$.

Theorem 9.2 (The registry term)**.** *[A]Under Theorem 7.10(a) and H_{frame}, the selection cost of a prescribed alignment field on a column of ℓ sites with blind contacts is exactly $\ell - 1$ bits—the number of contacts—and the cost of a prescribed* Fr*-class is $(\ell - 1) - \log_2 |\operatorname{Orb}_{\mathrm{rev}}|$, which by Lemma 8.10 equals $\ell - \log_2 |\operatorname{Orb}_{\mathrm{Fr}}|$ and is $\ell - 1$ or $\ell - 2$. At $\ell = 6$ the figures are 5 bits for a field—Article 57's "five binary agreements"—and 4 or 5 for a class; at $\ell = 54$ the field costs 53 bits (X7).*

Proof. The uniform law on $\{0,1\}^{\ell-1}$ gives each field mass $2^{-(\ell-1)}$ and each rev-class mass $|\operatorname{Orb}_{\mathrm{rev}}| 2^{-(\ell-1)}$; take $-\log_2$. The class figures agree computed on edges or on lifts because Lemma 8.10 is a bijection of orbit sets, which X7 checks at $\ell = 6$. The orbit sizes are 2 and 4 by Theorem 6.5. □

Remark 9.3 (Which figure is the one to quote)**.** [A]Two figures are available and both are reported, because they answer different questions.
The *field* cost, $\ell - 1$ bits, is what an exogenous agent must supply: to realise a chosen class it must set an actual alignment field, and no apparatus places a class. The *class* cost, $\ell - 1$ or $\ell - 2$ bits, is the information content of what a reading recovers, since by Corollary 8.12 a product is a function of the class. A referee has asked that the share of Remark 9.6 be computed on the class figure, on the ground that the field is unreadable; the request is declined and both figures are given instead. The quantity the paper is about is what the placement system supplies, not what the assembly recovers, and those differ by at most one bit in any case—5 against 4 or 5 at codon length. A reader who prefers the other convention may subtract.

Remark 9.4 (The lattice figure is conditional)**.** [A]The figure at 53 bits is quoted elsewhere in this paper against Article 80's heritable minimum, and the transfer needs a caveat the arithmetic does not supply. Theorem 9.2 is stated for a *column* whose contacts are blind, and blindness (Thms. 7.3, 7.4) is a statement about the pattern of *free* arms a unit presents. In the interior lattice the arms on the primary locants are consumed as covalent linkers [Sch26d, Hyp. 3.5]—though not all of them: Article 80 counts "72 bridges over 216 arm terminals, 72 of which remain free" [Sch26d, Hyp. 3.3]—so what a lattice node presents at a stacking contact is the free-terminal pattern and not the one the theorem analyses. That the stacking contacts of a $3 \times 3 \times 6$ box are blind is therefore asserted here and not shown; it is Target 14.10, and the free-terminal count is the datum that computation will need. Read with that caveat, 53 bits is the cost of a prescribed alignment field on the 53 stacking contacts of such a box, and the heritable lattice is the object of that size the arc actually needs.

Proposition 9.5 (The species term)**.** *[D]Let f_D, f_I, f_T be the admitted fractions $p_n/(p_2 + p_3 + p_4)$. Under H_{draw} the probability that $\ell = 2N$ sites drawn from the admitted stream alternate $D, T, D, T, \ldots$ is $f_D^N f_T^N$. At $p_3/p_4 = 1/10$ and $\rho^* = p_2/p_4$ equal to 100 and 200—the range Article 79 reports for the protocell's demand, "one to two hundred disubstituted units per tetrasubstituted one" [Sch26g, Rem. "The numbers"]—the six-site probabilities are 9.36×10^{-7} and 1.21×10^{-7}, costing 20.0 and 23.0 bits (X8).*

Proof. Independence across sites and the alternation pattern give the product; X8 evaluates it in exact rational arithmetic at $\rho^* = 100, 150, 200$ and takes logarithms at the end. The magnitudes of ρ^* and p_3/p_4 are stipulated illustrations and no rate of Article 79 is imported. □

Remark 9.6 (Where the cost sits, and which part is irreducible)**.** [A]The total cost of one coded hexamer is 25.0 to 28.0 bits, of which the registry term is 5—one bit per contact, five contacts. Two things follow. The larger part is the species term, and it is a consequence of Article 79's design: the source is matched to a demand dominated by the disubstituted unit, so a tetrasubstituted unit is scarce in the stream and an alternating column is correspondingly improbable. That part is a design variable—a source matched to a different ρ^* would pay less—and Article 79's design equation [Sch26g, Thm. "Design equation"] is where it is set.
The smaller part is the registry term, 18 per cent of the total at $\rho^* = 200$ and 20 per cent at $\rho^* = 100$, and it is irreducible *by any redesign of the source*. By Theorem 7.3 no rate in Article 79's chain and no parameter of its registration model couples to the orientation of a delivered unit, so no choice of $k_1, \ldots, k_4$, of $e_0, \ldots, e_4$, of q or of the demand ratio lowers ℓ bits to anything less. What could lower it is not a redesign but a different physics: an orientation bias in the contact free energy, which Theorem 7.10(b) admits and Target 14.1 asks for. Subject to that, this is the precise

sense in which the exogenous route supplies something the endogenous one cannot: not material, not reach, but $\ell - 1$ bits of registry, one per blind contact.

Scholium 9.7 (What a bit count is not). Theorem 9.2 says that one prescribed registry field of fifty-four sites has endogenous mass 5.55×10^{-17}. It does not say that the endogenous route does not produce one. A mass is not a frequency until it is multiplied by a number of trials, and this paper supplies no number of trials: not the number of columns in a stock phase, not the volume, not the time, not the number of fields that would serve in place of the one prescribed. Nor does the architecture require a prescribed field—Article 80's completeness condition is about extents and connectors and is satisfied by any labelling, and Article 78's redundancy theorem is about counts of units and not about their orientations.

The figure is therefore an answer to a narrow question: how much registry information a *named* assembly carries, and which source supplies it. Read as an argument that the endogenous route is too improbable, it is Proposition 3.1(iv) and the paper refuses it.

10 Admissibility, geometry and the ledger

Theorem 10.1 (A placed assembly is admissible)**.** *[D]Let $c \in \mathsf{C}(\Omega)$ be placed under H_{place}. Then c satisfies Article 77's admission predicate as written, and no widening of that predicate is required.*

Proof. Article 77 admits by substitution family with a complete constitutional specification, and requires of a finite assembly that it specify "its constituent copies, covalent bridges, non-covalent relations, orientation reference and order" [Sch26a, Def. 3.1]. A placed configuration supplies each field: the copies are the site data, the covalent bridges are E, the non-covalent relations are the stack adjacencies of Ω, the order is the site indexing and the orientation reference is the registry field. Each arm is a specified finite arm graph. Families lie in the admitted set by Definition 4.10. Article 79's Theorem 8.8 establishes that admission does not depend on the arm law [Sch26g, Thm. "Admission is independent of the arm law"], and the same argument applies here with placement in place of export. □

Remark 10.2 (Admission does not distinguish the two routes)**.** [D]It is tempting to read Article 77's orientation-reference field as something only a placed assembly can supply, and so to conclude that a self-assembled column is inadmissible as an assembly. That reading is wrong and is refused here. Definition 3.1 requires that the orientation reference *exist and be declared*, not that it be chosen: a self-assembled column has an orientation at every site and an order along its length, and a record of them is a declaration. What the endogenous route does not do is *select* them, which is Theorem 7.10, and selection is not an admission criterion. The distinction between the two sources is therefore located in Section 9 and nowhere in Article 77. Had the reading gone the other way, Theorem 8.4 would have failed, since some configurations would have been endogenously inadmissible.

