ACF 86 — Lean 4 Proof Dossier

Exchangeability and the Exclusion of Equilibrium Assembly

Necessary Conditions on Throughput, Entropy, and Fidelity in the Chrysene Formalism

Charles D. Schaper, Ph.D., Schaper Systems Press

Annals of the Chrysene Formalism, Vol. 1, Issue 4

Proof file: `acf-86-proof.lean` Verification harness: `verify_A86.py`

Kernel Certification

Item	Value
Proof file	`acf-86-proof.lean`
Lean version	`4.12.0` (core only, no Mathlib)
Namespace	`ChryseneFormalism.Article86`
Sections	S1–S12
Definitions	60
Theorems	137
Proved by `decide`	137
Depending on no axioms	**137**
Depending on `propext`	0
Depending on `Quot.sound` or `funext`	0
`sorry`	0
`native_decide`	0
Elaboration time	35 s
Exit code	`0`

What this file reaches, and why it reaches more than its predecessor

The Article 85 proof file could not touch that paper's information figures. They are binary entropies of $1/(1+e^{g\Delta})$, and no integer decision procedure reaches an entropy of a transcendental; the harness had to carry them as certified rational enclosures and the proof file said so before its theorem index.

Article 86 is a bounds paper, and its transcendentals are of a kinder shape. Three devices put almost all of it inside the kernel.

An integer root bracket. The error budget of S8 is four n-th roots of rationals, one of them a thousandth root, and all four are pinned exactly with no root ever formed. It is the third inversion of the family: Article 82 raises a bracket to *clear* a logarithm, Article 84 raises one to *form* a logarithm, this one raises one to *clear* a root.

$\ln 2$ **from its own series.** Landauer's constant is the one transcendental in Article 86's arithmetic rather than in its bounds, and S2 computes rational bounds for it inside the kernel from the inverse-hyperbolic-tangent series with an explicit remainder, in fixed point over `Nat` at a scale of 10^{40}. The bracket it produces has width 92 in units of 10^{-40}. So the thermodynamic floor of S7 is *derived here* and not imported, and every figure of it is a pair of cross-multiplied inequalities against those bounds.

A bracket in the exponent. The gap between the orientation floor and the error budget is 4.7 orders of magnitude. Rather than take a logarithm, S9 raises the already-certified bracket on the gap — (49,650, 49,750) — to the hundredth power and compares with 10^{465} and 10^{475}. No logarithm is formed anywhere in this file.

What is left outside is one quantity: Article 85's realised capacity at $\Delta = 1$, which is an entropy and stays with the harness. S10 says which figures depend on it, and three of the article's four corrected figures are among them.

The file assumes nothing at all. An `#print axioms` audit over every theorem reports *"does not depend on any axioms"* in all 137 cases. The audit runs inside the file, so a run prints 137 lines and exits 0; that transcript, not a silent run, is the certificate.

The correction, and why it belonged in a proof file

The submitted draft's generator reproduced the published figures of five other articles as floating-point literals, and its appendix said so. S3 of this file rebuilds two of them from the definitions instead: Article 85's sensitivity polynomial, by the same 2304-contact enumeration the Article 85 proof file performs, and Article 84's coded-hexamer bracket, by the binary-logarithm bracket on the same admitted fractions.
That was not bookkeeping. S10 certifies that

$$1/\sigma(0) = 2011^2 = 4{,}044{,}121$$

exactly — the sensing bound at the pure limit is an integer, and the same integer Articles 84 and 85 both print. The draft printed 4,044,162, the reciprocal of its own rounded literal 2.4727×10^{-7}, which exceeded the exact figure by 41. The kernel states the correction in three theorems: the exact value, that the superseded figure is not it, and the difference.

The article has since been revised. Its generator now derives σ and the realised capacities, it prints the integer, and its Remark "A correction: four reciprocals" records the four figures that changed. S10 keeps the superseded value beside the exact one all the same: a correction that disappears into the corrected text leaves a reader no way to tell a figure that was checked from one that was never wrong, and the corpus's practice is to record rather than absorb.

A kernel-checked arithmetic error is a stronger finding than a reported one, which is the argument for having a proof file derive what an article imports rather than restate it. The other three corrections — two reciprocals of three-figure literals and a two-place decimal given a third significant figure — live in the realised capacity and were therefore the harness's, not this file's.

Encoding

Nothing is subtracted anywhere. A rounding is a pair of strict inequalities rather than an absolute value. A ratio is cross-multiplied rather than divided. A complement is supplied as its own literal together with the equation that fixes it: the four sign-agnostic floors of S9 appear as `the_complements_of_sigma`, four equations of the form σ-numerator+complement = denominator, and the floors are then stated over the complements. The two endpoints of the root bracket are

passed in as `lo` and `hi` rather than computed as $2s - 2r \mp 1$, with `rootEndpoints` the equation that fixes them — so every use of the bracket in S8 comes in pairs, the endpoint equation and the bracket itself.

The root bracket.

$$\texttt{rootRounds}\ a\,b\,k\,lo\,hi\,s\ :\ lo^k\,b < a\,(2s)^k\ \wedge\ a\,(2s)^k < hi^k\,b$$

Writing $y = 1 - (a/b)^{1/k}$, the claim that y rounds to r/s is $\frac{2r-1}{2s} < y < \frac{2r+1}{2s}$, hence $\frac{2s-2r-1}{2s} < (a/b)^{1/k} < \frac{2s-2r+1}{2s}$, and $x \mapsto x^k$ is increasing on the positives, so raising the bracket to the k clears the root. S1 validates it on a case whose answer is exact, $1 - (1/4)^{1/2} = 1/2$, and on one whose answer is not, $1 - (1/2)^{1/3} = 0.2063$, and rejects the neighbours of both. The anchor certificate at $k = 1000$ compares two integers of about 7,300 decimal digits; the kernel's arbitrary-precision arithmetic settles them without approximation.

$\ln 2$ **in fixed point.** $\ln 2 = 2\,\mathrm{atanh}(1/3) = 2\sum_{k\geq 0} 1/\big((2k+1)3^{2k+1}\big)$. Each term is floored at the scale $P = 10^{40}$, so the partial sum underestimates by at most one unit per term; the tail beyond n terms is at most $(9/8)/\big((2n+1)3^{2n+1}\big)$, which is added to the upper bound with one further unit of slack so that the floor division cannot round the wrong way. At $n = 45$ the resulting bracket is $\texttt{LN2LO} \leq P \ln 2 \leq \texttt{LN2HI}$ with $\texttt{LN2LO} + 92 = \texttt{LN2HI}$, which S2 states as a theorem. Every Landauer figure of S7 is then two inequalities against those two naturals.

Signed coefficients, as a pair. Article 85's σ has coefficients of both signs, because the cleavage model makes $1 - \pi_h$ unavoidable. A polynomial is carried as a *pair* of naturals, each a packed base-2^{128} digit string of non-negative coefficients, and the polynomial is their difference; multiplication is multiplication because $(P_1 - N_1)(P_2 - N_2) = (P_1P_2 + N_1N_2) - (P_1N_2 + N_1P_2)$ keeps both halves non-negative. The two halves are written out by `packOf`, coefficient by coefficient, rather than as the three-hundred-digit naturals they evaluate to.

The derivation, performed once. The 2304-contact fold that builds σ is the expensive part of this file. The kernel evaluates it once, pins both halves to the literals of `SP` and `SN`, and every later certificate is stated over those. The derivation is still checked: if the enumeration gave anything else, `sigma_is_derived_from_the_enumeration` would fail and the rest would be about the wrong polynomial.

Connectivity by a spanning path. S11 needs a layer of Article 80's box to be connected under the in-plane connector. Rather than compute connected components, the file exhibits a snake path through the nine cells of a 3×3 layer, checks that consecutive cells are in-plane neighbours and that the path visits every cell, and then checks that no in-plane edge changes layer. Six layers, six components, $2^6 = 64$ states — a factor of 2^{48} below the unconstrained count.

What is not encoded. Two things. Article 85's *realised capacity* at $\Delta = 1$ is an entropy of a transcendental and stays with the harness; three of the four corrected figures depend on it. And the *bookkeeping* of the harness's block D12 — four named hypotheses, four corrections — has no section here on purpose: a count of the article's own bookkeeping is not a mathematical claim, and a kernel certificate of $4 = 4$ would be a decoration rather than a result.

Theorem Index

Section	Name	Statement (informal)
S1	`rounding_accepts_a_correct_value`	1/3 rounds to 0.33
	`rounding_rejects_the_value_below` … `…a_half_way_case`	and not to 0.32, 0.34, or a half-way case
	`log2_accepts_an_integral_logarithm`, `log2_rejects_its_neighbours`	Article 84's bracket, reused in S3
	`the_root_endpoints_of_one_half`	the endpoints, said without a subtraction
	`the_root_bracket_accepts_an_exact_root`	**the device**: $1-(1/4)^{1/2}=1/2$
	`and_rejects_the_value_below`, `and_rejects_the_value_above`	and both neighbours
	`the_root_bracket_accepts_an_inexact_root`	$1-(1/2)^{1/3}=0.2063$
	`and_rejects_the_neighbours_of_that`	so it is not vacuous
S2	`the_bracket_is_ordered`	`LN2LO` $<$ `LN2HI`
	`the_bracket_is_narrow`	width exactly 92 at the scale 10^{40}
	`ln_two_rounds_to_point_six_nine_three_one`	$\ln 2$ **from its own series**: 0.6931
	`and_not_to_point_six_nine_three_zero`, `…_two`	nor to either neighbour
	`ln_two_is_between_two_thirds_and_seven_tenths`	a coarse sanity bound
	`the_series_is_not_vacuous`, `the_terms_fall`	the terms are positive and decreasing
S3	`the_stream_has_twentyeight_unit_types`	28 of 48 index slots
	`the_admitted_numerators_sum_to_2011`	$2000+1+10$
	`f_I_squared_is_one_over_2011_squared`	4,044,121
	`sigma_is_derived_from_the_enumeration`	**the derivation**: the 2304-contact fold, pinned once
	`sigma_has_no_coefficient_above_degree_eight`, `no_packed_digit_carries`	the packing is faithful
	`sigma_at_zero_is_f_I_squared`	the point of contact with Articles 84 and 85
	`the_degree_is_exactly_eight`	leading coefficient -2400
	`sigma_at_pi_e_nine_tenths` … `…ninetynine_hundredths`	σ exactly at the three operating points
	`sigma_rounds_to_324e_minus_2_at_nine_tenths` …	and the four printed figures
	`the_hexamer_costs_twentyfive_bits_at_rho_one_hundred`	Article 84 re-derived: 25.0 bits
	`and_twentyeight_at_rho_two_hundred`	and 28.0
	`and_neither_to_the_neighbouring_tenth`	so the bracket pins them
	`the_registry_term_is_five_bits_and_needs_no_logarithm`	$2^5=32$

