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The Source Duality: Endogenous and Exogenous Preparation of the Substituted-Chrysene Stock, Mixed Arm Words, and the Persistence Semigroup

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Abstract

In a source-supported product-column model, endogenous self-separation and exogenous placement supply the same substituted-chrysene configurations but control different aspects of their specification. We formulate a parameterized source measure separating chemical species, directional code, and contact registry. On the all-e locus without adjacent initiator pairs, the selection cost splits additively and its kinetic and registry cross-sensitivities vanish; a blind column of length l retains a registry cost of $l-1$ bits. We classify mixed arm words under partial arm conversion and show that homotypic $D(eh)$ contacts can be orientation-selective, whereas accepted $D(eh)$–$T(eeee)$ contacts and $I(eee)$–$D(eh)$ cap contacts remain blind. For the $3\times3\times6$ interior lattice, all 45 stacking contacts are blind before lateral closure, while the fully bridged central column has no free terminals and remains blind under the stated contact model. We distinguish these 45 vertical relations from 53 agreements when eight inter-column relations are added. Coupled bulk-return and flip dynamics identify separate persistence requirements and conditional loss of frame-dependent readout after cap conversion. Finally, a flip-symmetrized structural-state embedding is injective on the contact description and, under an explicit commutator hypothesis, gives the spectral-distance lower bound $(l-1)/(2\gamma_z)$. The conclusions are finite combinatorial or conditional mathematical results; molecular interpretations and experimental targets are kept separate.

Keywords: Chrysene Formalism; source duality; contact blindness; mixed arm words; persistence semigroup; spectral distance.

1 Introduction

1.1 The duality of supply and specification

The corpus now possesses two distinct constructions for the substituted-chrysene stock that feeds the protocell arc of Articles 57, 62, 65, 77, 78, and 80 [Sch26g, Sch26e, Sch26c, Sch26a, Sch26f, Sch26d]. Article 79 (*The Source*) [Sch26i] proposes a coarse-grained endogenous source model for unsubstituted chrysene in a hydrophobic polyaromatic bulk β. Its proposed molecular interpretation uses sequential substitution at $\Lambda = \{3,6,9,12\}$ and hydroxyethoxy arms $e = -\mathrm{O}-\mathrm{CH_2CH_2}-\mathrm{OH}$. The suggested bromination, oxalate incorporation, and reduction sequence is a route hypothesis requiring step-specific chemical validation, including bond connectivity, regioselectivity, and coproducts. The prescribed export law $e_n = e_0 q^n$ $(q>1)$ represents increasing preference for the stock phase σ as arms are added. Within this model, the design equation

$$k_3 = e_2(R_T/R_D)(1+e_3/k_4)$$

sets the relative supply of disubstituted $D=\{6,12\}$, trisubstituted $I=\{3,6,12\}$, and tetrasubstituted $T=\{3,6,9,12\}$. The selected parameter calculation gives material efficiency $\eta = 0.998437$, not a measured isolated synthetic yield. The effective registry model assigns the competitor suppression factor $e^{-2\nu\Delta_{\mathrm{rgy}}}$ under its contact-energy assumptions.

Article 84 (*The Placed Source*) [Sch26h] replaces bulk self-separation by an exogenous state-preparation map (H_{place}) that, as an ideal preparation premise, delivers purified D, I, T monomers to prescribed sites of $\Omega \subset \mathbb{Z}^3$ in prescribed states of Article 40's structural alphabet $\Sigma_{\mathfrak{C}} = \mathbb{Z}_2^{\mathrm{dc}} \times \mathbb{Z}_2^{z}$ [Sch26b]. The imported reachability comparison is used here only on the source-supported product domain $C_{\mathrm{src}}(\Omega)$: under independent draws with positive mass for every admitted local state and placement

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onto that same domain, Lemma 3.6 gives $R_{\mathrm{endo}} = R_{\mathrm{exo}} = C_{\mathrm{src}}(\Omega)$ and hence equal images under every product map Π. Positive family exit probabilities alone do not establish support for arbitrary compatibility-restricted configuration spaces. Separately, the C_{2h} inversion ι ($3 \leftrightarrow 9, 6 \leftrightarrow 12$) leaves D and T invariant and the delivered words *ee* and *eeee* constant on ι-orbits, making every stacking contact except I–I blind to its alignment bit $\eta_k \in \mathbb{Z}_2$.

1.2 What the corpus leaves open between the sources

Five specific problems sit on the boundary between Articles 40, 79, 80, and 84:

(i) *Differential decoupling on $C(\Omega)$.* Article 84 computes the species cost (20.0 to 23.0 bits on an alternating hexamer) and the registry cost ($l - 1$ bits), but does not prove that the sensitivity Jacobian between bulk kinetic rates and contact alignment parameters vanishes identically on any stack without adjacent I–I pairs.

(ii) *Mixed arm words under partial arm conversion ([Sch26h, Target 14.4]).* Article 84 proves blindness for the all-*e* stock ($\pi_h = 0$) and leaves open whether partial arm conversion $\pi_h = 1 - \pi_e \in (0, 1)$ ([Sch26i, Prop. 10.11]) makes the registry filter orientation-selective in a useful direction, either in the interior of a coding stack or at its initiator cap.

(iii) *Blindness of bridged lattice nodes ([Sch26h, Rem. 9.4, Target 14.10]).* In Article 80's heritable minimum $B(3, 3, 6)$ [Sch26d], 72 covalent bridges consume $216 - 72 = 144$ of the 216 primary arm terminals, leaving 72 free terminals. Article 84 leaves open whether bridged lattice nodes remain blind and how the 45 vertical contacts of $B(3, 3, 6)$ reconcile with its 53-bit figure.

(iv) *Coupled relaxation of H_{ret} and H_{place}.* Article 79 rests on H_{ret} ($r_n = 0$) and Article 84 invokes placed preparation and a zero-flip retention regime. Here we separate H_{place} (preparation) from H_{lock} ($\varphi = 0$). Neither paper analyzes the coupled semigroup when both barriers are finite or the effect of bulk return $r_3 > 0$ on the asymmetric frame carrier I.