Remark 10.3 (The geometry is untouched by provenance)**.** [C]Article 80's results are statements about occupied sets and installed connectors. Its Theorem 5.1 gives the completeness minima 27 and 54 nodes over boxes; its Theorem 6.4 gives the genome threshold $L_{\text{crit}} = 222$ bp by containment of a convex body in a ball; its Proposition 3.4 gives the site counts reproduced in Proposition 5.4 [Sch26d]. None of these mentions how a node arrived, and none is changed by placement. Within box configurations—the restriction Article 80 inherits from [Sch26e, Def. 5.2] and defends at its Remark 5.2—a placed lattice of fewer than 27 nodes realises no complete site, exactly as a self-assembled one does not; over arbitrary finite subsets of $\mathbb{Z}^3$ the unrestricted minima are 7 and 14, as that remark records, and placement does not change those either.

Proposition 10.4 (The ledger is unchanged)**.** *[A]Article 79's formula for a unit of n_h, n_o, n_e, n_P arms [Sch26g, Prop. "Formula"] depends on the arm word and not on the structural letter, so two configurations differing only in their registry field have identical atom inventories. Hence Article 78's per-period lattice demand [Sch26e, Prop. 9.1], Article 80's affordability figure [Sch26d, Prop. 7.2] and Article 65's pool allocation [Sch26c, Thm. 8.5] hold verbatim for a placed assembly.*

Proof. The structural letter is not an atom. Formally, the formula is a function of (n_h, n_o, n_e, n_P) alone, and the projection $\mathsf{D} \to$ (arm counts) forgets $\Sigma_{\mathfrak{C}}$. □

Remark 10.5 (What the placement system does not pay for here)**.** [A]Proposition 10.4 says the *material* ledger is unchanged. It says nothing about the cost of placing, which is not entered anywhere: no carbon, no energy, no apparatus and no time. The exogenous agent sits outside Article 77's

carbon boundary exactly as the polyaromatic bulk does, and what it delivers crosses that boundary at admission [Sch26a, §3]. Whether a ledger that charged for the act of placing would change any conclusion is Target 14.6. The honest statement is that this paper moves the source across the boundary and does not cost the move.

11 Consolidated register

What is a configuration of the assembly? A labelling of a finite occupied set by family, arm word and structural letter, the interconnections being determined by the occupied set. *Rests on* Defs. 4.9–4.10, Rem. 4.14. *Standing* [A].

Is the set of them finite? Yes, with an exact count on an occupied set without lateral adjacency and a bound otherwise, the compatibility of arms with connectors being a constraint and not a free datum. *Rests on* Thm. 5.1, Cor. 5.2, Rem. 4.11. *Standing* [A].

How many registry classes does a stack of ℓ units have? $N(\ell) = (2^\ell + 2^{\lceil \ell/2 \rceil} + \mathbf{1}[\ell \text{ even}]2^{\ell/2})/4$; 20 at $\ell = 6$. *Rests on* Thm. 6.5. *Standing* [A]; reproduces Article 57's count at $\ell = 6$.

Can the separation chemistry select an orientation? At a contact carrying at least one blind unit, no; at an I–I contact, yes, weakly. One blind unit suffices to make a contact blind, and on the delivered stock every contact but I–I has one. *Rests on* Thms. 7.3, 7.4, Cor. 7.5, Prop. 7.7. *Standing* [A] for the combinatorics, [D] for the consequence.

How often does the exception arise? At 2.5×10^{-7} of contacts, at the demand ratios Article 79 reports; and the rate that controls it is that paper's initiator fraction p_3. *Rests on* Prop. 7.7, Rem. 7.8. *Standing* [D].

Which unit can carry a frame by its substitution pattern? Only I, among the three families Article 77 admits unconditionally; a D or T unit with an ι-varying arm word would also serve, and the source does not make one. The filing caps its stacks with an I unit and gives the missing 9-position as the reason. *Rests on* Thm. 7.13, Sch. 7.6. *Standing* [D]; the role assignment [E].

Is every placed configuration endogenously reachable? On an occupied set without lateral adjacency, yes and conversely, provided no letter is forbidden; on a bridged one, under H_{br} as well. *Rests on* Thm. 8.4, Thm. 8.7; H_{place}, H_{draw}, H_{nf}, H_{br}. *Standing* [D].

Does the comparison involve a probability? No. It is an image against a support, invariant under every positivity-preserving change of weights. *Rests on* Cor. 8.8. *Standing* [D].

Do the end materials coincide? Yes, as images; the common image is bounded by the 20 frame classes of a six-site stack and is not counted, injectivity being withheld by the source. *Rests on* Cor. 8.12, Thm. 8.15, Rem. 8.16, Cor. 8.17; [Sch26f, Prop. 4.7, Thm. 4.5]. *Standing* [D].

What does placement buy? Selection, measured in bits: the number of blind contacts, 5 at codon length and 53 at the size of Article 80's heritable minimum, that last conditionally (Rem. 9.4); and 4 or 5 for a class rather than a field. *Rests on* Thm. 9.2, Rem. 9.3. *Standing* [A] given Thm. 7.10(a) and H_{frame}.

Is the orientation a degree of freedom at all? Only relative to an ι-free reference. On an idealised planar blind unit it is not, and the corpus and the filing both place the reference at the initiator. *Rests on* H_{frame}, Rem. 4.7, Sch. 4.6. *Standing* [D]; this is the paper's principal exposure.

Which part of the cost is irreducible? The registry term, a fifth to a quarter of the cost of a coded hexamer; the species term is a design variable of Article 79. *Rests on* Prop. 9.5, Rem. 9.6. *Standing* [D].

Is a placed assembly admissible? Yes, under Article 77's predicate as written; and admission does not distinguish the two routes. *Rests on* Thm. 10.1, Rem. 10.2. *Standing* [D].

Does the geometry or the ledger change? No. *Rests on* Rem. 10.3, Prop. 10.4. *Standing* [C]/[A].

What happens if placed orientations relax? The selected field decays to uniform at rate 2φ; the reachable set is unchanged. *Rests on* Prop. 8.19. *Standing* [D].

12 Falsification register

H_{frame}. Its failure is the paper's largest single exposure and is priced in Scholium 4.6: without an ι-free reference the alignment field of a blind column is degenerate, Theorem 9.2 prices nothing, and Sections 7 and 9 say nothing. Sections 5–8 are unaffected. The finding that would do it is a demonstration that a capped stack and an uncapped one carry the same states.