Section	Name	Statement (informal)
S4	`the_middle_term_is_an_integer_at_alpha_one_tenth`	$N/9 = 28{,}000$, which is why the demand is an integer
	`the_demand_at_L_two_hundred_is_42200`	exactly
	`the_three_terms_are_...and_they_sum_to_the_demand`	1,000, 28,000, 13,200 exactly
	`the_anchor_share_rounds_to_two_point_four`	2.4 per cent
	`the_residue_share_...`, `the_lipid_share_...`	66.4 and 31.3 per cent
	`the_non_structural_share_...`	97.6 per cent
	`over_ninetyseven_per_cent_is_not_placed_at_all`	which is why only the anchor term is bounded
	`the_demand_at_alpha_one_fiftieth`, `and_it_rounds_to_19343`	135,400/7 exactly
	`the_demand_at_L_two_hundred_and_twentytwo`	43,762 at Article 80's critical length
	`the_anchor_term_there_is_1110`	$5L$
	`the_demand_rises_by_three_point_seven_per_cent`, `and_the_rise_...`	exactly 1,562 units
S5	`the_period_is_fortythree_thousand_two_hundred_seconds`	twelve hours
	`the_nominal_rate_rounds_to_point_nine_seven_seven`	0.977 units per second
	`the_critical_rate_...`, `the_nominal_interval_...`, `the_critical_interval_...`	1.013, 1.024, 0.987
	`at_a_six_hour_period_...`, `at_a_twentyfour_hour_period_...`	1.954 and 0.488
	`the_rate_is_of_order_one_per_second_at_every_period_tested`	**an order of magnitude, not a rounded equality**
	`and_the_spread_over_those_cases_is_exactly_fourfold`	so the claim is not fragile
S6	`the_lower_figure_is_four_point_one_seven_bits_per_unit`	25/6
	`the_upper_figure_is_four_point_six_seven`	28/6
	`the_registry_term_per_unit_rounds_to_point_eight_three_three`	5/6
	`the_box_has_fiftyfour_nodes_and_fortyfive_contacts`	$3 \times 3 \times 6$
	`and_is_nine_columns_of_six`	along the stacking axis
	`a_column_of_fiftyfour_would_carry_fiftythree`, `the_two_figures_are_different`	45 is not 53
	`the_registry_term_is_the_contacts_per_node_ratio`	$5 \cdot 54 = 45 \cdot 6$: exactly, not approximately
	`the_lower_bits_per_period_round_to_4167` ...	4,167 and 4,667 bits per period
	`the_lower_bit_rate_...`, `the_upper_bit_rate_...`	0.096 and 0.108 bits per second

Section	Name	Statement (informal)
S7	`the_selection_cost_is_two_point_eight_nine_kT_at_the_lower_figure`	$2.89\,kT$, against the series bracket
	`and_to_three_point_two_three_at_the_upper`	$3.23\,kT$
	`a_D_contact_carries_two_registrations_and_a_T_contact_four`	the two comparators
	`the_selection_cost_EXCEEDS_one_disubstituted_contact`	$> 2\,kT$
	`and_falls_below_one_tetrasubstituted_contact`	$< 4\,kT$
	`so_energy_is_comparable_and_not_slack`	both at once
	`the_ratio_to_a_disubstituted_contact_rounds_to_one_point_six_two`, `and_to_..._point_eight_one`	1.62 and 0.81
S8	`the_box_has_fortyfive_placements_and_a_period_one_thousand`	the two counts
	`the_box_budget_at_F_ninetynine`	**the root bracket**: 2.23×10^{-4}
	`and_not_the_neighbouring_mantissa_there`	nor either neighbour
	`the_box_budget_at_F_one_half`, `and_not..._either`	1.53×10^{-2}
	`the_anchor_budget_at_F_ninetynine`	1.01×10^{-5}, at $k = 1000$: two 7,300-digit integers
	`and_not_the_neighbouring_mantissa`	nor either neighbour
	`the_anchor_budget_at_F_one_half`	6.93×10^{-4}
	`the_budget_tightens_with_the_number_of_placements`	cross-multiplied, so no root is formed
S9	`every_delivered_D_or_T_contact_is_blind`	**equal registration numbers**, so uniform at every Δ
	`the_I_I_contact_reads_three_against_two`	the one exception
	`so_the_initiator_points_the_right_way`	and the only one
	`the_symmetric_sign_error_is_exactly_one_half`	identically in θ, over a hundred splits
	`and_the_identity_needs_no_value_of_Delta_or_of_the_gap`	which is the whole of the floor
	`the_complements_of_sigma`	four equations, so nothing is subtracted
	`the_floor_at_the_pure_limit_rounds_to_point_five` ...	the four sign-agnostic floors
	`and_to_point_four_eight_three_eight_at_four_places`	the four-place form
	`the_floor_with_the_initiator_rounds_to_nine_sixes`	0.499999876
	`it_falls_short_of_one_half_by_f_I_squared_over_two`, `and_the_shortfall_...`	1.24×10^{-7}

Section	Name	Statement (informal)
	`the_floor_is_below_one_half`	and the initiator is why
	`the_gap_endpoints_are_right`	the two literals, fixed
	`the_floor_exceeds_the_budget_by_four_point_nine_seven_e_four`	by the root bracket
	`the_gap_is_four_point_seven_orders_of_magnitude`	**certified in the exponent**, no logarithm taken
	`and_it_is_not_four_point_six_or_four_point_eight`	so the bracket is tight
	`under_correlated_failures_the_floor_still_exceeds_the_budget`	fiftyfold at $1 - F = 0.01$
	`so_the_comparison_does_not_rest_on_independence`	which is the point of that extreme
S10	`the_bound_at_the_pure_limit_is_exactly_2011_squared`	**the correction**: 4,044,121
	`and_the_superseded_figure_is_not_that_integer`	$4{,}044{,}162 \neq 4{,}044{,}121$
	`the_superseded_figure_exceeded_the_exact_one_by_fortyone`	the difference
	`the_bound_at_pi_e_ninetynine_rounds_to_2548` …	2,548, 111, 31
	`the_bound_rises_as_cleavage_falls`	the structural claim
S11	`there_are_two_new_forms_per_direction`, `so_four_in_all`	the repeated-locant states
	`and_each_has_a_repeated_locant`	which is what makes them new
	`the_relative_freedom_within_a_pair_is_zero`	eq. (107) fixes it
	`the_snake_steps_only_between_neighbours`	**connectivity by a spanning path**
	`and_visits_every_cell_of_the_layer`	all nine
	`so_a_layer_is_connected`	one component per layer
	`no_in_plane_edge_leaves_its_layer`	the connector joins cells at the same z
	`so_there_is_one_component_per_layer`	six of them
	`the_surviving_freedom_is_sixtyfour_states`	2^6
	`which_is_two_to_the_fortyeight_below_the_unconstrained_count`	$2^{54} = 2^6 \cdot 2^{48}$
S12	`energy_is_comparable`	between two contacts and four
	`rate_is_moderate`	order one per second
	`sensing_is_severe`	$\geq 1/\sigma$ contacts per bit
	`orientation_is_forced`	the floor exceeds the budget 10^4-fold
	`in_plane_is_structural`	one flip per layer
	`the_binding_constraint_is_not_energetic`	**the article's conclusion**, as one theorem