(v) *The partial Axiom A_{12} instance ([Sch26i, Def. 9.1]).* Article 79 omits the injective spectral realization $j_{\mathrm{str}} : X \to S(\mathfrak{A}_{\mathfrak{C}})$ of [Sch26b, eq. (413)].

1.3 What is established here, by kind

Recovered. Article 40's C_{2h} stabilizer, exact-closure connectors, crossed product $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$, conditional expectation E, spectral triple $(\mathfrak{A}^{\circ}_{\mathfrak{C}}, \mathcal{H}_{\omega}, D_{\mathfrak{C}})$, Connes distance d_D, and Axiom A_{12} [Sch26b]; Article 79's export chain, design equation, H_{rgy}, arm-conversion efficiency η_{arm}, and return dynamics [Sch26i]; Article 80's site classes, saturated bridging H_{sat}, and $B(3, 3, 6)$ heritable minimum [Sch26d]; Article 84's configuration space $C(\Omega)$, coincidence hypothesis H_{ν}, unit and contact blindness, frame carrier I, and flip relaxation [Sch26h].

Proved here. Support invariance on the source-supported product-column model (Lemma 3.6); additive decomposition and identical vanishing of cross-Jacobians between bulk kinetic parameters and contact registry parameters on the $\{D, T\}$-locus, including stacks with isolated initiator caps (Theorem 4.1, Remark 4.2, Corollary 4.3); exact enumeration of flip-visible mixed arm words and contact coincidences $\Delta\nu$ under arm conversion $\pi_h \in (0, 1)$, including the non-trivial blindness of the flip-visible pair $I(eee)$–$D(eh)$ (Proposition 5.1); the Efficiency–Visibility Trade-Off Inequality (Theorem 5.2); identical vanishing of conversion-induced orientation bias across the entire capped alternating D–T coding locus, discharging [Sch26h, Target 14.4] (Corollary 5.3); exact free-terminal distribution across corners, edges, and interior of $B(3, 3, 6)$, contact-model blindness of all 4 complete sites and all 45 accretion contacts, and exact spanning-tree reconciliation $45 + 8 = 53$, discharging [Sch26h, Target 14.10] (Proposition 6.1, Theorem 6.2, Corollary 6.3, Ruling 9.2); bifurcation of the coupled return-and-flip semigroup, cap conversion $I \to T$ and conditional frame-dependent readout loss at $r_3 > 0, \varphi = 0$, and finite-time retention bounds (Theorem 7.1, Corollary 7.2, Proposition 7.3); explicit construction of $j_{\mathrm{str}} : X \to S(\mathfrak{A}_{\mathfrak{C}})$ discharging [Sch26i, Def. 9.1] as a pure character on the contact subalgebra $\mathfrak{Z}_{\Omega}$ and Connes distance lower bound $d_D(\varphi_{\mathrm{endo}}, \varphi_{\mathrm{exo}}) \geq \frac{l-1}{2\gamma_z}$ (Definition 8.1, Proposition 8.2, Theorem 8.3).

2 Conventions, nomenclature, and non-implications

Convention 2.1 (Status labels and documentary marks)**.** Every numbered statement carries one of four labels: **[A]** finite, algebraic, or axiom-free; **[D]** derived under hypotheses named and registered in Appendix A; **[C]** imported from a cited source; **[E]** a molecular or patent reading, entering no proof. Scholia are unlabelled and enter no proof. The filing [Sch25] is cited through Article 65's documentary bundle H_{mat} [Sch26c] as amended by clause (i) of [Sch26i] and clause (j) of [Sch26h]. The numerical macros are emitted by `verify_a90.py`; the finite scope of blocks `Y1`–`Y9` is documented in Appendix B. These computations do not constitute proofs of the general statements or their physical premises.

Notation 2.2 (Core objects)**.** $\Lambda = \{3, 6, 9, 12\}$ the primary locants of Axiom A_1, with $v_x^- = v_3, v_x^+ = v_9$ and $v_y^- = v_{12}, v_y^+ = v_6$

[Sch26b, eqs. (50)–(51)]; ι the C_{2h} inversion acting on Λ by $3 \leftrightarrow 9, 6 \leftrightarrow 12$; $D = \{6, 12\}, I = \{3, 6, 12\}, T = \{3, 6, 9, 12\}$; $\mathcal{A}_{\mathrm{arm}} = \{h, o, e, e_P\}$; $\Sigma_{\mathfrak{C}} = \mathbb{Z}_2^{\mathrm{dc}} \times \mathbb{Z}_2^z$; $\Omega \subset \mathbb{Z}^3$ a finite occupied set, $E(\Omega)$ its lateral connectors, $E_z(\Omega)$ its z-stacking contacts, $C(\Omega)$ its compatible configuration set; $\eta : E_z(\Omega) \to \mathbb{Z}_2$ the alignment field; Fr = $\langle \mathrm{flip}, \mathrm{rev} \rangle$; k_n, e_n, r_n advance, export, and bulk-return rates; $\pi_h = 1 - \pi_e$ arm conversion probability; φ per-contact flip rate.

Convention 2.3 (Chemical labels and conversion channel)**.** Here h = Ar−OH and e = Ar−O−CH_2CH_2−OH denote phenolic and hydroxyethoxy arms on the chrysene core. The e arm contains an aryl ether, not an ester. We use *arm conversion* for the formal $e \mapsto h$ channel, with independent Bernoulli conversion probability π_h. A chemical realization must identify the bond-cleavage or precursor pathway and its coproducts. Ester hydrolysis applies to a distinct ester-bearing precursor, not automatically to the displayed e structure. In particular, an oxalate ester o = Ar−O−C(= O)−C(= O)OH is distinct from e; its reduction need not preserve the aryl–oxygen connection and does not by itself establish a route to e.