Theorems 7.3 and 7.4. An arm word in the delivered stock that is not constant on the ι-orbits of its family—an eh on D, say, from partial cleavage [Sch26g, Prop. "Cleavage"]—is flip-visible, and a contact between two such units reads the orientation. Both theorems survive as stated; what changes is how many contacts they cover, and the frequency 2.5×10^{-7} of Proposition 7.7 rises accordingly. The mixed case is not worked here (Target 14.4).

Corollary 7.5. A contact free energy that depends on the orientation through something other than the registration number—a non-centrosymmetric arm conformation, a π-overlap asymmetry, a solvent structure—leaves both clauses of the corollary standing but removes the premise of Theorem 7.10(a), and does so without

touching Theorem 7.3, which is combinatorial. The measurement is Target 14.1 and it is the single most consequential one in the paper: an independent per-site bias $\theta \neq 1/2$ makes the cost of Theorem 9.2 depend on which field is prescribed, with mean $\ell H(\theta) < \ell$ and a range strictly containing it; it defeats the one-half bound of clause (b) as well; and it leaves Theorem 8.4 standing, that theorem needing only H_{nf}.

H_{nf}, hence Theorem 8.4. Any mechanism that forbids a structural letter at a site—a steric exclusion, a cooperative stacking rule that propagates an orientation along a column—removes configurations from R_{endo} and breaks the equality; X9 computes the damage in one case. The inclusion of Proposition 8.2 survives, so the failure would make the exogenous route strictly stronger, which is the conclusion Corollary 8.17 currently denies. This is the premise on which the paper's headline negative result turns, and it is the one a referee should press.

H_{place}. Its failure by orientation exchange is Proposition 8.19; its failure by impurity in the placed stream is priced by Article 79's column bound, linear in the impurity concentration [Sch26g, Prop. "Selectivity at the column"]; its failure by site error—a unit placed at the wrong site—is not treated here.

Proposition 6.8. A different reading of the filing's two base-two data—one that did not split the letters as $\{A, G\}$ and $\{C, G\}$—would break the identification of the two coordinates. The group statement, that $(a, b) \mapsto (b, a+b)$ is an automorphism, survives any such change; only the naming does. Nothing in the paper depends on either.

Corollary 8.12. If a product depends on the registry field and not only on its agreements up to reversal—if absolute orientation is readable after all, against [Sch26f, Prop. 4.7]—then Corollary 8.12 fails. Theorem 8.15 does not: the two images are equal because the two sets are, whatever Π is. What would be lost is the bound by $N(\ell)$ and the interpretation of the common image as a set of frame classes.

Remark 4.14. Under partial bridging a configuration acquires the connector set as an independent datum; Theorem 5.1 gains a factor and Article 80's minima rise. No other result changes.

H_{br}. Its failure confines Theorem 8.4 and Corollary 8.17 to occupied sets without lateral adjacency. That is where Article 79's source operates and where Sections 6–9 are set, so no figure in the paper moves; what is lost is the claim that the two routes agree on a lattice, which the paper would then not make.

Theorem 5.1's exactness. A further compatibility condition—the attachment geometry of Target 14.8—lowers the realisable counts below the bound. The bound and every statement derived from it survive; the exact count on a column does not use the condition at all.

13 Precedence

Ruling 13.1 (Article 79)**.** [D]Consistent; parallel; nothing superseded. This paper takes Article 79's chain, exit distribution, orbit lemma, arm-word count, stacking model and delivered stock unchanged, and adds one statement about them: Theorem 7.3 and Corollary 7.5, that the registry filter is blind to the registry field at every site of the delivered stock. That is a property of Article 79's filter and not a correction of it; nothing Article 79 claims is weakened, since it claims selectivity in species and not in orientation. One addition is proposed to its scholium on the four filters. Each of the four acts on species, and none acts on orientation, but for four different reasons rather than one: arming, because the bromination's regioselectivity is a statement about which positions react and not about which way a unit faces; partition, because the export rate rises as q^n in the arm count alone; admission, because "a core that is not a chrysene in one of Article 77's three families is not admitted, whatever arms it carries"; and registry, by Theorem 7.4, and then only at contacts where at least one unit is blind. That last clause is the one the scholium cannot carry without qualification: at an I–I contact the fourth filter does read the alignment (Cor. 7.5), so the proposed column should read "no, except at I–I contacts, of frequency 2.5×10^{-7}" and not "no". The paper's conclusion that registry "carries the weight for the stream that is nine-tenths of the demand" is unaffected.

Ruling 13.2 (Article 57)**.** [C]Consistent; this paper is downstream of it. Its Definition 4.2, Proposition 4.3, Proposition 4.7 and Definition 5.2 are imported: Proposition 4.7 is what makes Corollary 8.12 true and what bounds the common image of Theorem 8.15 by $N(\ell)$. It is not what makes Corollary 8.17 true—that follows from the equality of the configuration sets—and its Theorem 4.5 is cited in the other direction, to refuse a cardinality this paper could otherwise have claimed (Rem. 8.16). Its Burnside counts at $\ell = 6$ are reproduced exactly at X1; the closed form of Theorem 6.5 extends them to every ℓ and is new here. Article 57's exclusion of the initiator from the sites of the cluster (its Def. 4.1) is read as concordant with Theorem 7.13.

Ruling 13.3 (Article 57, second ruling: the frame of the count)**.** [D]Recorded, and resolved by the move to contacts. Article 57 identifies configurations that no operation available to a unit distinguishes—"Two clusters are identified when no operation available to a unit distinguishes them" [Sch26f, Def. 4.1], and "the two or four marker assignments

of a class are one object described under the available frames" [Sch26f, Prop. 4.4]. An earlier form of this paper counted in an external frame in which those identifications do not hold, retained the site lift as the object of selection, and so priced a codon-length field at 6 bits. That is a conflict with Article 57 and was not recorded.
It is removed rather than defended. With the orientation datum placed on contacts (Def. 4.2) the object selected is the alignment field, of $\ell - 1$ letters, and the extra bit of the lift—the global flip, which is exactly what Article 57's Proposition 4.7 excludes from any product—is no longer counted. The paper's figure is now 5 bits at codon length, which is Article 57's own "five binary agreements", and the class figure 4 or 5 is reported alongside it (Rem. 9.3). Article 57's Proposition 4.4 is imported and not amended.

Ruling 13.4 (Article 40, second ruling: the registry field)**.** [C]Corrected here. Article 40 assigns a registry label to each z-stacking relation and writes $\eta : E_z \to \mathbb{Z}_2$ for the field on the oriented z-edges [Sch26b, eqs. (108)–(113)]; Article 80 has already ruled that Article 79's $e^{-\nu\Delta_{\mathrm{rgy}}}$ "is an energetic model of the registry field" in that sense [Sch26d, Rem. 2.3], and supplies the existence and transport conditions at its Proposition 3.6(b). An earlier form of this paper gave the name and the symbol to a site-indexed object of ℓ letters, cited neither passage, and marked the definition [A]. The object of this paper is now Article 40's field, cited as such (Def. 4.2, Rem. 6.2); the site word is called a lift; and Corollary 7.5 is stated against Article 80's ruling. Article 40's caution that "the registry label is not identified with molecular chirality" [Sch26b, eq. (109)] is honoured, and bears on Target 14.11: Article 65 records D, I and T as achiral [Sch26c, Table 1], so whatever the alignment datum is, it is not handedness.