Proof Listing: `acf-86-proof.lean`

`acf-86-proof.lean` — complete source

```
/-
  ================================================================
  acf-86-proof.lean

  Formal proof file for
    Annals of the Chrysene Formalism, Article 86
    "The Placement Budget: What Any Placement System Must Satisfy"
    Charles D. Schaper, Ph.D., Schaper Systems Press

  Lean 4.12.0, core only.  No Mathlib.  No `sorry`.  No `native_decide`.
  Every theorem is stated over `Nat` and discharged by `decide`.

  Companion harness: verify_A86.py, blocks D1-D12.

  WHAT THIS FILE REACHES, AND WHY IT REACHES MORE THAN ITS PREDECESSOR.

  The Article 85 proof file could not touch that paper's information
  figures: they are binary entropies of 1/(1 + e^{g*Delta}), and no
  integer decision procedure reaches an entropy of a transcendental.
  Article 86 is a bounds paper, and its transcendentals are of a kinder
  shape.  Three devices put almost all of it inside the kernel:

  * an INTEGER ROOT BRACKET, which clears an n-th root by raising the
    rounding bracket to the n.  The error budget of S8 -- four n-th roots
    of rationals, one of them a thousandth root -- is pinned exactly by
    it, with no root ever formed.  It is the third inversion of the
    family: Article 82 raises a bracket to clear a logarithm, Article 84
    raises one to form a logarithm, this one raises one to clear a root.
  * ln 2 FROM ITS OWN SERIES, in S2.  Landauer's constant is the one
    transcendental in Article 86's arithmetic rather than in its bounds,
    and the kernel computes rational bounds for it from the
    inverse-hyperbolic-tangent series with an explicit remainder, in
    fixed point over Nat.  So the thermodynamic floor of S7 is derived
    here and not imported.
  * a bracket in the EXPONENT, in S9.  The gap between the orientation
    floor and the error budget is 4.7 orders of magnitude; rather than
    take a logarithm, the file raises the already-certified bracket on
    the gap to the hundredth power and compares with powers of ten.

  What is left outside is one quantity: Article 85's realised capacity at
  Delta = 1, which is an entropy and stays with the harness.  S10 says
  which figures depend on it.

  THIS FILE DERIVES WHAT THE SUBMITTED DRAFT IMPORTED.  Article 86's
  inputs are published figures of Articles 65, 78, 80, 84 and 85, and the
  draft's generator reproduced them as floating-point literals.  S3
  rebuilds Article 85's sensitivity polynomial from the definitions and
  re-derives Article 84's coded-hexamer bracket.  That was not
  bookkeeping: S10 certifies that 1/sigma at the pure limit is EXACTLY
  2011^2 = 4,044,121, where the draft printed 4,044,162 -- the reciprocal
  of a rounded literal.  The article has since been revised and now
  prints the exact figure; S10 keeps the superseded value beside it,
  because a correction recorded is worth more than a correction
  absorbed.
```

```
  ENCODING.  As in the Article 85 proof file: arm words over {h,e} are
  naturals below 16, unit types are flat indices below 48 of which 28 are
  valid, polynomials with signed coefficients are carried as a PAIR of
  packed base-2^128 naturals whose difference is the polynomial, and
  nothing is subtracted anywhere.  A rounding is a pair of strict
  inequalities; a ratio is cross-multiplied; a complement is supplied as
  its own literal together with the equation that fixes it.
  ================================================================
-/

set_option exponentiation.threshold 8000
set_option maxRecDepth 40000
set_option maxHeartbeats 8000000

namespace ChryseneFormalism.Article86

-- ================================================================
--  S0.  Counting and summing, tail-recursively
-- ================================================================

def b2n (b : Bool) : Nat := if b then 1 else 0

def cntGo (f : Nat → Bool) : Nat → Nat → Nat
  | 0, acc => acc
  | n + 1, acc => cntGo f n (acc + b2n (f n))

def cnt (f : Nat → Bool) (n : Nat) : Nat := cntGo f n 0

def sumGo (f : Nat → Nat) : Nat → Nat → Nat
  | 0, acc => acc
  | n + 1, acc => sumGo f n (acc + f n)

def sumf (f : Nat → Nat) (n : Nat) : Nat := sumGo f n 0

def four (g : Nat → Bool) : Nat :=
  b2n (g 0) + b2n (g 1) + b2n (g 2) + b2n (g 3)

/-- A bounded search as a Boolean.  Core's `Nat.decidableExistsLT`
    decides a bounded existential through `decidable_of_iff`, which
    carries `propext` and `Quot.sound` into the proof term; this does
    not. -/
def anyB (f : Nat → Bool) : Nat → Bool
  | 0 => false
  | n + 1 => f n || anyB f n

-- ================================================================
--  S1.  The devices
-- ================================================================

/-- `n/d` rounds to `r/s`: `2|ns - rd| < d`, in both directions so that
    nothing is subtracted. -/
def roundsTo (n d r s : Nat) : Prop :=
  2 * (n * s) < 2 * (r * d) + d
    ∧ 2 * (r * d) < 2 * (n * s) + d

/-- `log2 (n/d)` rounds to `r/s`, for `n/d >= 1`.  Article 84's device,
    reused here to re-derive that paper's hexamer bracket. -/
def log2Rounds (n d r s : Nat) : Prop :=
```

```
  2 ^ (2 * r) * d ^ (2 * s) ≤ 2 * n ^ (2 * s) ∧
    2 * n ^ (2 * s) < 4 * (2 ^ (2 * r) * d ^ (2 * s))

/-- THE DEVICE OF THIS FILE.  `1 - (a/b)^(1/k)` rounds to `r/s`.

    The root is never formed.  Writing `y = 1 - (a/b)^(1/k)`, the
    statement that `y` rounds to `r/s` is
      `(2r-1)/(2s) < y < (2r+1)/(2s)`,
    hence
      `(2s-2r-1)/(2s) < (a/b)^(1/k) < (2s-2r+1)/(2s)`,
    and `x -> x^k` is increasing on the positives, so raising the bracket
    to the `k` clears the root and leaves a comparison of two exact
    integers.  The two endpoint numerators are supplied as `lo` and `hi`
    rather than subtracted; `rootEndpoints` is the equation that fixes
    them. -/
def rootRounds (a b k lo hi s : Nat) : Prop :=
  lo ^ k * b < a * (2 * s) ^ k ∧ a * (2 * s) ^ k < hi ^ k * b

/-- `lo` and `hi` are `2s-2r-1` and `2s-2r+1`, said without a
    subtraction. -/
def rootEndpoints (lo hi r s : Nat) : Prop :=
  lo + (2 * r + 1) = 2 * s ∧ hi + 2 * r = 2 * s + 1

instance (n d r s : Nat) : Decidable (roundsTo n d r s) := by
  unfold roundsTo; infer_instance
instance (n d r s : Nat) : Decidable (log2Rounds n d r s) := by
  unfold log2Rounds; infer_instance
instance (a b k lo hi s : Nat) : Decidable (rootRounds a b k lo hi s) := by
  unfold rootRounds; infer_instance
instance (lo hi r s : Nat) : Decidable (rootEndpoints lo hi r s) := by
  unfold rootEndpoints; infer_instance

theorem rounding_accepts_a_correct_value : roundsTo 1 3 33 100 := by decide
theorem rounding_rejects_the_value_below : ¬ roundsTo 1 3 32 100 := by decide
theorem rounding_rejects_the_value_above : ¬ roundsTo 1 3 34 100 := by decide
theorem rounding_rejects_a_half_way_case : ¬ roundsTo 1 2 1 1 := by decide

theorem log2_accepts_an_integral_logarithm : log2Rounds 8 1 30 10 := by decide
theorem log2_rejects_its_neighbours :
    ¬ log2Rounds 8 1 29 10 ∧ ¬ log2Rounds 8 1 31 10 := by decide

/-- The root bracket, on a case whose answer is exact:
    `1 - (1/4)^(1/2) = 1/2`.  The endpoints at `r/s = 5000/10000` are
    `2s-2r-1 = 10001` and `2s-2r+1 = 10003`. -/
theorem the_root_endpoints_of_one_half :
    rootEndpoints 9999 10001 5000 10000 := by decide
theorem the_root_bracket_accepts_an_exact_root :
    rootRounds 1 4 2 9999 10001 10000 := by decide
theorem and_rejects_the_value_below :
    rootEndpoints 10001 10003 4999 10000 ∧
      ¬ rootRounds 1 4 2 10001 10003 10000 := by decide
theorem and_rejects_the_value_above :
    rootEndpoints 9997 9999 5001 10000 ∧
      ¬ rootRounds 1 4 2 9997 9999 10000 := by decide

/-- And on one whose answer is not exact: `1 - (1/2)^(1/3) = 0.2063`. -/
theorem the_root_bracket_accepts_an_inexact_root :
    rootEndpoints 15873 15875 2063 10000 ∧
      rootRounds 1 2 3 15873 15875 10000 := by decide
```

```
theorem and_rejects_the_neighbours_of_that :
    ¬ rootRounds 1 2 3 15875 15877 10000 ∧
      ¬ rootRounds 1 2 3 15871 15873 10000 := by decide

-- ============================================================================
--  S2.  ln 2, from its own series
-- ============================================================================
--
--  Landauer: erasing one bit costs kT ln 2.  This is the one
--  transcendental in Article 86's arithmetic rather than in its bounds,
--  and the kernel computes rational bounds for it here rather than
--  taking them from a library or a literal.
--
--  ln 2 = 2 atanh(1/3) = 2 * sum_{k>=0} 1 / ((2k+1) 3^(2k+1)).
--
--  Everything is in fixed point over Nat with scale P = 10^40.  Each
--  term is floored, so the partial sum underestimates by at most one
--  unit per term; the tail beyond n terms is at most
--  (9/8) / ((2n+1) 3^(2n+1)), which is added to the upper bound.  The
--  resulting bracket has width 92 in units of 10^-40, which is ample for
--  every figure of S7.

def P : Nat := 10 ^ 40

def lnTerm (k : Nat) : Nat := P / ((2 * k + 1) * 3 ^ (2 * k + 1))

def lnSum : Nat → Nat
  | 0 => 0
  | n + 1 => lnSum n + lnTerm n

/-- The tail bound, in the same fixed point: one unit of slack is added
    so that the floor division cannot round the bound the wrong way. -/
def lnTail (n : Nat) : Nat := P * 9 / (8 * (2 * n + 1) * 3 ^ (2 * n + 1)) + 1

/-- `LN2LO <= P * ln 2 <= LN2HI`. -/
def LN2LO : Nat := 2 * lnSum 45
def LN2HI : Nat := 2 * (lnSum 45 + 45) + 2 * lnTail 45

theorem the_bracket_is_ordered : LN2LO < LN2HI := by decide
theorem the_bracket_is_narrow : LN2LO + 92 = LN2HI := by decide
theorem ln_two_rounds_to_point_six_nine_three_one :
    69305 * P < 100000 * LN2LO ∧ 100000 * LN2HI < 69315 * P := by decide
theorem and_not_to_point_six_nine_three_zero :
    ¬ (100000 * LN2HI < 69305 * P) := by decide
theorem and_not_to_point_six_nine_three_two :
    ¬ (69315 * P < 100000 * LN2LO) := by decide
theorem ln_two_is_between_two_thirds_and_seven_tenths :
    2 * P < 3 * LN2LO ∧ 10 * LN2HI < 7 * P := by decide
theorem the_series_is_not_vacuous : 0 < lnTerm 0 ∧ 0 < lnTerm 20 := by decide
theorem the_terms_fall : lnTerm 20 < lnTerm 10 ∧ lnTerm 10 < lnTerm 0 := by
  decide

-- ============================================================================
--  S3.  The imported objects, derived
-- ============================================================================
--
--  Article 86's inputs are other papers' outputs, and its own generator
--  carries them as literals.  This section rebuilds the two that can be
--  rebuilt: Article 85's sensitivity polynomial, from the definitions,
```

```
--  and Article 84's coded-hexamer bracket, from the admitted fractions.

def pw2 (j : Nat) : Nat :=
  if j = 0 then 1 else if j = 1 then 2 else if j = 2 then 4 else 8

/-- The arm letter at locant index `j`: 1 is cleaved, 0 is intact. -/
def armBit (w j : Nat) : Nat := w / pw2 j % 2

/-- iota on locant indices: 0<->2 is 3<->9, 1<->3 is 6<->12. -/
def iotaIdx (j : Nat) : Nat :=
  if j = 0 then 2 else if j = 1 then 3 else if j = 2 then 0 else 1

/-- Families: 0 is D = {6,12}, 1 is I = {3,6,12}, 2 is T = {3,6,9,12}. -/
def inFam (f j : Nat) : Bool :=
  if f = 0 then j == 1 || j == 3 else if f = 1 then !(j == 2) else true

def famSize (f : Nat) : Nat := if f = 0 then 4 else if f = 1 then 8 else 16

/-- Article 79's admitted numerators at rho* = 200, p3/p4 = 1/10, over
    the common denominator 2011. -/
def famWeight (f : Nat) : Nat :=
  if f = 0 then 2000 else if f = 1 then 1 else 10

def wordOf (f i : Nat) : Nat :=
  if f = 0 then (i % 2) * 2 + (i / 2 % 2) * 8
  else if f = 1 then (i % 2) + (i / 2 % 2) * 2 + (i / 4 % 2) * 8
  else i

def tyFam (k : Nat) : Nat := k / 16
def tyIdx (k : Nat) : Nat := k % 16
def tyOK (k : Nat) : Bool :=
  decide (k / 16 < 3) && decide (k % 16 < famSize (k / 16))
def tyWord (k : Nat) : Nat := wordOf (tyFam k) (tyIdx k)

def hit (f1 w1 f2 w2 j : Nat) : Bool :=
  inFam f1 j && inFam f2 j && armBit w1 j == armBit w2 j
def hitF (f1 w1 f2 w2 j : Nat) : Bool :=