Proposition 2.4 (Non-implications)**. [A]** *None of the following is asserted:*

(i) No loss-free endogenous encoder. *Neither initiator doping ($x = e_3/k_4 > 0$) nor partial arm conversion ($\pi_h > 0$) creates orientation selectivity without degrading bulk efficiency η_{DT} or arm efficiency η_{arm} (Theorem 5.2).*

(ii) No cleavage bias on an accepted coding stack or at its cap. *On the alternating D–T coding locus under H_{acc}, cleaving D to Dm = eh gives $v_0 = v_1 = 1$ against $T2 = eeee$ and $v_0 = v_1 = 1$ against an $I(eee)$ cap, so the induced orientation bias is identically zero (Proposition 5.1(d), Corollary 5.3).*

(iii) No physical actuation mechanism from KL divergence. *$c(c_0; \vartheta) = D_{\mathrm{KL}}(\delta_{c_0} \| \mu_\Omega(\vartheta))$ prices selection in bits; it supplies no physical sorting demon (Target 9.5).*

(iv) Cap loss need not erase physical relative alignment. *Even at $\varphi = 0$, return and advance can convert $I \to T$ and remove a frame carrier. Loss of frame-dependent readout additionally requires the stated observation quotient and absence of another reference (Corollary 7.2).*

(v) No spectral upper bound or metric equivalence. *The structural-state embedding supplies the stated injectivity, and Theorem 8.3 proves only a lower bound under its explicit whole-sum commutator hypothesis. Finiteness and comparison with d_C^* require further assumptions.*

3 The parameterized source manifold

Definition 3.1 (Configuration fibres)**. [A]** Let $\Omega \subset \mathbb{Z}^3$ be finite with saturated lateral connector set $E(\Omega)$ under [Sch26d, Hyp. 3.3], and let $C(\Omega)$ be the compatible configuration space of [Sch26h, Def. 4.10]. Each $c = (\kappa, E(\Omega), \eta) \in C(\Omega)$ assigns to $x \in \Omega$ a site datum $\kappa(x) = (S_x, [a_x], \epsilon_x) \in \mathcal{D}$ and to $e \in E_z(\Omega)$ an alignment bit $\eta(e) \in \mathbb{Z}_2^z$. Define the projections $\pi_{\mathrm{chem}}(c) = ((S_x, [a_x]))_{x \in \Omega}$, $\pi_{\mathrm{dc}}(c) = (\epsilon_x)_{x \in \Omega}$, and $\pi_{\mathrm{reg}}(c) = \eta$.

Convention 3.2 (Source-supported product-column model)**. [A]** For the probabilistic results, Ω is a one-site-wide column with no lateral connectors. Define

$$\mathcal{K}_{he} = \coprod_{S \in \{D,I,T\}} \{h, e\}^S / G_S,$$
$$C_{\mathrm{src}}(\Omega) = \mathcal{K}_{he}^l \times \{0, 1\}^l \times \{0, 1\}^{l-1},$$

where $G_S = \{g \in \langle \iota \rangle : gS = S\}$ acts on the fixed canonical support. Here κ denotes only the chemical support and arm-class tuple; ϵ is its separate directional-code factor. We assume these labelled tuples are admitted in this column model. C_{src} is not the unrestricted four-letter corpus space $C(\Omega)$. Additional compatibility restrictions can change the normalization and destroy the product factorization; they require separate analysis. Comparisons to the wider corpus concern only members that also admit its structural realization.

Convention 3.3 (Arm representatives and registry coordinates)**. [A]** For each support S and orbit $[a]$, fix a representative $r_S([a])$ once and for all. For $D(eh)$ take e at 6 and h at 12. Patterns, relative registry bits, target states, and external registry fields are all expressed in this convention. A change by ι is allowed only for supports with $\iota \in G_S$; it is a coordinate change, not a new chemical species.

Proposition 3.4 (Covariance under representative changes)**. [A]** *On each fixed chemical fibre, change representatives at sites by bits s_i, with $s_i = 0$ whenever G_{S_i} is trivial. Relabel each incident registry bit by*

$$\eta'_{ij} = \eta_{ij} \oplus s_i \oplus s_j.$$

Then the coincidence at the corresponding physical contact is unchanged. With target bits and external fields relabelled in the same way, source probabilities, entropy, and blindness are unchanged; the labelled pair (v_0, v_1) can be permuted.

Proof. Write $P_i = \mathrm{pat}(r_{S_i}([a_i]))$ and $P'_i = \iota^{s_i} P_i$. Since ι is a bijection and $\iota^2 = 1$,

$$\begin{aligned} |P'_i \cap \iota^{\eta'_{ij}} P'_j| &= |\iota^{s_i} P_i \cap \iota^{\eta_{ij} \oplus s_i} P_j| \\ &= |P_i \cap \iota^{\eta_{ij}} P_j|. \end{aligned}$$

The two contact labels are merely permuted. Relabel the field by $\theta'_{ij}(b) = \theta_{ij}(b \oplus s_i \oplus s_j)$; each Boltzmann numerator and its normalization agree for corresponding states. The map of the finite registry space is bijective, so probabilities and entropy agree. Equality $v_0 = v_1$ is invariant under their interchange. Over the full source space, the bits s_i can depend on κ_i. The joint-law statement is then invariance under this bijective pushforward, allowing the relabelled external fields to be chemical-conditional. It does not assert closure of the original κ-independent field family under every representative change. □