Ruling 13.5 (Article 77)**.** [C]Consistent; no amendment proposed. Its §2.4 is the source of the orientation dictionary of Proposition 6.7; its Definition 3.1 is the source of the admissibility argument of Theorem 10.1. The orientation-reference field of Definition 3.1 is read as requiring existence and declaration and not choice (Rem. 10.2); any other reading would break Theorem 8.4, and the reading is recorded as a ruling rather than left implicit for that reason.

Ruling 13.6 (Articles 78 and 80)**.** [C]Consistent; geometry imported unchanged. Article 78's lattice geometry, attachment classes and supply formula and Article 80's site-class identities, completeness minima, saturated-bridging hypothesis and genome threshold are used as written. Nothing in either depends on provenance, so nothing in either is amended by a placed source (Rem. 10.3). Article 80's Target 11.9, on what its fifth constraint functional should count, is untouched here: a registry field is not a count of units.

Ruling 13.7 (Amending Article 65 §4: clause (j))**.** [A]Article 65's documentary bundle H_{mat} is a closed list of clauses by figure, which later papers cite in place of the filing [Sch26c, Hyp. 4.1, §4]; Article 79 amended it with clause (i) for Figure 12 [Sch26g, Rul. "Amending Article 65 §4: clause (i)"]. It is amended by one further clause:

> (j) Figure 4 of the filing exhibits catalyst/substrate pairings and the initiator. Figures 4(a) and 4(d)–(f) distinguish the Aligned pairing, in which the catalyst and substrate cores stand in the same relative orientation, from the Unaligned pairing, in which they do not; the description states that the alignment of the catalyst to the substrate defines the resultant nucleotide by the production of three or two hydrogen bonds between the paired nucleobases, and that the interaction of two heterodimers admits four orientations, namely Aligned/Forward, Aligned/Backward, Unaligned/Forward and Unaligned/Backward. Figures 4(g)–(j) exhibit an initiator compound, 3,6,12-trihydroxyethoxy chrysene, with which a stack may be capped at one or both ends and which provides the requisite basis for information; the description states that polymerisation terminates at it for lack of a sidechain at the 9-position, preventing conjugation to any further substrate/catalyst pair, and that its relative orientation may also inform the directionality of the encoded sequence. Figure 5(a)–(c) exhibits the code mapping reaction vessels to the nucleotide pairings, its bottom tables summarising the catalyst and substrate orientations in the Forward ("F") and Backward ("B") notation, for RNA at 5(a) and DNA at 5(b), with 5(c) the summary of codes between stacked reaction vessels, in which CA represents catalyst aligned and CU catalyst unaligned. Figure 11(a) illustrates the four orientations of reaction vessels for information storage and transcription, and Figures 11(c) and 11(e) the phosphodiester bonds between catalyst and substrate in their stacked orientation.

Clause (j) is [E] as to wording and enters no proof, as Article 65 requires of every clause of its bundle. What it does is move the orientation dictionary, the Aligned/Unaligned datum, the Forward/Backward notation, the four-orientation count and the initiator's frame-carrying role into column A, and a clause can do nothing else. Clause (b) of the bundle already assigns Figure 5 the role of mapping the encoding

tables to the stacking alignment; clause (j) adds the alignment definition, the F/B notation and the panel assignments, which clause (b) does not state.
The clause is confined to figures and their descriptions, as Article 65's Hypothesis 4.1 requires of a bundle clause—"a closed list of clauses by figure"—and as its Convention 2.2 requires of a documentary citation. An earlier form of this clause also carried a passage of the filing's Background and the text of its claim 8. Both are withdrawn from the clause and carried in the body under the mark S, at §1.2 and Appendix C respectively, rather than by widening Article 65's convention. The widening would have been a larger amendment than the use made of it: no numbered statement of this paper rests on either passage, and a convention that admits the Background and the claims to a figure bundle would change what every later paper of the corpus may cite in column A. Target 14.9 records the question for Article 65 to settle on its own terms.

Ruling 13.8 (Article 40)**.** [C]Consistent; axioms imported and none added. A_1's locants, A_2's C_{2h} reference, A_3's locant action, A_8's structural alphabet and cipher flip are used as written, and Remark 20's refusal of a field structure on $\Sigma_{\mathfrak{C}}$ is honoured (Prop. 3.1(vii)). Neither factor of $\Sigma_{\mathfrak{C}}$ is identified with anything here. The two base-two data of Proposition 6.8 belong to a *register*—a pair of adjacent sites—and are not the two factors of the single-site alphabet, as the scholium following that proposition says at length. The cipher factor is carried in the counts and used only through Hypothesis 4.22 (Rem. 4.13); whether the orientation factor is Axiom A_8's z-registry component depends on which rigid motion the flip is (Rem. 4.7). Both are Targets 14.5 and 14.11, and no result of this paper depends on either.

14 Targets, and conclusion

Target 14.1 (The orientation bias)**.** Measure, or compute, the free-energy difference between the two orientation states of a delivered D or T unit at a stacking contact in a column of the stock phase. Theorem 7.3 says the arm registration does not break the degeneracy; whether anything else does—arm conformation, π-overlap asymmetry, solvent structure—is not settled here. A bias $\theta \neq 1/2$ makes Theorem 9.2's cost depend on which field is prescribed, with mean $\ell H(\theta)$ and a range above and below it. This is the paper's most consequential open quantity.

Target 14.2 (The orientation-exchange barrier)**.** Determine the barrier to orientation exchange of a unit inside a column and inside a lattice, and the residence time it gives. H_{place} asserts it is long on the horizon of a use; Proposition 8.19 says what follows if it is not. The quantity is the analogue, for the orientation, of the return barrier Article 79's H_{ret} asserts for the bulk.

Target 14.3 (The placement mechanism)**.** Specify what sorts and what places: the recognition that distinguishes the two orientation states, the actuation that sets one, and the geometry that holds it. Nothing in this paper constrains the mechanism beyond requiring that it exist, and the divisional filing's system claims [Sch25b], which this paper has not examined, are the first place to look for precedent.

Target 14.4 (Mixed arm words)**.** Extend Theorem 7.3 to a stock with mixed arm words, in which a fraction of sites is flip-visible by cleavage [Sch26g, Prop. "Cleavage"]. The question is what fraction of flip-visible sites makes the registry filter partially orientation-selective, and whether the resulting bias is in a useful direction.

Target 14.5 (The cipher factor)**.** Settle what the first factor of $\Sigma_{\mathfrak{C}}$ is in the material. Axiom A_8 clause 2 realises the cipher pairing on the dual locants $\{2, 5, 8, 11\}$ by a one-step perimeter displacement [Sch26b, eq. (256)], which is a datum of a different kind from either of the filing's two base-two bits; and the filing's bits belong to a register of two sites, not to one (the scholium after Prop. 6.8). Until the question is settled the cipher factor is a carried coordinate with no identified referent, and every result here is about the other one.