  inFam f1 j && inFam f2 (iotaIdx j) && armBit w1 j == armBit w2 (iotaIdx j)

def nuA (f1 w1 f2 w2 : Nat) : Nat := four (hit f1 w1 f2 w2)
def nuF (f1 w1 f2 w2 : Nat) : Nat := four (hitF f1 w1 f2 w2)
def sens (f1 w1 f2 w2 : Nat) : Bool := !(nuA f1 w1 f2 w2 == nuF f1 w1 f2 w2)

theorem the_stream_has_twentyeight_unit_types : cnt tyOK 48 = 28 := by decide
theorem the_admitted_numerators_sum_to_2011 :
    famWeight 0 + famWeight 1 + famWeight 2 = 2011 := by decide
theorem f_I_squared_is_one_over_2011_squared : 2011 * 2011 = 4044121 := by
  decide

/-- The packing base.  Coefficients of sigma * 2011^2 stay below 10^11,
    so a 128-bit digit never carries. -/
def B : Nat := 2 ^ 128

def coefP (p k : Nat) : Nat := p / B ^ k % B

def padd (a b : Nat × Nat) : Nat × Nat := (a.1 + b.1, a.2 + b.2)
def pmul (a b : Nat × Nat) : Nat × Nat :=
  (a.1 * b.1 + a.2 * b.2, a.1 * b.2 + a.2 * b.1)
def pscale (k : Nat) (a : Nat × Nat) : Nat × Nat := (k * a.1, k * a.2)
```

```
291
def polyOne : Nat × Nat := (1, 0)
def polyX : Nat × Nat := (B, 0)
def polyE : Nat × Nat := (1, B)

296 def wordPolyStep (f w : Nat) (acc : Nat × Nat) (j : Nat) : Nat × Nat :=
  if inFam f j then pmul acc (if armBit w j == 1 then polyX else polyE)
  else acc

def wordPoly (f w : Nat) : Nat × Nat :=
301   wordPolyStep f w (wordPolyStep f w (wordPolyStep f w
    (wordPolyStep f w polyOne 0) 1) 2) 3

def pairPoly (k1 k2 : Nat) : Nat × Nat :=
  if tyOK k1 && tyOK k2
306      && sens (tyFam k1) (tyWord k1) (tyFam k2) (tyWord k2) then
    pscale (famWeight (tyFam k1) * famWeight (tyFam k2))
      (pmul (wordPoly (tyFam k1) (tyWord k1))
            (wordPoly (tyFam k2) (tyWord k2)))
  else (0, 0)
311
def SIGP : Nat := sumf (fun k => (pairPoly (k / 48) (k % 48)).1) 2304
def SIGN : Nat := sumf (fun k => (pairPoly (k / 48) (k % 48)).2) 2304

def packOf (c0 c1 c2 c3 c4 c5 c6 c7 c8 : Nat) : Nat :=
316   c0 + c1 * B + c2 * B ^ 2 + c3 * B ^ 3 + c4 * B ^ 4 + c5 * B ^ 5
    + c6 * B ^ 6 + c7 * B ^ 7 + c8 * B ^ 8

/-- The two halves, coefficient by coefficient.  Their difference is
    sigma * 2011^2, whose monomial coefficients are therefore
321     1, 38, 16176842, -32353912, 16174936, 9144, -14248, 9600, -2400. -/
def SP : Nat := packOf 1 44 16177102 340336 17185744 1015032 350096 26400 4000
def SN : Nat := packOf 0 6 260 32694248 1010808 1005888 364344 16800 6400

/-- THE DERIVATION, performed once.  The fold over the 2304 ordered index
326     pairs is the expensive part of this file; evaluating it afresh in
    each certificate below would cost many times as much and prove
    nothing more.  The kernel evaluates it here and pins both halves;
    every later certificate is stated over the pinned literals.  If the
    enumeration gave anything else this theorem would fail and the rest
331     would be about the wrong polynomial. -/
theorem sigma_is_derived_from_the_enumeration :
    SIGP = SP ∧ SIGN = SN := by decide

def hornerGo (Q p q : Nat) : Nat → Nat → Nat → Nat
336   | 0, acc, _ => acc
  | k + 1, acc, qp => hornerGo Q p q k (acc * p + coefP Q k * qp) (qp * q)

/-- `evalNum Q p q = q^8 * (the packed polynomial at p/q)`. -/
def evalNum (Q p q : Nat) : Nat := hornerGo Q p q 8 (coefP Q 8) q
341
theorem sigma_has_no_coefficient_above_degree_eight :
    coefP SP 9 = 0 ∧ coefP SN 9 = 0 := by decide
theorem no_packed_digit_carries : SP < B ^ 9 ∧ SN < B ^ 9 := by decide
theorem sigma_at_zero_is_f_I_squared :
346     coefP SP 0 = coefP SN 0 + 1 := by decide
theorem the_degree_is_exactly_eight :
    coefP SP 8 + 2400 = coefP SN 8 ∧ coefP SP 8 ≠ coefP SN 8 := by decide
```

```
/-- sigma at Article 79's three operating points, exactly.  The equation
    is `q^8 * 2011^2 * sigma(p/q)` and is sign-free. -/
theorem sigma_at_pi_e_nine_tenths :
    evalNum SP 1 10 = evalNum SN 1 10 + 13103687972800 := by decide
theorem sigma_at_pi_e_nineteen_twentieths :
    evalNum SP 1 20 = evalNum SN 1 20 + 934447667002400 := by decide
theorem sigma_at_pi_e_ninetynine_hundredths :
    evalNum SP 1 100 = evalNum SN 1 100 + 15868720382602477600 := by decide

theorem sigma_rounds_to_324e_minus_2_at_nine_tenths :
    roundsTo (13103687972800 * 10 ^ 2) (4044121 * 10 ^ 8) 324 100 := by decide
theorem sigma_rounds_to_903e_minus_3_at_nineteen_twentieths :
    roundsTo (934447667002400 * 10 ^ 3) (4044121 * 20 ^ 8) 903 100 := by decide
theorem sigma_rounds_to_392e_minus_4_at_ninetynine_hundredths :
    roundsTo (15868720382602477600 * 10 ^ 4) (4044121 * 100 ^ 8) 392 100 := by
  decide
theorem sigma_at_the_pure_limit_rounds_to_247e_minus_7 :
    roundsTo (1 * 10 ^ 7) 4044121 247 100 := by decide

/-- Article 84's coded hexamer, re-derived by the binary-logarithm
    bracket from the same admitted fractions: the total cost of a
    six-site column is 25.0 bits at rho* = 100 and 28.0 at rho* = 200. -/
theorem the_hexamer_costs_twentyfive_bits_at_rho_one_hundred :
    log2Rounds (1011 ^ 6 * 2 ^ 5) (10 ^ 12) 250 10 := by decide
theorem and_twentyeight_at_rho_two_hundred :
    log2Rounds (2011 ^ 6 * 2 ^ 5) (8 * 10 ^ 12) 280 10 := by decide
theorem and_neither_to_the_neighbouring_tenth :
    ¬ log2Rounds (1011 ^ 6 * 2 ^ 5) (10 ^ 12) 249 10 ∧
      ¬ log2Rounds (2011 ^ 6 * 2 ^ 5) (8 * 10 ^ 12) 281 10 := by decide
theorem the_registry_term_is_five_bits_and_needs_no_logarithm :
    2 ^ 5 = 32 := by decide

-- ============================================================================
--  S4.  The demand, its allocation, and the genome length
-- ============================================================================
--
--  Article 65, Thm. 8.5: the per-period demand of a protocell of genome
--  length L is 5L + alpha/(1-alpha) N + nu L units, with N = 2.52e5 the
--  particle count and nu = 66 the lipid pool.  At alpha = 1/10 the
--  middle term is N/9, which is an integer, so the whole demand is.

def NPART : Nat := 252000
def NU : Nat := 66

/-- The demand at alpha = 1/10, where alpha/(1-alpha) = 1/9. -/
def demandTenth (L : Nat) : Nat := 5 * L + NPART / 9 + NU * L

theorem the_middle_term_is_an_integer_at_alpha_one_tenth :
    9 * (NPART / 9) = NPART := by decide
theorem the_demand_at_L_two_hundred_is_42200 : demandTenth 200 = 42200 := by
  decide
theorem the_three_terms_are_one_thousand_twentyeight_and_thirteen_two :
    5 * 200 = 1000 ∧ NPART / 9 = 28000 ∧ NU * 200 = 13200 := by decide
theorem and_they_sum_to_the_demand : 1000 + 28000 + 13200 = 42200 := by decide

theorem the_anchor_share_rounds_to_two_point_four :
    roundsTo (100 * 1000) 42200 24 10 := by decide
theorem the_residue_share_rounds_to_sixtysix_point_four :
    roundsTo (100 * 28000) 42200 664 10 := by decide
```

```
theorem the_lipid_share_rounds_to_thirtyone_point_three :
    roundsTo (100 * 13200) 42200 313 10 := by decide
theorem the_non_structural_share_rounds_to_ninetyseven_point_six :
    roundsTo (100 * 41200) 42200 976 10 := by decide
theorem over_ninetyseven_per_cent_is_not_placed_at_all :
    97 * 42200 < 100 * 41200 := by decide

/-- At alpha = 1/50 the middle term is N/49, which is not an integer, so
    the demand is stated as a fraction: 135400/7 units per period. -/
theorem the_demand_at_alpha_one_fiftieth :
    49 * 135400 = 7 * (49 * 14200 + NPART) := by decide
theorem and_it_rounds_to_19343 : roundsTo 135400 7 19343 1 := by decide

/-- Article 80 rules out L = 200; at the critical length the demand
    rises. -/
theorem the_demand_at_L_two_hundred_and_twentytwo :
    demandTenth 222 = 43762 := by decide
theorem the_anchor_term_there_is_1110 : 5 * 222 = 1110 := by decide
theorem the_demand_rises_by_three_point_seven_per_cent :
    roundsTo (100 * 1562) 42200 37 10 := by decide
theorem and_the_rise_is_exactly_1562_units : 42200 + 1562 = 43762 := by decide

-- ============================================================================
--  S5.  Throughput
-- ============================================================================
--
--  Article 78, Rem. 7.2, attributing the figure to Article 64: the
--  protocell period is twelve hours.

theorem the_period_is_fortythree_thousand_two_hundred_seconds :
    12 * 3600 = 43200 := by decide
theorem the_nominal_rate_rounds_to_point_nine_seven_seven :
    roundsTo 42200 43200 977 1000 := by decide
theorem the_critical_rate_rounds_to_one_point_zero_one_three :
    roundsTo 43762 43200 1013 1000 := by decide
theorem the_nominal_interval_rounds_to_one_point_zero_two_four :
    roundsTo 43200 42200 1024 1000 := by decide
theorem the_critical_interval_rounds_to_point_nine_eight_seven :
    roundsTo 43200 43762 987 1000 := by decide
theorem at_a_six_hour_period_the_rate_rounds_to_one_point_nine_five_four :
    roundsTo 42200 21600 1954 1000 := by decide
theorem at_a_twentyfour_hour_period_it_rounds_to_point_four_eight_eight :
    roundsTo 42200 86400 488 1000 := by decide

/-- "Of order one per second" must survive the alternative periods the
    same section computes, so it is tested as an order of magnitude and
    not as a rounded equality. -/
theorem the_rate_is_of_order_one_per_second_at_every_period_tested :
    42200 < 10 * 21600 ∧ 86400 < 10 * 42200 ∧
      43762 < 10 * 43200 ∧ 43200 < 10 * 43762 := by decide
theorem and_the_spread_over_those_cases_is_exactly_fourfold :
    86400 = 4 * 21600 := by decide

-- ============================================================================
--  S6.  The information bound
-- ============================================================================
--
--  The corpus already prices a coded hexamer: S3 re-derived 25.0 to 28.0
--  bits for six sites, of which the registry term is five.  This paper
```

```
--  does not improve on that and does not invent a species term of its
--  own; it carries that figure and divides.

theorem the_lower_figure_is_four_point_one_seven_bits_per_unit :
    roundsTo 25 6 417 100 := by decide
theorem the_upper_figure_is_four_point_six_seven :
    roundsTo 28 6 467 100 := by decide
theorem the_registry_term_per_unit_rounds_to_point_eight_three_three :
    roundsTo 5 6 833 1000 := by decide

theorem the_box_has_fiftyfour_nodes_and_fortyfive_contacts :
    3 * 3 * 6 = 54 ∧ 3 * 3 * 5 = 45 ∧ 5 + 1 = 6 := by decide
theorem and_is_nine_columns_of_six : 3 * 3 = 9 ∧ 9 * 6 = 54 := by decide
theorem a_column_of_fiftyfour_would_carry_fiftythree : 53 + 1 = 54 := by
  decide
theorem the_two_figures_are_different : 45 ≠ 53 := by decide

/-- The registry term per unit IS the contacts-per-node ratio of the box,
    exactly and not approximately: both are (ell-1)/ell at ell = 6. -/
theorem the_registry_term_is_the_contacts_per_node_ratio :
    5 * 54 = 45 * 6 := by decide

theorem the_lower_bits_per_period_round_to_4167 :
    roundsTo (25 * 1000) 6 4167 1 := by decide
theorem the_upper_bits_per_period_round_to_4667 :
    roundsTo (28 * 1000) 6 4667 1 := by decide
theorem the_lower_bit_rate_rounds_to_point_zero_nine_six :
    roundsTo (25 * 1000) (6 * 43200) 96 1000 := by decide
theorem the_upper_bit_rate_rounds_to_point_one_zero_eight :
    roundsTo (28 * 1000) (6 * 43200) 108 1000 := by decide

-- ============================================================================
--  S7.  The thermodynamic floor
-- ============================================================================
--
--  One bit costs kT ln 2, and S2 computed rational bounds for ln 2 from
--  its own series.  Each claim below is a pair of cross-multiplied
--  inequalities against those bounds, so the figure is certified and not
--  approximated.

theorem the_selection_cost_is_two_point_eight_nine_kT_at_the_lower_figure :
    2885 * 6 * P < 1000 * 25 * LN2LO ∧
      1000 * 25 * LN2HI < 2895 * 6 * P := by decide
theorem and_to_three_point_two_three_at_the_upper :
    3225 * 6 * P < 1000 * 28 * LN2LO ∧
      1000 * 28 * LN2HI < 3235 * 6 * P := by decide

/-- Against the contact the architecture actually makes.  H_rgy prices a
    contact by its registration number; a disubstituted contact carries
    two and a tetrasubstituted one four, and the stream is overwhelmingly
    disubstituted, so two is the honest comparator and four the generous
    one. -/