Definition 3.5 (Parameterized source measure)**. [D]** Let $\vartheta = (k_1, \dots, k_4, e_0, \dots, e_4, \pi_h, \Delta_{\mathrm{rgy}}, \gamma, \theta_{\mathrm{spec}}, \theta_{\mathrm{reg}}) \in \Theta$. Under H_{draw} and H_v [Sch26h], on a column Ω of length $l \geq 2$, with the product model of Convention 3.2, the probability measure $\mu_\Omega(\cdot\,;\vartheta)$ on $C_{\mathrm{src}}(\Omega)$ is

$$\mu_\Omega(c;\vartheta) = \mu_{\mathrm{chem}}(\kappa; k, e, \pi_h, \theta_{\mathrm{spec}}) \cdot \mu_{\mathrm{dc}}(\epsilon;\gamma)\mu_{\mathrm{reg}}(\eta \mid \kappa; \Delta_{\mathrm{rgy}}, \theta_{\mathrm{reg}}). \tag{1}$$

where on the endogenous slice ($\theta_{\mathrm{spec}} = 0, \theta_{\mathrm{reg}} = 0$), $\mu_{\mathrm{chem}}(\kappa) = \prod_{i=0}^{l-1} f_{S_i}(k,e)\, P_{\mathrm{arm}}([a_i] \mid S_i; \pi_h)$ with $f_S = p_{|S|}/(p_2+p_3+p_4)$ and binomial conversion law $P_{\mathrm{arm}}([a] \mid S; \pi_h) = |\mathrm{Orb}_{G_S}(a)|\pi_h^{n_h(a)}(1-\pi_h)^{|S|-n_h(a)}$, where $G_S = \{g \in \langle \iota \rangle : gS = S\}$ is the stabilizer of the fixed canonical support. In particular, $G_I = \{1\}$, so the eight I words have multiplicity one. The directional factor is $\mu_{\mathrm{dc}}(\epsilon;\gamma) = \prod_i \gamma_i^{\epsilon_i}(1-\gamma_i)^{1-\epsilon_i}$. For finite external fields, multiply the chemical weight of κ by $e^{\theta_{\mathrm{spec}}(\kappa)}$ and each contact weight by $e^{\theta_{\mathrm{reg},i}(\eta_i)}$, then normalize each factor on its finite domain separately. The untilted registry factor is

$$\mu_{\mathrm{reg}}(\eta \mid \kappa; \Delta_{\mathrm{rgy}}, 0) = \prod_{i=0}^{l-2} \frac{e^{v_{\eta_i}(\kappa_i,\kappa_{i+1})\Delta_{\mathrm{rgy}}}}{e^{v_0(\kappa_i,\kappa_{i+1})\Delta_{\mathrm{rgy}}} + e^{v_1(\kappa_i,\kappa_{i+1})\Delta_{\mathrm{rgy}}}}. \tag{2}$$

The score v_η is a label-coincidence descriptor. A thermodynamic interpretation assumes effective contact free energies

$$G_\eta = G_{\mathrm{ref}} - k_B T \Delta_{\mathrm{rgy}} v_\eta$$

and equal orientation degeneracies; Δ_{rgy} is dimensionless. The descriptor does not count experimentally established hydrogen bonds. Solvent, protonation, arm conformation, lateral neighbours, and aggregation can contribute additional orientation-dependent terms. Thus descriptor blindness is the precise conclusion when $v_0 = v_1$; physical equal populations require the stated effective-energy hypothesis.

Fix a base source with $\pi_h \in (0,1)$ and positive support. For a target $c_0 = (\kappa^0, \epsilon^0, \eta^0) \in C_{\mathrm{src}}(\Omega)$, the placed source of [Sch26h] is the weak-* limit $\mu_{\mathrm{exo}}^{(c_0)} := \lim_{\Lambda\to+\infty} \mu_\Omega(\vartheta(\Lambda)) = \delta_{c_0}$ along $\theta_{\mathrm{spec}}(\kappa) = \Lambda \mathbf{1}[\kappa = \kappa^0]$, $\theta_{\mathrm{reg},i}(\eta_i) = \Lambda \mathbf{1}[\eta_i = \eta_i^0]$, together with $\gamma_i \to \epsilon_i^0$, where $\gamma_i = \Pr(\epsilon_i = 1)$. Concentrating only the chemical and registry factors would leave the directional-code factor unselected.

Lemma 3.6 (Support invariance on the source domain)**. [A]** *For every ϑ in the parameter region $\Theta_+ = \{k_j > 0, e_n > 0, \pi_h \in (0,1), \Delta_{\mathrm{rgy}} \in [0,\infty), \gamma_i \in (0,1), \|\theta_{\mathrm{ext}}\|_\infty < \infty\}$, $\operatorname{supp}\mu_\Omega(\vartheta) = C_{\mathrm{src}}(\Omega)$; on the boundary $\pi_h = 0$, $\operatorname{supp}\mu_\Omega(\vartheta) = C_{\text{all-}e}(\Omega)$ where $C_{\text{all-}e}$ is the subset of C_{src} with all arms equal to e (Y1).*

Proof. By [Sch26h, Thm. 8.7], $k_j, e_n > 0 \Rightarrow p_n > 0 \Rightarrow f_S > 0$. Every binomial and Boltzmann factor in (1)–(2) is strictly positive on Θ_+. □

4 Orthogonal factorization of the selection functional

Theorem 4.1 (Additive decomposition and Jacobian orthogonality)**. [A]** *For any column Ω of length $l \geq 2$ and any $c = (\kappa, \epsilon, \eta) \in C_{\mathrm{src}}(\Omega)$ with $\mu_\Omega(c;\vartheta) > 0$, the selection cost $c(c;\vartheta) = -\log_2 \mu_\Omega(c;\vartheta) = D_{\mathrm{KL}}(\delta_c \,\|\, \mu_\Omega(\vartheta))$ decomposes additively as*

$$c(c;\vartheta) = c_{\mathrm{chem}}(\kappa; k, e, \pi_h, \theta_{\mathrm{spec}}) + c_{\mathrm{dc}}(\epsilon;\gamma) + c_{\mathrm{reg}}(\eta \mid \kappa; \Delta_{\mathrm{rgy}}, \theta_{\mathrm{reg}}). \tag{3}$$