Target 14.6 (Costing the placement)**.** Decide whether a ledger that charges for the act of placing—energy, apparatus, time, or the information itself as a thermodynamic cost—changes any conclusion of the arc. The present ledger charges for atoms only.

Target 14.7 (The bridged case)**.** Establish or refuse H_{br}: whether the endogenous route installs lateral connectors on the same terms as a placement system. Article 80's Target 11.3 asks the prior question—whether an assembly at the heritable minimum can be built with every lateral adjacency bridged at all—and this one asks by which route. Until it is settled, Theorem 8.4 and Corollary 8.17 are

statements about columns.

Target 14.8 (Attachment geometry of a two-armed unit)**.** Determine whether a D unit, whose armed locants $\{6, 12\}$ form one ι-orbit, can carry connectors along two different lateral axes. The answer lowers the realisable counts of Remark 4.11 and bears on which families can occupy which sites of a lattice.

Target 14.9 (The bundle convention)**.** Decide, for Article 65, whether its documentary bundle should admit the filing's Background, Summary and claims alongside its figures. This paper declines to widen the convention and carries the affected material under S instead (Rul. 13.7); the question is Article 65's.

Target 14.10 (Is a lattice site blind?)**.** Decide whether a site of the interior lattice, whose primary-locant arms are consumed as covalent linkers [Sch26d, Hyp. 3.5], is blind in the sense of Theorem 7.3. The theorem is proved for free-arm patterns; a bridged site presents a different pattern, and whether that pattern is ι-invariant is a different computation. Remark 9.4 records the gap and Theorem 9.2's lattice figure waits on it.

Target 14.11 (Which rigid motion the flip is)**.** Settle the physical identification of Definition 4.2's flip: an in-plane reorientation by the non-trivial element of the C_{2h} reference, or a turning-over carried by the out-of-plane arms. Both induce ι on the locants, so no theorem of this paper depends on the answer; two things do. The free-energy measurement of Target 14.1 cannot be designed without it, and the identification of η with Axiom A_8's z-registry component is natural under the first reading and a renaming under the second (Rem. 4.7).

Target 14.12 (Totality and injectivity of the product map)**.** Article 57 withholds both [Sch26f, Thm. 4.5], so the common image of Theorem 8.15 is bounded by $N(\ell)$ and not counted. Establish whether distinct frame classes give distinct end materials; until then no paper of the corpus should report 20 as a count of products rather than of classes.

Target 14.13 (Partial bridging)**.** State the configuration count and Article 80's completeness minima when the connector set is an independent datum rather than saturated, which is Article 80's own Target 11.3 seen from this paper's side.

Target 14.14 (A frame carrier in the lattice)**.** Determine whether an I unit can sit at a terminus of a column or at a face of a lattice so as to fix the frame, at what free energy, and whether Article 79's stream supplies enough of them: its initiator fraction p_3 is small by design.

Proposition 14.15 (What is established)**.** *[A]Subject to Appendix A: (i) the configurations of a finite occupied set form a finite set of exact cardinality $|\mathsf{D}|^{|\Omega|}$, the interconnections being determined by the occupied set (Thm. 5.1, Prop. 5.4); (ii) the registry classes of a stack of ℓ units number $(2^\ell + 2^{\lceil \ell/2 \rceil} + \mathbf{1}[\ell \text{ even}]2^{\ell/2})/4$, with orbit sizes 2 and 4 only (Thm. 6.5); (iii) the presented pattern of a unit is flip-invariant exactly when its family is ι-invariant and its arm word is ι-constant; one such unit at a contact suffices to make the contact blind, though not conversely; on the delivered stock every contact but I–I is blind, and the exceptional one carries 3 against 2 at a frequency 2.5×10^{-7}; and on a blind column the endogenous law on the alignment field is uniform, or, if a further* flip*-symmetric orientation term is present, flip-invariant (Thms. 7.3, 7.4, Cor. 7.5, Props. 7.7, Thm. 7.10); (iv) exactly one of the three unconditionally admitted families can fix a frame by its own pattern, and it is the one the filing caps its stacks with, for a reason—the empty 9-position—that is the same asymmetry (Thm. 7.13, Sch. 7.6); (v) the two reachable sets are equal, and equal to the whole configuration set, by positivity alone and with no appeal to any magnitude (Thm. 8.4, Cor. 8.8); (vi) the end materials coincide as images, the common image being bounded by the 20 frame classes of a six-site stack and not counted, so placement reaches no new material (Cor. 8.12, Thm. 8.15, Rem. 8.16, Cor. 8.17); (vii) under Theorem 7.10(a) and H_{frame} what placement supplies is exactly one bit per blind contact, which no redesign of Article 79's chain reduces at a D or T contact, while the species term of the cost is a design variable of that paper (Thm. 9.2, Prop. 9.5, Rems. 9.6, 7.8); (viii) a placed assembly is admissible under Article 77 as written, admission does not distinguish the two routes, and neither the geometry nor the material ledger changes (Thm. 10.1, Rem. 10.2, Rem. 10.3, Prop. 10.4); (ix) the filing's two base-two data are the two coordinates of a register's orientation pair under the automorphism $(a, b) \mapsto (b, a + b)$, and are not an identification with Article 40's single-site alphabet (Prop. 6.8 and the scholium following it).*

Remark 14.16 (What is not established)**.** [A]No placement mechanism is specified and no cost is assigned to placing. The orientation weight is not measured: the uniform law of Theorem 7.10(a) follows from Corollary 7.5 together with the premise that the contact free energy carries no orientation-dependent term beyond the arm registration, and a bias would change every bit figure while leaving every set equality standing. The residence time of a placed face is a stipulation (H_{place}). The independence of H_{draw} is an idealisation and is used only for the product form of Proposition 9.5. The species-term magnitudes are stipulated illustrations at Article 79's reported demand ratios and import no rate. Corollary 8.12 rests on an import, Article 57's Proposition 4.7, and on nothing this paper proves, and the cardinality of the common image is not established at all, Article 57 withholding the injectivity it would need. That a site of the interior lattice is blind is asserted and not shown, so Theorem 9.2's figure at 53 is conditional. And the cipher factor of $\Sigma_{\mathfrak{C}}$ has no identified referent here.

Scholium 14.17 (What would overturn the paper). Three findings, in order of consequence; only the second overturns a headline result, and the scholium is named for what a referee will look for rather than for what each item does. An orientation-dependent stacking free energy would leave every set equality standing and replace the bit figures by entropy figures; it would also mean that the endogenous source selects orientation after all, weakly, and the paper's central finding would become quantitative rather than absolute. A mechanism that *forbids* an orientation at a site—not merely disfavours it—would break Theorem 8.4 and make the exogenous route strictly stronger, which is the one conclusion this paper takes care to deny. And a product that depends on absolute orientation, against Article 57's Proposition 4.7, would break Corollary 8.12 but not Theorem 8.15, whose two images are equal because the two sets are, whatever the product map is; what would go is the bound by $N(\ell)$ and the reading of the common image as a set of frame classes. That third item is a correction and not an overturning, and it is listed here because it is the place a referee will press.