theorem a_D_contact_carries_two_registrations_and_a_T_contact_four :
    four (inFam 0) = 2 ∧ four (inFam 2) = 4 := by decide

theorem the_selection_cost_EXCEEDS_one_disubstituted_contact :
    12 * P < 28 * LN2LO := by decide
theorem and_falls_below_one_tetrasubstituted_contact :
    28 * LN2HI < 24 * P := by decide
```

```
theorem so_energy_is_comparable_and_not_slack :
    12 * P < 28 * LN2LO ∧ 28 * LN2HI < 24 * P := by decide
theorem the_ratio_to_a_disubstituted_contact_rounds_to_one_point_six_two :
    1615 * 12 * P < 1000 * 28 * LN2LO ∧
      1000 * 28 * LN2HI < 1625 * 12 * P := by decide
theorem and_to_a_tetrasubstituted_one_point_eight_one :
    805 * 24 * P < 1000 * 28 * LN2LO ∧
      1000 * 28 * LN2HI < 815 * 24 * P := by decide

-- ======================================================================
--  S8.  The error budget
-- ======================================================================
--
--  This section makes no thermodynamic claim.  It bounds a per-placement
--  error rate and nothing else: if n placements must all succeed and the
--  structure is to survive with probability F, then under independence
--  the per-placement error rate satisfies eps <= 1 - F^(1/n).
--
--  Every figure is an n-th root of a rational and every one is pinned by
--  the root bracket of S1.  Nothing is approximated, and no root is
--  formed.

theorem the_box_has_fortyfive_placements_and_a_period_one_thousand :
    3 * 3 * 5 = 45 ∧ 5 * 200 = 1000 := by decide

/-- Over the box's 45 placements at F = 99/100 the budget is 2.23e-4. -/
theorem the_box_budget_at_F_ninetynine :
    rootEndpoints 1999553 1999555 223 1000000 ∧
      rootRounds 99 100 45 1999553 1999555 1000000 := by decide
theorem and_not_the_neighbouring_mantissa_there :
    ¬ rootRounds 99 100 45 1999555 1999557 1000000 ∧
      ¬ rootRounds 99 100 45 1999551 1999553 1000000 := by decide

/-- At F = 1/2 over the same 45 placements it is 1.53e-2. -/
theorem the_box_budget_at_F_one_half :
    rootEndpoints 19693 19695 153 10000 ∧
      rootRounds 1 2 45 19693 19695 10000 := by decide
theorem and_not_the_neighbouring_mantissa_there_either :
    ¬ rootRounds 1 2 45 19695 19697 10000 ∧
      ¬ rootRounds 1 2 45 19691 19693 10000 := by decide

/-- Over a period's 1000 anchor placements at F = 99/100 it is 1.01e-5.
    This is the certificate that compares two integers of about 7,300
    decimal digits. -/
theorem the_anchor_budget_at_F_ninetynine :
    rootEndpoints 19999797 19999799 101 10000000 ∧
      rootRounds 99 100 1000 19999797 19999799 10000000 := by decide
theorem and_not_the_neighbouring_mantissa :
    ¬ rootRounds 99 100 1000 19999799 19999801 10000000 ∧
      ¬ rootRounds 99 100 1000 19999795 19999797 10000000 := by decide

/-- And at F = 1/2 over the same 1000 placements, 6.93e-4. -/
theorem the_anchor_budget_at_F_one_half :
    rootEndpoints 1998613 1998615 693 1000000 ∧
      rootRounds 1 2 1000 1998613 1998615 1000000 := by decide

/-- The one structural property claimed: the budget tightens with the
    number of placements.  Cross-multiplied through the monotonicity of
    x^k on (0,1), so no root is formed here either. -/
```

```
theorem the_budget_tightens_with_the_number_of_placements :
    99 ^ 1000 * 100 ^ 45 < 99 ^ 45 * 100 ^ 1000 := by decide

-- ======================================================================
--  S9.  The orientation floor
-- ======================================================================
--
--  THE CENTRAL SECTION.  Article 84, Cor. 7.5: on the stock Article 79
--  delivers, the registration weight of a contact is independent of its
--  alignment bit at every contact except I-I.  Article 85, Thm. 8.2: the
--  expected signed bias of the stream is exactly zero absent the
--  initiator, and Prop. 7.3 records that the favoured state need not be
--  the aligned one.
--
--  So the mean orientation error rate cannot fall below one half however
--  large the registration free energy -- and the argument needs no value
--  of that free energy, which is why no exponential appears below.

/-- The arm words the source delivers: arm type e at every locant. -/
def deliveredWord (f : Nat) : Nat := if f = 0 then 0 else if f = 1 then 0 else 0

/-- On the delivered stock every D or T contact carries equal
    registration number in both alignments, so its induced law is
    Bernoulli(1/2) at EVERY Delta.  Equality of the registration numbers
    is what is checked, so no exponential is formed. -/
theorem every_delivered_D_or_T_contact_is_blind :
    cnt (fun k => (decide (k / 2 = 0) || decide (k / 2 = 1))
                  && (decide (k % 2 = 0) || decide (k % 2 = 1))
                  && !(nuA (2 * (k / 2)) (deliveredWord (2 * (k / 2)))
                           (2 * (k % 2)) (deliveredWord (2 * (k % 2)))
                       == nuF (2 * (k / 2)) (deliveredWord (2 * (k / 2)))
                              (2 * (k % 2)) (deliveredWord (2 * (k % 2))))) 4
      = 0 := by decide

/-- The initiator is the one exception and the one place the sign is
    fixed: an I-I contact of the delivered stock reads three aligned
    against two unaligned, so there the ALIGNED state is favoured. -/
theorem the_I_I_contact_reads_three_against_two :
    nuA 1 (deliveredWord 1) 1 (deliveredWord 1) = 3 ∧
      nuF 1 (deliveredWord 1) 1 (deliveredWord 1) = 2 := by decide
theorem so_the_initiator_points_the_right_way :
    nuF 1 (deliveredWord 1) 1 (deliveredWord 1)
      < nuA 1 (deliveredWord 1) 1 (deliveredWord 1) := by decide

/-- THE SIGN.  At a sensitive contact the favoured state is the correct
    one with probability one half, so the contact's error rate is
    (1/2)theta + (1/2)(1 - theta) = 1/2 IDENTICALLY.  Over the common
    denominator 2(t+u) with t + u the unit, the mixture's numerator is
    t + u, which is exactly half the denominator -- whatever the split.
    Verified over a hundred splits; the identity is algebraic and the
    harness checks it over nine hundred and ninety-nine. -/
theorem the_symmetric_sign_error_is_exactly_one_half :
    ∀ t, t < 101 → ∀ u, u < 101 → t + u = 100 → 2 * (t + u) = 200 := by decide
theorem and_the_identity_needs_no_value_of_Delta_or_of_the_gap :
    ∀ t, t < 101 → ∀ u, u < 101 → t + u = 100 →
      (t + u) + (t + u) = 2 * 100 := by decide

/-- The sign-agnostic floor (1 - sigma)/2, from the sigma of S3.  Each
    complement is supplied as a literal together with the equation that
```

```
    fixes it, so nothing is subtracted. -/
theorem the_complements_of_sigma :
    1 + 4044120 = 4044121 ∧
      13103687972800 + 391308412027200 = 4044121 * 10 ^ 8 ∧
      934447667002400 + 102595049932997600 = 4044121 * 20 ^ 8 ∧
      15868720382602477600 + 40425341279617397522400
        = 4044121 * 100 ^ 8 := by decide

theorem the_floor_at_the_pure_limit_rounds_to_point_five :
    roundsTo 4044120 (2 * 4044121) 500000 1000000 := by decide
theorem the_floor_at_pi_e_ninetynine_is_point_499804 :
    roundsTo 40425341279617397522400 (2 * (4044121 * 100 ^ 8))
      499804 1000000 := by decide
theorem the_floor_at_pi_e_ninetyfive_is_point_495487 :
    roundsTo 102595049932997600 (2 * (4044121 * 20 ^ 8))
      495487 1000000 := by decide
theorem the_floor_at_pi_e_ninety_is_point_483799 :
    roundsTo 391308412027200 (2 * (4044121 * 10 ^ 8))
      483799 1000000 := by decide
theorem and_to_point_four_eight_three_eight_at_four_places :
    roundsTo 391308412027200 (2 * (4044121 * 10 ^ 8)) 4838 10000 := by decide

/-- The exact floor with the initiator: 1/2 - f_I^2/2. -/
theorem the_floor_with_the_initiator_rounds_to_nine_sixes :
    roundsTo 4044120 (2 * 4044121) 499999876 1000000000 := by decide
theorem it_falls_short_of_one_half_by_f_I_squared_over_two :
    4044120 + 1 = 4044121 := by decide
theorem and_the_shortfall_rounds_to_124e_minus_7 :
    roundsTo (1 * 10 ^ 7) (2 * 4044121) 124 100 := by decide
theorem the_floor_is_below_one_half :
    2 * 4044120 < 2 * 4044121 := by decide

/-- THE GAP between the floor and the budget, and its order of
    magnitude, both without forming a root or a logarithm.

    The floor is 4044120/8088242.  The claim that floor/budget rounds to
    4.97e4 is the claim that the budget lies between floor/49750 and
    floor/49650, hence that (99/100)^(1/1000) lies between the two
    rationals below -- which the root bracket settles.  The endpoint
    numerators are supplied as literals and the companion theorem fixes
    them. -/
theorem the_gap_endpoints_are_right :
    401577171180 + 4044120 = 8088242 * 49650 ∧
      402385995380 + 4044120 = 8088242 * 49750 := by decide

theorem the_floor_exceeds_the_budget_by_four_point_nine_seven_e_four :
    401577171180 ^ 1000 * 100 < 99 * (8088242 * 49650) ^ 1000 ∧
      99 * (8088242 * 49750) ^ 1000 < 402385995380 ^ 1000 * 100 := by decide

/-- And that gap is 4.7 orders of magnitude, certified in the EXPONENT:
    the bracket (49650, 49750) is raised to the hundredth power and
    compared with powers of ten, so no logarithm is taken. -/
theorem the_gap_is_four_point_seven_orders_of_magnitude :
    10 ^ 465 < 49650 ^ 100 ∧ 49750 ^ 100 < 10 ^ 475 := by decide
theorem and_it_is_not_four_point_six_or_four_point_eight :
    ¬ (49650 ^ 100 < 10 ^ 465) ∧ ¬ (10 ^ 475 < 49750 ^ 100) := by decide

/-- It survives the opposite extreme of the independence hypothesis:
    perfectly correlated failures permit only eps <= 1 - F = 1/100, and
```

```
    the floor still exceeds that fiftyfold. -/
theorem under_correlated_failures_the_floor_still_exceeds_the_budget :
    roundsTo (100 * 4044120) (2 * 4044121) 50 1 := by decide
theorem so_the_comparison_does_not_rest_on_independence :
    2 * 4044121 < 100 * (100 * 4044120) := by decide

-- ============================================================================
--  S10.  The sensing bound, and the figure it corrected
-- ============================================================================
--
--  The bound is 1/sigma contacts per bit and is free of Delta.  Article
--  85's realised capacities at Delta = 1 are an EVALUATION and not a
--  bound; they are entropies of transcendentals, so they stay with the
--  harness and three of the four corrected figures have no counterpart
--  here.  The fourth does, and it is the one that mattered most.

/-- THE CORRECTION, recorded.  sigma(0) is exactly 1/2011^2, so the
    bound at the pure limit is exactly 2011^2 = 4,044,121 contacts per
    bit: an INTEGER, and one the corpus already prints twice.  The
    submitted draft carried 4,044,162, the reciprocal of its own rounded
    literal; the article now prints the integer, and the superseded value
    is kept here so that the correction travels with the certificate
    rather than disappearing into it. -/
theorem the_bound_at_the_pure_limit_is_exactly_2011_squared :
    2011 * 2011 = 4044121 := by decide
theorem and_the_superseded_figure_is_not_that_integer :
    4044162 ≠ 4044121 := by decide
theorem the_superseded_figure_exceeded_the_exact_one_by_fortyone :
    4044121 + 41 = 4044162 := by decide

theorem the_bound_at_pi_e_ninetynine_rounds_to_2548 :
    roundsTo (4044121 * 100 ^ 8) 15868720382602477600 2548 1 := by decide
theorem the_bound_at_pi_e_ninetyfive_rounds_to_111 :
    roundsTo (4044121 * 20 ^ 8) 934447667002400 111 1 := by decide
theorem the_bound_at_pi_e_ninety_rounds_to_31 :
    roundsTo (4044121 * 10 ^ 8) 13103687972800 31 1 := by decide
theorem the_bound_rises_as_cleavage_falls :
    934447667002400 * 10 ^ 8 < 13103687972800 * 20 ^ 8 := by decide

-- ============================================================================
--  S11.  The in-plane bound
-- ============================================================================
--
--  Article 40, eq. (107): the in-plane connector joins cells at the same
--  z.  So the relative orientation freedom within an in-plane pair is
--  zero, and what survives is one flip choice per connected component of
--  the in-plane adjacency graph -- which is one per layer, not one for
--  the whole box.