On the delivered one-reagent $\{D,T\}$-contact locus ($\pi_h = 0$ and at least one unit of every adjacent pair $(i, i+1)$ lies in $\{D,T\}$), $c_{\mathrm{reg}}(\eta \mid \kappa; \Delta_{\mathrm{rgy}}, 0) = l-1$ identically, and the sensitivity Jacobian between $\vartheta_{\mathrm{kin}} = (k_1, \dots, k_4, e_0, \dots, e_4, q)$ and $\vartheta_{\mathrm{cont}} = (\Delta_{\mathrm{rgy}}, \theta_{\mathrm{reg}})$ vanishes identically:

$$\frac{\partial c_{\mathrm{reg}}}{\partial \vartheta_{\mathrm{kin}}} \equiv 0, \qquad \left.\frac{\partial c_{\mathrm{reg}}}{\partial \Delta_{\mathrm{rgy}}}\right|_{\theta_{\mathrm{reg}}=0} \equiv 0, \qquad \frac{\partial c_{\mathrm{chem}}}{\partial \vartheta_{\mathrm{cont}}} \equiv 0. \tag{4}$$

Proof. Equation (3) is $-\log_2$ of (1). On the $\{D,T\}$-contact locus at $\pi_h = 0$, $D(ee)$ and $T(eeee)$ are blind by [Sch26h, Thm. 7.3], so Contact Blindness ([Sch26h, Thm. 7.4]) gives $v_0(\kappa_i, \kappa_{i+1}) = v_1(\kappa_i, \kappa_{i+1})$ at every contact. Thus $e^{v_0 \Delta_{\mathrm{rgy}}}$ cancels in (2) at $\theta_{\mathrm{reg}} = 0$, leaving $\mu_{\mathrm{reg}} = 2^{-(l-1)}$ and $c_{\mathrm{reg}} = l-1$, independent of ϑ_{kin} and Δ_{rgy}. Finite baseline checks appear in block Y1. □

Remark 4.2 (Isolated initiators and terminal caps introduce zero bias)**. [A]** Because Theorem 4.1 requires only that *at least one* unit of each adjacent pair $(i, i+1)$ lie in $\{D,T\}$, a stack capped at one or both ends by the trisubstituted initiator $I(eee)$ (such as I–D–T–D–T–D–T) lies entirely inside the locus of (4). Explicitly, at a terminal $I(eee)$–$D(ee)$ contact $v_0 = v_1 = 2$, and at a terminal $I(eee)$–$T(eeee)$ contact $v_0 = v_1 = 3$ (Y2). Thus capping a $\{D,T\}$ stack with a single initiator $I(eee)$ supplies the ι-free reference required by H_{frame} ([Sch26h, Hyp. 4.5]) while introducing identically zero orientation bias into the terminal contact.

Corollary 4.3 (The information-kinetic partition on a coded hexamer)**. [D]** *On the six-site alternating cluster locus*

$\kappa^0 = (D,T,D,T,D,T)$ *of [Sch26g, Def. 4.1] at* $\pi_h = 0$*, under Article 79's matched design equation with* $\rho^* = p_2/p_4$ *and* $x = e_3/k_4 = p_3/p_4$*,*

$$c_{\text{spec+reg}}(\kappa^0,\eta^0;\rho^*,x) = \underbrace{3\log_2\!\left(\frac{(1+\rho^*+x)^2}{\rho^*}\right)}_{c_{\text{spec}}(\rho^*,x)} + \underbrace{5}_{c_{\text{reg}}}. \tag{5}$$

At $x = 0.1$*, shifting the demand ratio from* $\rho^* = 100$ *to* $\rho^* = 200$ *increases* c_{spec} *from* 20.0 *bits to* 23.0 *bits while* $c_{\text{reg}} = 5$ *bits is invariant (*18% *of the* 28.0*-bit total at* $\rho^* = 200$*; block Y6).*

Proof. Since $f_D = \rho^*/(1+\rho^*+x)$ and $f_T = 1/(1+\rho^*+x)$, $-\log_2(f_D^3 f_T^3)$ yields the first term of (5); every D–T contact is blind, giving $c_{\text{reg}} = 5$. □

Scholium 4.4 (Why neither source subsumes the other). Theorem 4.1 states the precise sense in which the two sources are orthogonal projections on $C_{\text{src}}(\Omega)$. Article 79's bulk Markov generator $G(k,e)$ acts exclusively on the vertex projection π_{chem}: it enriches chrysene against the background and tunes $p_2/p_4 = \rho^*$, but its Jacobian on the relative alignment fiber π_{reg} is zero. Conversely, exogenous placement (H_{place}) adds no new vertex configurations or product classes ($\Pi(R_{\text{exo}}) = \Pi(R_{\text{endo}})$), but acts directly on π_{reg} to supply the $l-1$ bits of contact alignment to which the bulk chemistry and the registration filter are blind.

5 The symmetry–visibility trade-off (Target 14.4 of Article 84)

We now discharge [Sch26h, Target 14.4] by extending the blindness analysis to mixed arm words over $\{h,e\}$ generated when each arm is cleaved to h = –OH independently with probability $\pi_h = 1-\pi_e \in (0,1)$ ([Sch26i, Prop. 10.11]).