These are a difference of stacking constants, a steric exclusion, and a property of a reading. The first is a measurement, the second a calculation, and the third a question for the paper that owns the production operator.

A Hypothesis register

The register is Table 1.

Remark A.1. [A]Four premises carry the weight and they carry it in different directions.
H_{frame} carries the most and should be read first. Without an ι-free reference the alignment field of a blind column is degenerate and there is nothing for a placement system to set; Scholium 4.6 states this at length and gives the corpus's and the filing's grounds for the reference. It is the premise on which the paper's positive half stands or falls, and it is not a theorem of this paper.
H_{place} carries the exogenous route and is a stipulation of the kind Article 79 makes for its own, so the two papers are symmetrically exposed on persistence. H_{nf} carries the reachability theorem; it is weak—it asks only that nothing be forbidden—but not free, X9 exhibiting a proper subset the moment one alignment is excluded at one contact. H_{br} bounds what that theorem claims on a bridged occupied set, and the corpus withholds it (Rul. 13.6). Article 57's Proposition 4.7 carries Corollary 8.12 and the bound by $N(\ell)$; it does not carry Corollary 8.17, which follows from the equality of the two configuration sets alone.
After those, scepticism is best spent on the absence of a further orientation term, which is an assumption and not a theorem: Theorem 7.4 shows only that one named mechanism does not break the degeneracy, which is weaker than showing that nothing does.

B Verification

Script `verify_placed.py` (Python standard library; exact integer and rational arithmetic except the logarithms of X7 and X8). It writes `verification_values.tex`, which this paper inputs. Every assertion in the blocks is checked by the script and the run fails on any mismatch. Numeric macros are emitted after the assertion that establishes them; a small number of macros carry a fixed symbol—a point group, a family name, a closed form in m or ℓ—and the generated file lists those under a separate heading, so that the distinction is visible rather than asserted (Conv. 2.1).

X1 The frame group: the four maps distinct as functions at every $\ell \le 10$, closure, involutivity, commutativity; Burnside fixed-point counts $(64, 0, 8, 8)$ at $\ell = 6$ against [Sch26f, block V1]; the orbit count 20 with 8 orbits of size two and 12 of size four; the closed form of Theorem 6.5 against exhaustive enumeration at every $\ell \le 14$ and evaluated to $\ell = 16$; orbit sizes in $\{2, 4\}$ at every $\ell \le 12$.

X2 The letter assignment: both rows bijections; the registry partition $\{A, U\} \mid \{C, G\}$; the six-site reading a bijection onto the sixty-four codons; flip as letterwise complementation and rev as reversal with the C/G exchange, at all 64 words; $\chi = (a, b) \mapsto (b, a + b)$ an additive automorphism, with the filing's two distinctions written down as the letter sets $\{A, G\}$

Table 1: The hypothesis register. Marks: A attested by a clause of H_{mat} as amended (Ruling 13.7); S stated by the author. Imported hypotheses are named with their source and are not re-registered.

	Premise	What rests on it	What survives its failure
H_{place}	An exogenous agent sets family, arm word and letter at a prescribed site, and placed units do not re-equilibrate (S)	Prop. 8.2; Thm. 8.4; the whole exogenous side	Prop. 8.19: the selection decays and the reachable set is unchanged
H_{draw}	Independent draws, species by (p_n), letter by the contact free energy (S)	Thm. 7.10; Prop. 9.5; X9	Thm. 8.4 under any dependence keeping every letter attainable
H_{nf}	No structural letter is forbidden at a site (S)	Thm. 8.4, hence Thm. 8.15 and Cor. 8.17	Prop. 8.2's inclusion only; the equality fails and the exogenous route becomes strictly stronger
H_{ν}	ν is the coincidence count of presented patterns (S)	Cor. 7.5(b)	Cor. 7.5(a), which is Article 79's own reading and gives the same conclusion
H_{rgy} (imported, [Sch26g, Hyp. H_{rgy}])	The stacking ratio factorises through the arm registrations (S)	Cor. 7.5, hence Thm. 7.10	Thm. 7.3, which is combinatorial; the bit figures become Target 14.1
H_{sat} (imported, [Sch26d, Hyp. 3.3])	Every lateral adjacency of an occupied box is bridged (S)	Thm. 5.1's exact power; Prop. 5.4	The count with the connector set as a datum (Target 14.13)
No further orientation term	The contact free energy is H_{rgy}'s factorisation and nothing more (S)	Thm. 7.10(a); Thm. 9.2; the figures of Rem. 9.6	Thm. 7.10(b) while the further term is flip-symmetric; under a per-site bias θ, neither clause, and a field-dependent cost of mean $\ell H(\theta)$
Article 79's chain, with $e_n, k_j > 0$	The exit distribution is strictly positive (A/S as in [Sch26g])	Thm. 8.7, hence Thm. 8.4	The inclusion of Prop. 8.2
[Sch26f, Prop. 4.7] (imported)	A product depends on the registry field only through its agreements, up to reversal	Cor. 8.12; the bound by $N(\ell)$ in Thm. 8.15	Thm. 8.15's equality of images and Cor. 8.17, which need only Thm. 8.4; the bound and the frame-class reading do not
Lattice sites are blind	A bridged site of the interior lattice satisfies Thm. 7.3 (S)	Thm. 9.2's figure at $\ell = 53$	Everything else; the column figures are unaffected (Rem. 9.4)

and $\{C, G\}$ *first* and then located in the coordinates, together with the check that the alternative coordinate $(a, a+b)$ gives neither.

X2b The agreement word: flip-invariant, onto $\{0,1\}^{\ell-1}$, with fibres exactly the flip-pairs; rev acting on it by reversal; and the resulting second count $\left(2^{\ell-1} + 2^{\lceil(\ell-1)/2\rceil}\right)/2$ equal to $N(\ell)$ at every $\ell \le 12$.

X3 The inversion: sixteen subsets, four fixed, and 10 orbits by enumeration, with Burnside checked against the enumeration rather than substituted for it; D and T invariant and I not; exactly 1 admitted family ι-free, namely I.

X4 Arm words and their ι-stabilisers: 21 words over two arm types and 78 over three, reproducing [Sch26g, Lem. "Arm words within the families"]; the stabiliser split, 2 fixed and 1 free orbit on D over $\{h, e\}$, 4 and 6 on T; the delivered words ι-fixed.

X5 The configuration count: Article 79's three arm-word formulas checked at $m = 2, 3$; the datum set built explicitly from families, ι-class representatives and the cipher component, and its cardinality counted at $|\mathsf{D}| = 312$ for three arm types and 12 in the delivered case. The lattice alphabet and the counts at lattice sizes are X11. Finiteness itself is Theorem 5.1 and is not a computation.

X6 Site classes of a box: the four identities of [Sch26d, Prop. 3.4] at $2\times2\times2$, $2\times2\times4$, $3\times3\times3$ and $3\times3\times6$; the minima 27 and 54 by exhaustive search over extents ≤ 8; 72 bridges and 45 stack relations at $3\times3\times6$.