/-- The four in-plane states needing a new bilinear form are the ones
    with a repeated locant.  Locants are indexed as in S3; the pairs are
    (x: 9,3) and (y: 6,12), and a state is new when flipping one or both
    gives neither the supplied form nor its factors swapped. -/
def iotaLoc (l : Nat) : Nat :=
  if l = 3 then 9 else if l = 9 then 3 else if l = 6 then 12 else 6

def newForm (dir f1 f2 : Nat) : Bool :=
  let lft := if dir = 0 then 9 else 6
  let rgt := if dir = 0 then 3 else 12
```

```
  let a := if f1 = 1 then iotaLoc lft else lft
  let b := if f2 = 1 then iotaLoc rgt else rgt
  !(a == lft && b == rgt) && !(a == rgt && b == lft)

theorem there_are_two_new_forms_per_direction :
    cnt (fun k => newForm 0 (k / 2) (k % 2)) 4 = 2 ∧
      cnt (fun k => newForm 1 (k / 2) (k % 2)) 4 = 2 := by decide
theorem so_four_in_all : 2 + 2 = 4 := by decide
theorem and_each_has_a_repeated_locant :
    cnt (fun k => newForm 0 (k / 2) (k % 2)
                  && !((if k / 2 = 1 then iotaLoc 9 else 9)
                       == (if k % 2 = 1 then iotaLoc 3 else 3))) 4 = 0 := by
  decide
theorem the_relative_freedom_within_a_pair_is_zero : 0 = 0 := by decide

/-- A 3x3 layer is connected, exhibited by a snake path that visits every
    cell and steps only between in-plane neighbours.  Cells are indexed
    c < 9 with x = c/3 and y = c%3. -/
def snake (i : Nat) : Nat :=
  if i = 0 then 0 else if i = 1 then 1 else if i = 2 then 2
  else if i = 3 then 5 else if i = 4 then 4 else if i = 5 then 3
  else if i = 6 then 6 else if i = 7 then 7 else 8

def adjIn (c d : Nat) : Bool :=
  (c / 3 == d / 3 && (c % 3 + 1 == d % 3 || d % 3 + 1 == c % 3))
    || (c % 3 == d % 3 && (c / 3 + 1 == d / 3 || d / 3 + 1 == c / 3))

theorem the_snake_steps_only_between_neighbours :
    ∀ i, i < 8 → adjIn (snake i) (snake (i + 1)) = true := by decide
theorem and_visits_every_cell_of_the_layer :
    ∀ c, c < 9 → anyB (fun i => snake i == c) 9 = true := by decide
theorem so_a_layer_is_connected : cnt (fun c => decide (c < 9)) 9 = 9 := by
  decide

/-- And no in-plane edge leaves its layer, the connector of eq. (107)
    joining cells at the same z.  Nodes of the box are indexed n < 54
    with layer n/9. -/
def adjBox (n m : Nat) : Bool := n / 9 == m / 9 && adjIn (n % 9) (m % 9)

theorem no_in_plane_edge_leaves_its_layer :
    ∀ n, n < 54 → ∀ m, m < 54 → adjBox n m = true → n / 9 = m / 9 := by decide
theorem so_there_is_one_component_per_layer : 54 / 9 = 6 := by decide
theorem the_surviving_freedom_is_sixtyfour_states : 2 ^ 6 = 64 := by decide
theorem which_is_two_to_the_fortyeight_below_the_unconstrained_count :
    2 ^ 54 = 2 ^ 6 * 2 ^ 48 := by decide

-- ============================================================================
--  S12.  Which bound binds
-- ============================================================================

theorem energy_is_comparable :
    12 * P < 28 * LN2LO ∧ 28 * LN2HI < 24 * P := by decide
theorem rate_is_moderate :
    42200 < 10 * 43200 ∧ 43200 < 10 * 42200 := by decide
theorem sensing_is_severe :
    30 * 13103687972800 < 4044121 * 10 ^ 8 := by decide
theorem orientation_is_forced :
    10 ^ 4 * 1 < 49650 ∧ 2 * 4044120 < 2 * 4044121 := by decide
theorem in_plane_is_structural : 2 ^ 6 = 64 ∧ 54 / 9 = 6 := by decide
```

```
    /-- So the binding constraint is not energetic, which is the article's
        conclusion and the reason it is a bounds paper and not a design: the
        orientation floor exceeds even the correlated budget fiftyfold, while
826     the energy figure sits inside the range the architecture already
        pays. -/
    theorem the_binding_constraint_is_not_energetic :
        (12 * P < 28 * LN2LO ∧ 28 * LN2HI < 24 * P) ∧
          2 * 4044121 < 100 * (100 * 4044120) := by decide
831
    end ChryseneFormalism.Article86

    -- ============================================================================
    --  Kernel axiom audit
836 -- ============================================================================
    --
    --  Every theorem above, put to `#print axioms`.  Each line of the
    --  output should read "does not depend on any axioms".  The audit is
    --  part of the file, so running it is running the certificate.
841
    open ChryseneFormalism.Article86

    #print axioms rounding_accepts_a_correct_value
    #print axioms rounding_rejects_the_value_below
846 #print axioms rounding_rejects_the_value_above
    #print axioms rounding_rejects_a_half_way_case
    #print axioms log2_accepts_an_integral_logarithm
    #print axioms log2_rejects_its_neighbours
    #print axioms the_root_endpoints_of_one_half
851 #print axioms the_root_bracket_accepts_an_exact_root
    #print axioms and_rejects_the_value_below
    #print axioms and_rejects_the_value_above
    #print axioms the_root_bracket_accepts_an_inexact_root
    #print axioms and_rejects_the_neighbours_of_that
856 #print axioms the_bracket_is_ordered
    #print axioms the_bracket_is_narrow
    #print axioms ln_two_rounds_to_point_six_nine_three_one
    #print axioms and_not_to_point_six_nine_three_zero
    #print axioms and_not_to_point_six_nine_three_two
861 #print axioms ln_two_is_between_two_thirds_and_seven_tenths
    #print axioms the_series_is_not_vacuous
    #print axioms the_terms_fall
    #print axioms the_stream_has_twentyeight_unit_types
    #print axioms the_admitted_numerators_sum_to_2011
866 #print axioms f_I_squared_is_one_over_2011_squared
    #print axioms sigma_is_derived_from_the_enumeration
    #print axioms sigma_has_no_coefficient_above_degree_eight
    #print axioms no_packed_digit_carries
    #print axioms sigma_at_zero_is_f_I_squared
871 #print axioms the_degree_is_exactly_eight
    #print axioms sigma_at_pi_e_nine_tenths
    #print axioms sigma_at_pi_e_nineteen_twentieths
    #print axioms sigma_at_pi_e_ninetynine_hundredths
    #print axioms sigma_rounds_to_324e_minus_2_at_nine_tenths
876 #print axioms sigma_rounds_to_903e_minus_3_at_nineteen_twentieths
    #print axioms sigma_rounds_to_392e_minus_4_at_ninetynine_hundredths
    #print axioms sigma_at_the_pure_limit_rounds_to_247e_minus_7
    #print axioms the_hexamer_costs_twentyfive_bits_at_rho_one_hundred
    #print axioms and_twentyeight_at_rho_two_hundred
```

```
#print axioms and_neither_to_the_neighbouring_tenth
#print axioms the_registry_term_is_five_bits_and_needs_no_logarithm
#print axioms the_middle_term_is_an_integer_at_alpha_one_tenth
#print axioms the_demand_at_L_two_hundred_is_42200
#print axioms the_three_terms_are_one_thousand_twentyeight_and_thirteen_two
#print axioms and_they_sum_to_the_demand
#print axioms the_anchor_share_rounds_to_two_point_four
#print axioms the_residue_share_rounds_to_sixtysix_point_four
#print axioms the_lipid_share_rounds_to_thirtyone_point_three
#print axioms the_non_structural_share_rounds_to_ninetyseven_point_six
#print axioms over_ninetyseven_per_cent_is_not_placed_at_all
#print axioms the_demand_at_alpha_one_fiftieth
#print axioms and_it_rounds_to_19343
#print axioms the_demand_at_L_two_hundred_and_twentytwo
#print axioms the_anchor_term_there_is_1110
#print axioms the_demand_rises_by_three_point_seven_per_cent
#print axioms and_the_rise_is_exactly_1562_units
#print axioms the_period_is_fortythree_thousand_two_hundred_seconds
#print axioms the_nominal_rate_rounds_to_point_nine_seven_seven
#print axioms the_critical_rate_rounds_to_one_point_zero_one_three
#print axioms the_nominal_interval_rounds_to_one_point_zero_two_four
#print axioms the_critical_interval_rounds_to_point_nine_eight_seven
#print axioms at_a_six_hour_period_the_rate_rounds_to_one_point_nine_five_four
#print axioms at_a_twentyfour_hour_period_it_rounds_to_point_four_eight_eight
#print axioms the_rate_is_of_order_one_per_second_at_every_period_tested
#print axioms and_the_spread_over_those_cases_is_exactly_fourfold
#print axioms the_lower_figure_is_four_point_one_seven_bits_per_unit
#print axioms the_upper_figure_is_four_point_six_seven
#print axioms the_registry_term_per_unit_rounds_to_point_eight_three_three
#print axioms the_box_has_fiftyfour_nodes_and_fortyfive_contacts
#print axioms and_is_nine_columns_of_six
#print axioms a_column_of_fiftyfour_would_carry_fiftythree
#print axioms the_two_figures_are_different
#print axioms the_registry_term_is_the_contacts_per_node_ratio
#print axioms the_lower_bits_per_period_round_to_4167
#print axioms the_upper_bits_per_period_round_to_4667
#print axioms the_lower_bit_rate_rounds_to_point_zero_nine_six
#print axioms the_upper_bit_rate_rounds_to_point_one_zero_eight
#print axioms the_selection_cost_is_two_point_eight_nine_kT_at_the_lower_figure
#print axioms and_to_three_point_two_three_at_the_upper
#print axioms a_D_contact_carries_two_registrations_and_a_T_contact_four
#print axioms the_selection_cost_EXCEEDS_one_disubstituted_contact
#print axioms and_falls_below_one_tetrasubstituted_contact
#print axioms so_energy_is_comparable_and_not_slack
#print axioms the_ratio_to_a_disubstituted_contact_rounds_to_one_point_six_two
#print axioms and_to_a_tetrasubstituted_one_point_eight_one
#print axioms the_box_has_fortyfive_placements_and_a_period_one_thousand
#print axioms the_box_budget_at_F_ninetynine
#print axioms and_not_the_neighbouring_mantissa_there
#print axioms the_box_budget_at_F_one_half
#print axioms and_not_the_neighbouring_mantissa_there_either
#print axioms the_anchor_budget_at_F_ninetynine
#print axioms and_not_the_neighbouring_mantissa
#print axioms the_anchor_budget_at_F_one_half
#print axioms the_budget_tightens_with_the_number_of_placements
#print axioms every_delivered_D_or_T_contact_is_blind
#print axioms the_I_I_contact_reads_three_against_two
#print axioms so_the_initiator_points_the_right_way
#print axioms the_symmetric_sign_error_is_exactly_one_half
```

```
#print axioms and_the_identity_needs_no_value_of_Delta_or_of_the_gap
#print axioms the_complements_of_sigma
#print axioms the_floor_at_the_pure_limit_rounds_to_point_five
#print axioms the_floor_at_pi_e_ninetynine_is_point_499804
#print axioms the_floor_at_pi_e_ninetyfive_is_point_495487
#print axioms the_floor_at_pi_e_ninety_is_point_483799
#print axioms and_to_point_four_eight_three_eight_at_four_places
#print axioms the_floor_with_the_initiator_rounds_to_nine_sixes
#print axioms it_falls_short_of_one_half_by_f_I_squared_over_two
#print axioms and_the_shortfall_rounds_to_124e_minus_7
#print axioms the_floor_is_below_one_half
#print axioms the_gap_endpoints_are_right
#print axioms the_floor_exceeds_the_budget_by_four_point_nine_seven_e_four
#print axioms the_gap_is_four_point_seven_orders_of_magnitude
#print axioms and_it_is_not_four_point_six_or_four_point_eight
#print axioms under_correlated_failures_the_floor_still_exceeds_the_budget
#print axioms so_the_comparison_does_not_rest_on_independence
#print axioms the_bound_at_the_pure_limit_is_exactly_2011_squared
#print axioms and_the_superseded_figure_is_not_that_integer
#print axioms the_superseded_figure_exceeded_the_exact_one_by_fortyone
#print axioms the_bound_at_pi_e_ninetynine_rounds_to_2548
#print axioms the_bound_at_pi_e_ninetyfive_rounds_to_111
#print axioms the_bound_at_pi_e_ninety_rounds_to_31
#print axioms the_bound_rises_as_cleavage_falls
#print axioms there_are_two_new_forms_per_direction
#print axioms so_four_in_all
#print axioms and_each_has_a_repeated_locant
#print axioms the_relative_freedom_within_a_pair_is_zero
#print axioms the_snake_steps_only_between_neighbours
#print axioms and_visits_every_cell_of_the_layer
#print axioms so_a_layer_is_connected
#print axioms no_in_plane_edge_leaves_its_layer
#print axioms so_there_is_one_component_per_layer
#print axioms the_surviving_freedom_is_sixtyfour_states
#print axioms which_is_two_to_the_fortyeight_below_the_unconstrained_count
#print axioms energy_is_comparable
#print axioms rate_is_moderate
#print axioms sensing_is_severe
#print axioms orientation_is_forced
#print axioms in_plane_is_structural
#print axioms the_binding_constraint_is_not_energetic
```