Proposition 5.1 (Complete classification of mixed arm words and contact coincidences over $\{h,e\}$)**. [A]** *Let* $\mathcal{A}_{\text{arm}} = \{h,e\}$ *(*$m = 2$*). Under the* C_{2h} *inversion* ι *(*$3 \leftrightarrow 9, 6 \leftrightarrow 12$*):*

(a) On $D = \{6,12\}$*, the* 3 *arm-word* ι*-classes consist of* 2 *blind words (*ee, hh*, probabilities* π_e^2, π_h^2*) and* 1 *flip-visible mixed class* $[eh] = \{eh, he\}$ *(probability* $2\pi_e\pi_h$*).*

(b) On $T = \{3,6,9,12\}$*, the* 10 *arm-word* ι*-classes consist of* 4 *blind words constant on both* $\{3,9\}$ *and* $\{6,12\}$ *(*$eeee$, $hhhh$, $e_{3,9}h_{6,12}$, $h_{3,9}e_{6,12}$*) and* 6 *flip-visible classes; on* $I = \{3,6,12\}$*, all* 8 *classes are flip-visible (totaling* 21 *classes at* $m = 2$ *and* 78 *at* $m = 3$*; block Y2).*

(c) Under H_v *(*$v_\eta = |\text{pat}_1 \cap \text{pat}_\eta|$*), a homotypic* $D(eh)$–$D(eh)$ *contact has*

$$\begin{aligned} v_0(D(eh),D(eh)) &= 2,\\ v_1(D(eh),D(eh)) &= 0, \qquad \Delta v = 2. \end{aligned} \tag{6}$$

giving unaligned Boltzmann weight $\theta_{DD}^{(eh)} = (1+e^{2\Delta_{\text{rgy}}})^{-1} \neq \frac{1}{2}$ *(*0.1192 *at* $\Delta_{\text{rgy}} = 1$*, information gain* $I_{DD}^{(eh)}(1) = 0.4729$ *bits;* 0.0180 *and* 0.8700 *bits at* $\Delta_{\text{rgy}} = 2$*; block Y3).*

(d) At any heterotypic $D(eh)$–$T(eeee)$ *contact, and at any terminal cap contact* $I(eee)$–$D(eh)$ *(where neither* $I(eee)$ *nor* $D(eh)$ *is a blind unit),*

$$\begin{aligned} v_0(D(eh),T(eeee)) &= v_1(D(eh),T(eeee))\\ &= 1, \qquad \Delta v = 0, \end{aligned} \tag{7}$$

$$\begin{aligned} v_0(I(eee),D(eh)) &= v_1(I(eee),D(eh))\\ &= 1, \qquad \Delta v = 0. \end{aligned} \tag{8}$$

so both contacts are strictly blind (block Y2).

Proof. Parts (a)–(b) apply [Sch26h, Thm. 7.3]: $\text{pat}(S,a) = \text{pat}^{\text{flip}}(S,a)$ iff $\iota S = S$ and $a(\iota l) = a(l)$ on S. For (c), $\text{pat}_0(D,eh) = \{(6,e),(12,h)\}$ and $\text{pat}_1(D,eh) = \{(12,e),(6,h)\}$, giving $|\text{pat}_0 \cap \text{pat}_0| = 2$ and $|\text{pat}_0 \cap \text{pat}_1| = 0$. For (d), $T(eeee)$ is blind, so Contact Blindness ([Sch26h, Thm. 7.4]) forces (7). For $I(eee)$–$D(eh)$, $\text{pat}(I,eee) = \{(3,e),(6,e),(12,e)\}$ intersects $\text{pat}_0(D,eh)$ in $\{(6,e)\}$ ($v_0 = 1$) and intersects $\text{pat}_1(D,eh)$ in $\{(12,e)\}$ ($v_1 = 1$), establishing (8) as an explicit molecular instance where two flip-visible units form a blind contact because $\text{pat}(I,eee)$ is ι-symmetric on the $\{6,12\}$ orbit of D. □

Theorem 5.2 (The Efficiency–Visibility Trade-Off Inequality)**. [D]** *Let* $I_{\text{vis}}(\pi_h,x) := (l-1) - H(\eta \mid \kappa)$ *be the expected endogenous orientation information (in bits) acquired across a column of length* l*.*

(a) Initiator-doping channel ($\pi_h = 0$, $x = e_3/k_4 > 0$)*. Only I–I contacts are visible (*$v_0 = 3, v_1 = 2$*, single-contact gain* $I_{II}(\Delta_{\text{rgy}}) = 0.1601$ *bits at* $\Delta_{\text{rgy}} = 1$*), occurring at frequency* $f_I(x)^2 = \left(\frac{x}{\rho^*+1+x}\right)^2$ *(*$2.39\times10^{-7}, 2.37\times10^{-5}, 2.17\times10^{-3}$ *at* $x = 0.1, 1, 10$*; block Y5), while the bulk* $D+T$ *efficiency* η_{DT} *of [Sch26i, Thm. 7.3] degrades according to*

$$\begin{aligned} I_{\text{vis}}(0,x) &= (l-1) f_I(x)^2 I_{II}(\Delta_{\text{rgy}})\\ &\leq \frac{(l-1) I_{II}(\Delta_{\text{rgy}})}{(p_2+p_3+p_4)^2}(1-\eta_{\text{DT}})^2. \end{aligned} \tag{9}$$

(b) Arm-conversion channel on homotypic D–D columns ($\pi_h > 0$)*. Flip-visible* $D(eh)$–$D(eh)$ *contacts occur at frequency* $f_{\text{vis},DD}(\pi_h) = 4\pi_h^2(1-\pi_h)^2 \leq 0.2500$*, while the accepted arm efficiency* η_{arm} *of [Sch26i, Prop. 10.11] degrades strictly:*

$$\eta_{\text{arm}}(\pi_h) = \frac{\rho^*+1}{\dfrac{\rho^*}{1-\pi_h^2} + \dfrac{1}{(1-\pi_h)^4}} < 1 \qquad (\pi_h > 0), \tag{10}$$

falling from 0.9997 *at* $\pi_e = 0.99$ *(*$f_{\text{vis}} = 3.92\times10^{-4}$*) to* 0.9964 *at* $\pi_e = 0.95$ *(*$f_{\text{vis}} = 9.03\times10^{-3}$*),* 0.9875 *at* $\pi_e = 0.90$ *(*$f_{\text{vis}} = 3.24\times10^{-2}$*), and* 0.7117 *at* $\pi_e = 0.50$ *(*$f_{\text{vis}} = 0.2500$*; block Y4).*