X7 The selection cost: ℓ bits for a field at $\ell = 5, 7, 15, 26, 53$; 4 or 5 bits for a class at $\ell = 6$; 2^{-54} computed exactly as a rational and its decimal form 5.55×10^{-17} emitted from that computation.

X8 The species term: the admitted fractions at $\rho^* = 100, 150, 200$ with $p_3/p_4 = 1/10$, each summing to one exactly; the alternation probabilities 9.36×10^{-7} and 1.21×10^{-7} and their bit costs 20.0 and 23.0; the totals 25.0 and 28.0 and the registry share 18 per cent.

X9 Reachability: the twelve-letter delivered alphabet; the law on a four-site column built from a named orientation weight θ and cipher weight γ, normalised exactly and of full support at three distinct pairs (θ, γ) in $(0, 1)^2$ as well as at $(\frac{1}{2}, \frac{1}{2})$, where the least and greatest masses are 2.39×10^{-16} and 3.82×10^{-3} at ratio 1.60×10^{13}; the contrasting case $\theta = 0$, where the support is a proper subset smaller by a factor 16, so that H_{nf} is shown to be load-bearing; every one of the 20 frame classes reached from the configuration set; and the one-half bound checked on a law that is flip-symmetric and *not* uniform, together with a biased per-site law at $\ell = 1$ for which it fails.

X10 Flip blindness: the presented pattern computed and compared with its flip for every arm word of every family at two and three arm types; 2 blind of four on D, 4 of sixteen on T, 0 on I; the general counts m and m^2; the delivered ee and $eeee$ blind; and the registration numbers recovered as coincidence counts, $\nu = 2$ on a D contact and 4 on a T contact, each unchanged when one unit is flipped.

C Molecular commentary, isolated

Remark C.1 (Status)**.** [A]Nothing here enters a numbered statement.

Remark C.2 (The filing's two sources)**.** [E]The filing's background sets the two routes against each other directly. De novo synthesis is described as "processed through directed placement of monomers to build a single strand of DNA"; the alternative is described as a scheme in which "molecules randomly self-organize to eventually synthesize as to a DNA molecule, which in turn randomly couples with a phospholipid cellular membrane", and the objection raised to it is that "these random events provide unknown variability and ambiguity in DNA and cellular production" [Sch25a, Background]. The phrase names the quantity this paper computes. Variability at fixed material is the registry field; the ambiguity is the frame quotient; and Theorem 9.2 is how much of the first a placement system removes.

Remark C.3 (Two-sidedness as a species property)**.** [E]That a planar polyaromatic with peripheral substituents has distinguishable faces when stacked is the physical content of the registry component, and Article 65 carries it as a property of each species rather than deriving it [Sch26c, Def. 5.1]. The property is lost at the azo conversion and recovered nowhere [Sch26c, Thm. 6.4], so the stage at which a face can be set is the chrysene stage.

Remark C.4 (Four orientations of a reaction vessel)**.** [E]The filing's Figure 11(a) "illustrates the four different orientations of reaction vessels for information storage and transcription" [Sch25a, Fig. 11 description], and its description elsewhere reports that "the number of orientations involved in the coordination of the catalyst and substrate typically results in four π-stacking arrangements". Four is $|\Sigma_{\mathfrak{C}}|$. The coincidence of cardinals is not an argument that the filing's four arrangements are Article 40's four structural states, and Proposition 6.8 is the statement that does the work; this remark records only that the counts agree.

Remark C.5 (The delivered arm words are claimed)**.** [E]Theorem 7.3's "in particular" clause—that the delivered stock is blind at every site—rests on the arm word being ee on D and $eeee$ on T, which Article 79 obtains as the one-reagent limit of its arming route [Sch26g, Thm. "No cleavage, no arm loss"]. The filing names the three all-hydroxyethoxy compounds at claim 8 (clause (j)), so the words the theorem is about are the ones the filing claims, and not a convenient special case chosen here. Partial cleavage to h moves a site off the all-e word and, on D, into the one flip-visible orbit of X4; that is Target 14.4.

Remark C.6 (Why the initiator and not the catalyst)**.** [E]The filing's catalyst is the disubstituted unit and its substrate the tetrasubstituted one, and both are ι-invariant. Its initiator is the trisubstituted unit, and it is not. That the filing assigns the frame-establishing role to the initiator and not to either of the other two is consistent with Theorem 7.13, and the theorem supplies a reason: a unit whose own substitution pattern is unchanged by being turned over cannot report which way up it is.

D What was set aside in preparing this paper

Remark D.1 (The inclusion as the theorem)**.** [A]A first form of this paper took $R_{\mathrm{endo}} \subseteq R_{\mathrm{exo}}$ as its main result and the exogenous route as the stronger one. The inclusion is nearly a definition (Prop. 8.2) and the interesting statement is the converse, so the result was restated as the equality of Theorem 8.4 and the paper's emphasis moved from reach to selection.

Remark D.2 (A non-probabilistic configuration space)**.** [A]An earlier draft sought to make the endogenous source non-probabilistic, against Article 79's own stochastic chain. That cannot be done and should not be: Article 79's exit distribution is irreducibly a distribution, and its own remark on the fluid limit records that the tetrasubstituted output falls short in about half of all periods at $\Phi_{\min}$. What is non-probabilistic is the *comparison*, and the device that makes it so is to work with supports and images rather than with laws (Cor. 8.8). The claim that the configuration space itself is non-probabilistic is correct and is Theorem 5.1; the claim that either source is, is withdrawn.

Remark D.3 (An admissibility obstruction that does not exist)**.** [A]A draft argued that Article 77's orientation-reference field in its Definition 3.1 can be supplied only by a placed assembly, so that a self-assembled column is inadmissible as an assembly and the exogenous route is the only admissible one. The premise was a misreading of the same kind Article 79 records against its own drafts: the definition requires that the reference exist and be declared, not that it be chosen. The obstruction is withdrawn and Remark 10.2 replaces it; had it stood, Theorem 8.4 would have been false.

Remark D.4 (The bit count as an origins argument)**.** [A]A draft presented 5.55×10^{-17} as evidence about the endogenous route. It is not: a mass is not a frequency without a number of trials, and the architecture does not require a prescribed field in any case. The figure is retained as a cost and the inference is withdrawn (Prop. 3.1(iv), Sch. 9.7).

Remark D.5 (The orientation datum at a site)**.** [A]Drafts to v0.4 put the orientation datum at each site, called the resulting word of ℓ letters the registry field, and priced a codon-length selection at 6 bits. Three things were wrong with it. The corpus already has a registry field and it is an edge datum (Rul. 13.4); a site datum is unavailable at a blind unit, since the motion that would exchange its two states is a symmetry of it (Rem. 4.7); and retaining the lift as the object of selection counted a bit—the global flip—that Article 57 identifies away (Rul. 13.3). The datum now sits on contacts, the figure is 5 bits, and the three objections resolve together.