Console Transcript: the Audit Itself

What a run of the file prints

The 137 lines below are the output of `lean acf-86-proof.lean`, not a separate measurement. Every one of them reads *"does not depend on any axioms"*; there is no other kind of line in the transcript, no warning, and no error. The elapsed time and the exit code are at the foot of it.

$ lean acf-86-proof.lean

```
$ lean --version
Lean (version 4.12.0, x86_64-unknown-linux-gnu, commit dc2533473114, Release)

$ lean acf-86-proof.lean
'ChryseneFormalism.Article86.rounding_accepts_a_correct_value' does not depend on any axioms
'ChryseneFormalism.Article86.rounding_rejects_the_value_below' does not depend on any axioms
'ChryseneFormalism.Article86.rounding_rejects_the_value_above' does not depend on any axioms
'ChryseneFormalism.Article86.rounding_rejects_a_half_way_case' does not depend on any axioms
'ChryseneFormalism.Article86.log2_accepts_an_integral_logarithm' does not depend on any axioms
'ChryseneFormalism.Article86.log2_rejects_its_neighbours' does not depend on any axioms
'ChryseneFormalism.Article86.the_root_endpoints_of_one_half' does not depend on any axioms
'ChryseneFormalism.Article86.the_root_bracket_accepts_an_exact_root' does not depend on any axioms
'ChryseneFormalism.Article86.and_rejects_the_value_below' does not depend on any axioms
'ChryseneFormalism.Article86.and_rejects_the_value_above' does not depend on any axioms
'ChryseneFormalism.Article86.the_root_bracket_accepts_an_inexact_root' does not depend on any axioms
'ChryseneFormalism.Article86.and_rejects_the_neighbours_of_that' does not depend on any axioms
'ChryseneFormalism.Article86.the_bracket_is_ordered' does not depend on any axioms
'ChryseneFormalism.Article86.the_bracket_is_narrow' does not depend on any axioms
'ChryseneFormalism.Article86.ln_two_rounds_to_point_six_nine_three_one' does not depend on any axioms
'ChryseneFormalism.Article86.and_not_to_point_six_nine_three_zero' does not depend on any axioms
'ChryseneFormalism.Article86.and_not_to_point_six_nine_three_two' does not depend on any axioms
'ChryseneFormalism.Article86.ln_two_is_between_two_thirds_and_seven_tenths' does not depend on any axioms
'ChryseneFormalism.Article86.the_series_is_not_vacuous' does not depend on any axioms
'ChryseneFormalism.Article86.the_terms_fall' does not depend on any axioms
'ChryseneFormalism.Article86.the_stream_has_twentyeight_unit_types' does not depend on any axioms
```

```
'ChryseneFormalism.Article86.the_admitted_numerators_sum_to_2011' does not
    depend on any axioms
'ChryseneFormalism.Article86.f_I_squared_is_one_over_2011_squared' does not
    depend on any axioms
'ChryseneFormalism.Article86.sigma_is_derived_from_the_enumeration' does not
    depend on any axioms
'ChryseneFormalism.Article86.sigma_has_no_coefficient_above_degree_eight' does
     not depend on any axioms
'ChryseneFormalism.Article86.no_packed_digit_carries' does not depend on any
    axioms
'ChryseneFormalism.Article86.sigma_at_zero_is_f_I_squared' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_degree_is_exactly_eight' does not depend on
    any axioms
'ChryseneFormalism.Article86.sigma_at_pi_e_nine_tenths' does not depend on any
     axioms
'ChryseneFormalism.Article86.sigma_at_pi_e_nineteen_twentieths' does not
    depend on any axioms
'ChryseneFormalism.Article86.sigma_at_pi_e_ninetynine_hundredths' does not
    depend on any axioms
'ChryseneFormalism.Article86.sigma_rounds_to_324e_minus_2_at_nine_tenths' does
     not depend on any axioms
'ChryseneFormalism.Article86.
    sigma_rounds_to_903e_minus_3_at_nineteen_twentieths' does not depend on
    any axioms
'ChryseneFormalism.Article86.
    sigma_rounds_to_392e_minus_4_at_ninetynine_hundredths' does not depend on
    any axioms
'ChryseneFormalism.Article86.sigma_at_the_pure_limit_rounds_to_247e_minus_7'
    does not depend on any axioms
'ChryseneFormalism.Article86.
    the_hexamer_costs_twentyfive_bits_at_rho_one_hundred' does not depend on
    any axioms
'ChryseneFormalism.Article86.and_twentyeight_at_rho_two_hundred' does not
    depend on any axioms
'ChryseneFormalism.Article86.and_neither_to_the_neighbouring_tenth' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    the_registry_term_is_five_bits_and_needs_no_logarithm' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_middle_term_is_an_integer_at_alpha_one_tenth'
     does not depend on any axioms
'ChryseneFormalism.Article86.the_demand_at_L_two_hundred_is_42200' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    the_three_terms_are_one_thousand_twentyeight_and_thirteen_two' does not
    depend on any axioms
'ChryseneFormalism.Article86.and_they_sum_to_the_demand' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_anchor_share_rounds_to_two_point_four' does
    not depend on any axioms
'ChryseneFormalism.Article86.the_residue_share_rounds_to_sixtysix_point_four'
    does not depend on any axioms
'ChryseneFormalism.Article86.the_lipid_share_rounds_to_thirtyone_point_three'
    does not depend on any axioms
'ChryseneFormalism.Article86.
    the_non_structural_share_rounds_to_ninetyseven_point_six' does not depend
    on any axioms
'ChryseneFormalism.Article86.over_ninetyseven_per_cent_is_not_placed_at_all'
```

```
    does not depend on any axioms
'ChryseneFormalism.Article86.the_demand_at_alpha_one_fiftieth' does not depend
    on any axioms
'ChryseneFormalism.Article86.and_it_rounds_to_19343' does not depend on any
    axioms
'ChryseneFormalism.Article86.the_demand_at_L_two_hundred_and_twentytwo' does
    not depend on any axioms
'ChryseneFormalism.Article86.the_anchor_term_there_is_1110' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_demand_rises_by_three_point_seven_per_cent'
    does not depend on any axioms
'ChryseneFormalism.Article86.and_the_rise_is_exactly_1562_units' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    the_period_is_fortythree_thousand_two_hundred_seconds' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_nominal_rate_rounds_to_point_nine_seven_seven
    ' does not depend on any axioms
'ChryseneFormalism.Article86.
    the_critical_rate_rounds_to_one_point_zero_one_three' does not depend on
    any axioms
'ChryseneFormalism.Article86.
    the_nominal_interval_rounds_to_one_point_zero_two_four' does not depend on
    any axioms
'ChryseneFormalism.Article86.
    the_critical_interval_rounds_to_point_nine_eight_seven' does not depend on
    any axioms
'ChryseneFormalism.Article86.
    at_a_six_hour_period_the_rate_rounds_to_one_point_nine_five_four' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    at_a_twentyfour_hour_period_it_rounds_to_point_four_eight_eight' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    the_rate_is_of_order_one_per_second_at_every_period_tested' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    and_the_spread_over_those_cases_is_exactly_fourfold' does not depend on
    any axioms
'ChryseneFormalism.Article86.
    the_lower_figure_is_four_point_one_seven_bits_per_unit' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_upper_figure_is_four_point_six_seven' does
    not depend on any axioms
'ChryseneFormalism.Article86.
    the_registry_term_per_unit_rounds_to_point_eight_three_three' does not
    depend on any axioms
'ChryseneFormalism.Article86.
    the_box_has_fiftyfour_nodes_and_fortyfive_contacts' does not depend on any
    axioms
'ChryseneFormalism.Article86.and_is_nine_columns_of_six' does not depend on
    any axioms
'ChryseneFormalism.Article86.a_column_of_fiftyfour_would_carry_fiftythree'
    does not depend on any axioms
'ChryseneFormalism.Article86.the_two_figures_are_different' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_registry_term_is_the_contacts_per_node_ratio'
    does not depend on any axioms
'ChryseneFormalism.Article86.the_lower_bits_per_period_round_to_4167' does not
```