Proof. In (a), $1-\eta_{\mathrm{DT}} = p_0+p_1+p_3 \geq p_3 = f_I(p_2+p_3+p_4)$ by [Sch26i, Thm. 7.3]; squaring yields (9). In (b), independent draws of $[eh]$ at two adjacent D sites have probability $(2\pi_e\pi_h)^2 = 4\pi_h^2(1-\pi_h)^2$, and (10) is [Sch26i, Prop. 10.11] divided by R_T. □

Corollary 5.3 (Discharge of Article 84, Target 14.4: Arm conversion yields zero bias on the capped coding locus)**. [D]** *Under H_{acc} ([Sch26i, Hyp. 10.9]) and H_v ([Sch26h, Hyp. 4.18]), on the alternating D–T coding locus where accepted $D \in \{D2(ee), Dm(eh)\}$ and accepted $T \in \{T2(eeee)\}$—and including a terminal initiator cap $I(eee)$ adjacent to $D \in \{ee, eh\}$ or $T(eeee)$—the orientation information gained from partial arm conversion is identically zero:*

$$I_{\mathrm{vis},coding}(\pi_h) = 0 \qquad \forall\, \pi_h \in [0,1). \tag{11}$$

Moreover, replacing $D2(ee)$ by cleaved $Dm(eh)$ at a D–T2 or $I(eee)$–D contact lowers the registration count from $v=2$ to $v=1$, weakening competitor suppression by $e^{\Delta_{\mathrm{rgy}}}$ while reducing $\eta_{\mathrm{arm}}(\pi_h)$ (Y2, Y4).

Proof. Under H_{acc}, the nucleotide transformation accepts T only as $T2 = eeee$, while D accepts $\{D2, Dm\} = \{ee, eh\}$ and the initiator use accepts $I2 = eee$. By Proposition 5.1(d) and Remark 4.2, every interior $D(ee/eh)$–$T(eeee)$ contact and every terminal $I(eee)$–$D(ee/eh)$ or $I(eee)$–$T(eeee)$ contact satisfies $v_0 = v_1$ ($\Delta v = 0$). □

6 Terminal blindness on the 3D interior lattice (Target 14.10 of Article 84)

Article 84's Remark 9.4 and Target 14.10 observe that in the heritable minimum box $B(3,3,6)$ of Article 80 [Sch26d], primary arms are consumed as covalent lateral bridges: "Article 80 counts '72 bridges over 216 arm terminals, 72 of which remain free' [[Sch26d, Hyp. 3.3]]—so what a lattice node presents at a stacking contact is the free-terminal pattern and not the one the theorem analyses. That the stacking contacts of a $3\times3\times6$ box are blind is therefore asserted here and not shown; it is Target 14.10" [Sch26h, Rem. 9.4]. We now perform that exact computation and discharge Target 14.10.

Proposition 6.1 (Exact free-terminal distribution of $B(3,3,6)$)**. [A]** *Let $\Omega = B(3,3,6) = \{0,1,2\}\times\{0,1,2\}\times\{0,\dots,5\}$ carry T units at all 54 nodes under saturated bridging H_{sat} ([Sch26d, Hyp. 3.3]), with exact-closure connectors joining $(\mathbf{k}, v_\alpha^+)$ to $(\mathbf{k}+e_\alpha, v_\alpha^-)$ where $v_x^- = v_3$, $v_x^+ = v_9$ and $v_y^- = v_{12}$, $v_y^+ = v_6$ ([Sch26b, eqs. (50)–(51), (107)]). In each 3×3 planar layer $k_z \in \{0,\dots,5\}$, the set of unbridged (free) primary locants $S_{\mathrm{free}}(k_x,k_y) \subseteq \Lambda$ is determined by (k_x,k_y) as follows:*

(a) Central node $(1,1)$ (1 per layer, 6 in Ω): *Lateral degree 4; all four primary locants are bridged, so*

$$S_{\mathrm{free}}(1,1) = \emptyset \quad (0 \textit{ free terminals}),$$
$$S_{\mathrm{bridged}}(1,1) = \{3,6,9,12\}.$$

(b) Four edge nodes (4 per layer, 24 in Ω): *Lateral degree 3;* $S_{\mathrm{free}}(0,1) = \{3\}$, $S_{\mathrm{free}}(2,1) = \{9\}$, $S_{\mathrm{free}}(1,0) = \{12\}$, $S_{\mathrm{free}}(1,2) = \{6\}$ *(1 free terminal each, 4 per layer, 24 in Ω).*

(c) Four corner nodes (4 per layer, 24 in Ω): *Lateral degree 2;* $S_{\mathrm{free}}(0,0) = \{3,12\}$, $S_{\mathrm{free}}(0,2) = \{3,6\}$, $S_{\mathrm{free}}(2,0) = \{9,12\}$, $S_{\mathrm{free}}(2,2) = \{6,9\}$ *(2 free terminals each, 8 per layer, 48 in Ω).*

Summing across the 6 layers gives $6\times(0+4+8) = 72$ free arm terminals and $216-72 = 144$ bridged terminals (72 bridges) alongside 45 vertical stacking contacts (block Y9).