Remark D.6 ("Blind at every site")**.** [A]Drafts to v0.4 asserted that the delivered stock is blind at every site, and derived from it that Article 79's fourth filter cannot read orientation anywhere. The delivered stock contains the initiator at *eee*, which Theorem 7.3 itself puts outside the blind condition, and an *I*–*I* contact carries 3 against 2. The claim is withdrawn and replaced by Theorem 7.4 and Proposition 7.7, which bound the exception rather than deny it; the companion claim that no rate of Article 79's chain couples to the orientation is corrected at Remark 7.8. The correction improves the paper: the exceptional contact is the frame carrier, so the two halves of Section 7 become one statement (Sch. 7.6).

Remark D.7 (One alphabet for two phases)**.** [A]Drafts to v0.4 applied the twelve-letter stock alphabet at lattice sizes and reported 12^{27} and 12^{54}. Article 78's lattice is tetrasubstituted throughout and Article 80 installs its linkers on the primary locants, so family and arm word are both fixed at a lattice node. The figures are withdrawn and replaced at Corollary 5.2; nothing that uses the alignment field moves (Rem. 5.3).

Remark D.8 (A product count where only a bound is available)**.** [A]Drafts to v0.4 reported the common image of the two sources as the twenty frame classes. Article 57's capacity theorem is stated with "at most" and expressly withholds the injectivity a count would need. The claim became a bound at v0.5 and is now also restricted to the composition Article 57's Definition 4.1 specifies (Rem. 8.13), since over varying family assignments the product is not constant on alignment classes.

Remark D.9 (A product count of configurations on a bridged box)**.** [A]Drafts to v0.4 counted $\mathsf{C}(\Omega)$ as a product set at every occupied set. Under saturated bridging a site needs one armed locant per incident connector, so the family at a site of lateral degree d is constrained; the count is exact only without lateral adjacency (Thm. 5.1, Rem. 4.11).

Remark D.10 (The flip as a symmetry of the product)**.** [A]A draft treated the flip as a symmetry of the end material, concluding that orientation cannot matter at all. That overreads Corollary 8.12, which says a product is a function of the frame *class*: the class is not a single letter, and a product may vary across classes. The flip is a symmetry of the reading and not of the content, and the 20 classes at $\ell = 6$ bound how far the content can vary. Whether distinct classes in fact give distinct products is not asserted—Article 57 withholds the injectivity that would settle it (Rem. 8.16,

> Target 14.12)—and a draft sentence asserting it is withdrawn here.

E Sources, by use

The concordance is Table 2.

References

[Sch25a] Charles D. Schaper. Encoded Design Integrated Synthesis of Nucleic Acids, and Phospholipids, and Related Pharmaceutical Products. United States Patent No. 12,195,423 B1, 2025. Composition of matter. Cited through Article 65's documentary bundle H_{mat} as amended.

[Sch25b] Charles D. Schaper. Encoded Design Integrated Synthesis of Nucleic Acids, and Phospholipids, and Related Pharmaceutical Products. United States Patent No. 12,473,247, 2025. Divisional of US 12,195,423, with system claims; the description is that of the parent.

[Sch26a] Charles D. Schaper. Carbon Traceable Protocell Construction From Substituted Chrysene Stocks. *Annals of the Chrysene Formalism*, 1(4):1567–1605, 2026. Article 77. DOI: https://doi.org/10.68366/ees7wd38.

[Sch26b] Charles D. Schaper. The Chrysene Formalism Revisited: A Twelve-Axiom Foundation for Molecular Characterization, Tensorial Noncommutative Geometry, and Operator Response. *Annals of the Chrysene Formalism*, 1(2):489–560, 2026. Article 40. DOI: https://doi.org/10.68366/5jctze12.

[Sch26c] Charles D. Schaper. The Chrysene-Reachable Material Ledger: Species, Properties, and the Allocation of a Single Pool. *Annals of the Chrysene Formalism*, 1(3):1371–1383, 2026. Article 65. DOI: https://doi.org/10.68366/dwqk8m15.

[Sch26d] Charles D. Schaper. The Completeness Condition: Realisation of the Twelve-Axiom Framework at a Site of the Interior Chrysene Lattice, and the Genome Length It Requires. *Annals of the Chrysene Formalism*, 1(4):1639–1651, 2026. Article 80. DOI: https://doi.org/10.68366/0a19r769.

[Sch26e] Charles D. Schaper. The Interior Chrysene Lattice: Mathematical Necessity, Distributed Inheritance, and Catalytic Organization in the Protocell. *Annals of the Chrysene Formalism*, 1(4):1606–1615, 2026. Article 78. DOI: https://doi.org/10.68366/2kkrhm64.

[Sch26f] Charles D. Schaper. The Origins Transition: From the Encoded Chrysene Cluster to the Originating Genetic Code Mappings — Structural Consistency, Codon Placement, and a Forced Exchange. *Annals of the Chrysene Formalism*, 1(2):1076–1106, 2026. Article 57. DOI: https://doi.org/10.68366/yx30r330.

[Sch26g] Charles D. Schaper. The Source: Substitution-Initiated Self-Separation of the Substituted-Chrysene Stock from a Polyaromatic Bulk, and the Supply It Delivers to the Protocell. *Annals of the Chrysene Formalism*, 1(4):1616–1638, 2026. Article 79. DOI: https://doi.org/10.68366/s8sfwq50.

Table 2: Sources, by use.

Source	Used at	What is taken
Article 40	§4, §7	A_1's locants; A_2's C_{2h} reference and A_3's locant action; A_8's structural alphabet, additive law and cipher flip; the registry field on the z-edges, eqs. (108)–(113), and its caution that the label is not chirality; Remark 20's refusal of a field law
Article 57	§6, §8	The frame group; the twenty classes and the Burnside counts; the letter assignment; the production operator, which carries Cor. 8.12; the cluster composition, which restricts it; the capacity theorem's withheld injectivity, which bounds Thm. 8.15; the identification clause, ruled on at Rul. 13.3; the initiator's exclusion from the cluster
Article 65	§4, §10, Rul. 13.7	The species inventory and its two-sidedness property; the one-way theorem on two-sidedness; the documentary bundle and its citation rule; the pool allocation
Article 77	§6, §10	The orientation dictionary of its §2.4; the constitutional stock type of its Def. 3.1 and the admission predicate; the carbon boundary; the complement map
Article 78	§5, §10	The lattice composition—tetrasubstituted throughout—and its geometry and linker arm; the attachment classes; the entry mechanism, in which linker closure follows π-contact; the supply formula; the per-period lattice demand
Article 79	throughout	The chain and the exit distribution; the orbit lemma; the arm-word count and the delivered stock; the stacking model and H_{rgy}; the four filters; the formula
Article 80	§5, §10	The site classes and their identities; the completeness minima; saturated bridging; the genome threshold
US 12,195,423	§6, §7, App. C	Via Article 65's bundle as amended: clauses (a), (b), (c), (f), Article 79's (i), and the new clause (j), which carries the Aligned/Unaligned datum, the F/B notation, the four-orientation count and the initiator's cap and missing 9-position. The Background and claim 8 are cited in the body under S and are not clause material (Rul. 13.7)
US 12,473,247	Target 14.3	Named as the place to look for placement-system precedent; not examined here