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    depend on any axioms
'ChryseneFormalism.Article86.the_upper_bits_per_period_round_to_4667' does not
    depend on any axioms
'ChryseneFormalism.Article86.the_lower_bit_rate_rounds_to_point_zero_nine_six'
    does not depend on any axioms
'ChryseneFormalism.Article86.the_upper_bit_rate_rounds_to_point_one_zero_eight
   ' does not depend on any axioms
'ChryseneFormalism.Article86.
   the_selection_cost_is_two_point_eight_nine_kT_at_the_lower_figure' does
   not depend on any axioms
'ChryseneFormalism.Article86.and_to_three_point_two_three_at_the_upper' does
   not depend on any axioms
'ChryseneFormalism.Article86.
   a_D_contact_carries_two_registrations_and_a_T_contact_four' does not
   depend on any axioms
'ChryseneFormalism.Article86.
   the_selection_cost_EXCEEDS_one_disubstituted_contact' does not depend on
   any axioms
'ChryseneFormalism.Article86.and_falls_below_one_tetrasubstituted_contact'
   does not depend on any axioms
'ChryseneFormalism.Article86.so_energy_is_comparable_and_not_slack' does not
   depend on any axioms
'ChryseneFormalism.Article86.
   the_ratio_to_a_disubstituted_contact_rounds_to_one_point_six_two' does not
    depend on any axioms
'ChryseneFormalism.Article86.and_to_a_tetrasubstituted_one_point_eight_one'
   does not depend on any axioms
'ChryseneFormalism.Article86.
   the_box_has_fortyfive_placements_and_a_period_one_thousand' does not
   depend on any axioms
'ChryseneFormalism.Article86.the_box_budget_at_F_ninetynine' does not depend
   on any axioms
'ChryseneFormalism.Article86.and_not_the_neighbouring_mantissa_there' does not
    depend on any axioms
'ChryseneFormalism.Article86.the_box_budget_at_F_one_half' does not depend on
   any axioms
'ChryseneFormalism.Article86.and_not_the_neighbouring_mantissa_there_either'
   does not depend on any axioms
'ChryseneFormalism.Article86.the_anchor_budget_at_F_ninetynine' does not
   depend on any axioms
'ChryseneFormalism.Article86.and_not_the_neighbouring_mantissa' does not
   depend on any axioms
'ChryseneFormalism.Article86.the_anchor_budget_at_F_one_half' does not depend
   on any axioms
'ChryseneFormalism.Article86.the_budget_tightens_with_the_number_of_placements
   ' does not depend on any axioms
'ChryseneFormalism.Article86.every_delivered_D_or_T_contact_is_blind' does not
    depend on any axioms
'ChryseneFormalism.Article86.the_I_I_contact_reads_three_against_two' does not
    depend on any axioms
'ChryseneFormalism.Article86.so_the_initiator_points_the_right_way' does not
   depend on any axioms
'ChryseneFormalism.Article86.the_symmetric_sign_error_is_exactly_one_half'
   does not depend on any axioms
'ChryseneFormalism.Article86.
   and_the_identity_needs_no_value_of_Delta_or_of_the_gap' does not depend on
    any axioms
'ChryseneFormalism.Article86.the_complements_of_sigma' does not depend on any
   axioms
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'ChryseneFormalism.Article86.the_floor_at_the_pure_limit_rounds_to_point_five'
    does not depend on any axioms
'ChryseneFormalism.Article86.the_floor_at_pi_e_ninetynine_is_point_499804'
   does not depend on any axioms
'ChryseneFormalism.Article86.the_floor_at_pi_e_ninetyfive_is_point_495487'
   does not depend on any axioms
'ChryseneFormalism.Article86.the_floor_at_pi_e_ninety_is_point_483799' does
   not depend on any axioms
'ChryseneFormalism.Article86.
   and_to_point_four_eight_three_eight_at_four_places' does not depend on any
    axioms
'ChryseneFormalism.Article86.the_floor_with_the_initiator_rounds_to_nine_sixes
   ' does not depend on any axioms
'ChryseneFormalism.Article86.
   it_falls_short_of_one_half_by_f_I_squared_over_two' does not depend on any
    axioms
'ChryseneFormalism.Article86.and_the_shortfall_rounds_to_124e_minus_7' does
   not depend on any axioms
'ChryseneFormalism.Article86.the_floor_is_below_one_half' does not depend on
   any axioms
'ChryseneFormalism.Article86.the_gap_endpoints_are_right' does not depend on
   any axioms
'ChryseneFormalism.Article86.
   the_floor_exceeds_the_budget_by_four_point_nine_seven_e_four' does not
   depend on any axioms
'ChryseneFormalism.Article86.the_gap_is_four_point_seven_orders_of_magnitude'
   does not depend on any axioms
'ChryseneFormalism.Article86.and_it_is_not_four_point_six_or_four_point_eight'
    does not depend on any axioms
'ChryseneFormalism.Article86.
   under_correlated_failures_the_floor_still_exceeds_the_budget' does not
   depend on any axioms
'ChryseneFormalism.Article86.so_the_comparison_does_not_rest_on_independence'
   does not depend on any axioms
'ChryseneFormalism.Article86.
   the_bound_at_the_pure_limit_is_exactly_2011_squared' does not depend on
   any axioms
'ChryseneFormalism.Article86.and_the_superseded_figure_is_not_that_integer'
   does not depend on any axioms
'ChryseneFormalism.Article86.
   the_superseded_figure_exceeded_the_exact_one_by_fortyone' does not depend
   on any axioms
'ChryseneFormalism.Article86.the_bound_at_pi_e_ninetynine_rounds_to_2548' does
    not depend on any axioms
'ChryseneFormalism.Article86.the_bound_at_pi_e_ninetyfive_rounds_to_111' does
   not depend on any axioms
'ChryseneFormalism.Article86.the_bound_at_pi_e_ninety_rounds_to_31' does not
   depend on any axioms
'ChryseneFormalism.Article86.the_bound_rises_as_cleavage_falls' does not
   depend on any axioms
'ChryseneFormalism.Article86.there_are_two_new_forms_per_direction' does not
   depend on any axioms
'ChryseneFormalism.Article86.so_four_in_all' does not depend on any axioms
'ChryseneFormalism.Article86.and_each_has_a_repeated_locant' does not depend
   on any axioms
'ChryseneFormalism.Article86.the_relative_freedom_within_a_pair_is_zero' does
   not depend on any axioms
'ChryseneFormalism.Article86.the_snake_steps_only_between_neighbours' does not
    depend on any axioms
```

```
'ChryseneFormalism.Article86.and_visits_every_cell_of_the_layer' does not depend on any axioms
'ChryseneFormalism.Article86.so_a_layer_is_connected' does not depend on any axioms
'ChryseneFormalism.Article86.no_in_plane_edge_leaves_its_layer' does not depend on any axioms
'ChryseneFormalism.Article86.so_there_is_one_component_per_layer' does not depend on any axioms
'ChryseneFormalism.Article86.the_surviving_freedom_is_sixtyfour_states' does not depend on any axioms
'ChryseneFormalism.Article86.
    which_is_two_to_the_fortyeight_below_the_unconstrained_count' does not depend on any axioms
'ChryseneFormalism.Article86.energy_is_comparable' does not depend on any axioms
'ChryseneFormalism.Article86.rate_is_moderate' does not depend on any axioms
'ChryseneFormalism.Article86.sensing_is_severe' does not depend on any axioms
'ChryseneFormalism.Article86.orientation_is_forced' does not depend on any axioms
'ChryseneFormalism.Article86.in_plane_is_structural' does not depend on any axioms
'ChryseneFormalism.Article86.the_binding_constraint_is_not_energetic' does not depend on any axioms

exit code 0, elapsed 31 s
```

Kernel Axiom Audit

Axiom Inventory

Running `#print axioms` on each of the 137 theorems in `acf-86-proof.lean` yields:

Result	Meaning	Count
no axioms	The proof term reduces to primitive inference steps alone. Nothing is assumed beyond the type theory.	**137**
`propext`	Not used.	0
`Quot.sound`	Not used.	0
`funext`	Not used.	0
`Classical.choice`	Not used.	0
Extra axioms introduced by this proof:		**none**

The empty audit is what the three devices buy. A general proof of the monotonicity of x^k, or of the convergence of the atanh series, would need a nonlinear or analytic tactic and those are Mathlib; the core tactics that come close, `simp` and `omega`, both carry `propext` and `Quot.sound` into the proof term. Every claim here is instead a decidable proposition about particular naturals, and the bounds that make it decidable — the root bracket's endpoints, the series' truncation and remainder — are in the statements rather than in the tactics. The same discipline rules out the bounded existential, whose core instance `Nat.decidableExistsLT` goes through `decidable_of_iff`; S11's surjectivity claim is written with a Boolean search `anyB` instead.

The proof files of the corpus, and what their counts mean

Article	Kind	Checks	Theorems	Axiom-free
79	formal	88	55	54
80	formal	99	61	61
81	matching	91	55	55
82	concordance	120	109	109
84	formal	206	171	171
85	formal	283	129	129
86	bounds	157	137	**137**

The first six rows are the figures the Article 82, 84 and 85 proof dossiers report, carried here unchanged; they were measured there and have not been re-measured. The last row was measured here, by the audit above.

The useful comparison is not the theorem count but the fraction of the article a file covers, and on that measure this is the most complete of the four. The Article 85 file has 129 theorems and leaves that paper's whole analytic half outside; this one has 137 and leaves one quantity outside. Article 86 is a shorter paper with fewer figures, so its harness runs 157 checks against Article 85's 283 — and the ratio of theorems to checks runs the other way, which is the measure to read.

One theorem here carries more than any other: `the_bound_at_the_pure_limit_is_exactly_2011_squared`, together with its two companions, is a kernel-checked correction to a figure the submitted draft carried. That is a different kind of result from a certificate that a figure is right, and the count does not distinguish them.

Verification Note

The proof file makes no use of `native_decide`, which bypasses the kernel by running compiled code; of `sorry`; of `Classical.choice` or `Classical.em`; or of any Mathlib import. The `decide` tactic in Lean 4 core reduces a goal to a kernel-evaluated Boolean expression, and the kernel performs the evaluation.

Three `set_option` lines appear at the head of the file and none of them weakens anything. `exponentiation.threshold` raises the point at which the *elaborator* declines to pre-evaluate a power; past it Lean emits a warning, not an error, and the kernel evaluates the power either way. This file needs it generously: the root bracket of S8 raises eight-digit numerators to the thousandth power, S9 raises twelve-digit ones to the thousandth and a five-digit one to the hundredth, and the packing bases are 2^{128} and 10^{40}. `maxRecDepth` is raised because the default refuses the 2304-fold of S3 outright, and `maxHeartbeats` because the elaborator's default budget is smaller than that fold. All three are budgets, not assumptions, and the audit above is the evidence.

To reproduce: install Lean 4.12.0 and run `lean acf-86-proof.lean`. It takes about 35 seconds, most of it in the pinned σ derivation of S3, prints 137 audit lines and exits 0. There are no dependencies beyond the Lean core, so no `lakefile` and no package fetch are required.

Scope: four limits, stated plainly

First, Δ is nowhere measured in the corpus. Article 79's registration free energy is its least constrained parameter, and that is exactly why the two bounds Article 86 calls binding are the two that do not contain it: the sensing bound is $1/\sigma$ and the orientation floor is one half. S9 and S10 certify both without a value for Δ, and the one figure that does depend on it — the realised capacity at $\Delta = 1$ — is not in this file at all.

Second, no placement mechanism is proposed, anywhere. The article bounds what a placement system must satisfy; it does not exhibit one. S9 certifies that the orientation error rate is one half at every Δ, which says equilibrium cannot do the job; it does not say what can, and nothing in this file or in the article suggests a sorter.

Third, two of the six bounds are not of the same kind as the other four. S8 makes no thermodynamic claim: it bounds a per-placement error rate under independence, and the translation into a free energy is S9's subject, where the finding is that for the orientation component no translation exists. And S5's throughput bound is an order of magnitude, not a figure — which is why it is stated as four inequalities over four combinations of period and genome length rather than as a rounded rate.

Fourth, both artifacts encode the article's conclusions. The harness fails if the orientation floor does not exceed the budget; this file's S9 and S12 do the same. That is

deliberate and the article's appendix states it: a published verification artifact should stop passing when the claims it supports stop holding. But it means neither file is an independent test of the conclusions it encodes — with the four corrections of S10 and the harness's block D12 as the exception, since those were places where the artifact disagreed with the draft and the draft was what needed changing, as it has been.