Proof. At site (k_x,k_y,k_z), vertex $v_3 = v_x^-$ is bridged to (k_x-1,k_y,k_z) iff $k_x \in \{1,2\}$, hence free iff $k_x = 0$; $v_9 = v_x^+$ is bridged to (k_x+1,k_y,k_z) iff $k_x \in \{0,1\}$, hence free iff $k_x = 2$. Similarly, $v_{12} = v_y^-$ is free iff $k_y = 0$, and $v_6 = v_y^+$ is free iff $k_y = 2$. Evaluating these four conditions at the nine pairs $(k_x,k_y) \in \{0,1,2\}^2$ gives (a)–(c) and the exact sum $6\times12 = 72$. □

Theorem 6.2 (Discharge of Article 84, Target 14.10: Lattice terminal blindness)**. [D]** *On the heritable complete lattice $\Omega = B(3,3,6)$ of [Sch26d, Thm. 5.1(b)]:*

(a) Complete-site and central-column blindness (post-bridging): *At the central column $(1,1,k_z)$, which contains all $|K(B(3,3,6))| = \jmath(3)\jmath(3)\jmath(6) = 4$ complete sites of the lattice ($k_z \in \{1,2,3,4\}$) and all 5 central stacking contacts, both $S_{\mathrm{free}}(1,1) = \emptyset$ and $S_{\mathrm{bridged}}(1,1) = \Lambda$ are ι-invariant ($\iota\emptyset = \emptyset$, $\iota\Lambda = \Lambda$). Hence every complete site of $B(3,3,6)$ is strictly flip-blind ($v_0 = v_1$) under both the free-terminal pattern and the bridged-terminal pattern.*

(b) Accretion blindness across all 45 stacking contacts: *Under [Sch26f, Hyp. 7.1], where π-stacking precedes covalent linker closure ($e_P + e_P \to$ phosphodiester bridge), each node enters its stacking contact as an unbridged $T(e_P^4)$ or $T(e^4)$ unit with ι-invariant pattern Λ. Consequently, at the moment of stacking, all 45 stacking contacts of $B(3,3,6)$ are blind ($v_0 = v_1 = 4$).*

(c) Boundary locking after linker closure: *Once the 72 covalent lateral bridges are closed, each peripheral column $(k_x,k_y) \neq (1,1)$ (20 edge contacts with $|S_{\mathrm{free}}| = 1$ and 20 corner contacts with $|S_{\mathrm{free}}| = 2$) has $\iota S_{\mathrm{free}}(k_x,k_y) = S_{\mathrm{free}}(2-k_x,2-k_y) \neq S_{\mathrm{free}}(k_x,k_y)$. Flipping a single peripheral node in place after linker closure would map its bridged locants $S_{\mathrm{bridged}}(k_x,k_y)$ to $\iota S_{\mathrm{bridged}}(k_x,k_y)$, an inadmissible endpoint relabelling in the fixed-connector graph, leaving $C(\Omega)$ (block Y9).*

Proof. (a) By Proposition 6.1(a), $S_{\text{free}}(1,1) = \emptyset$ and $S_{\text{bridged}}(1,1) = \{3,6,9,12\}$. Both sets are fixed by ι, and all four arms are identical linker arms, so Theorem 7.3 of [Sch26h] gives unit and contact blindness on the entire central column. By [Sch26d, Prop. 3.4], the complete sites of $B(3,3,6)$ are $(1,1,k_z)$ for $1 \le k_z \le 4$, which all lie in this column. (b) Prior to linker closure, every node carries four identical unbridged arms on $\Lambda = \{3,6,9,12\}$, which is blind by [Sch26h, Thm. 7.3]. (c) Since $\iota(3) = 9$ and $\iota(12) = 6$, ι exchanges $v_x^- \leftrightarrow v_x^+$ and $v_y^- \leftrightarrow v_y^+$, which maps $S_{\text{free}}(k_x,k_y)$ to $S_{\text{free}}(2-k_x, 2-k_y)$. This equals $S_{\text{free}}(k_x,k_y)$ iff $(k_x,k_y) = (1,1)$. □

Corollary 6.3 (Selection costs under an independent unbiased registry law)**. [D]** *Under Theorem 6.2(b), the independent-contact source law, and a retained reference convention, the vertical alignment field $\eta : E_z(B(3,3,6)) \to \mathbb{Z}_2$ is uniform on $\{0,1\}^{45}$. Selecting one field therefore costs* 45 *bits. Adding eight inter-column relations along a spanning tree gives* 53 *relations. The selection cost is* 53 *bits only if these eight additional observables are independent and unbiased, conditional on the vertical field. Geometry alone does not establish that probability law (Ruling 9.2; block Y9).*

7 Dual persistence dynamics: coupled return and flip semigroups

Theorem 7.1 (Bifurcation of the coupled persistence semigroup)**. [D]** *Under H_{cov} and H_{joint}, let an l-site column prepared in state $c_0 = (\kappa^0, \epsilon^0, \eta^0)$ evolve in the closed source ($u_n = 0$) under the coupled generator $\mathcal{L}_{\text{tot}} = \mathcal{L}_{\text{ret}}(r_n, a_n, d_n) + \mathcal{L}_{\text{flip}}(\varphi)$, Here H_{lock} denotes $\varphi = 0$; H_{place} denotes initial preparation only and does not assert retention. Here μ_t in (i)–(ii) denotes the projected law of chemical labels, directional code, and registry, not the phase-resolved chain; phase exchange may continue. The rate $r_n \ge 0$ is the bulk return rate ([Sch26i, §9.1]) and $\varphi \ge 0$ is the contact flip rate ([Sch26h, Prop. 8.19]). For (iii), use a finite fixed-slot chain with phases $\{\kappa_n, \sigma_n, \beta_n\}$ for each accessible degree n. Assume $d_n, r_n, k_{n+1} > 0$ for $n < 4$, $e_n > 0$, and $a_n > 0$ when reassembly is required. Substitution advances degree irreversibly, degree 4 is closed as a species family, and no destructive reaction, vacancy loss, or replacement by another molecule is included. Thus a slot retains its molecular identity during phase exchange. As $t \to \infty$:*

(i) Double persistence ($r_n = 0, \varphi = 0$; $H_{\text{ret}} \wedge H_{\text{lock}}$): $\mu_t = \delta_{c_0}$ *for all* $t \ge 0$.

(ii) Rotational relaxation ($r_n = 0, \varphi > 0$; $H_{\text{ret}} \wedge \neg H_{\text{lock}}$): $\pi_{\text{chem}}(\mu_t) = \delta_{\kappa^0}$ *is preserved forever, while* $\|\pi_{\text{reg}}(\mu_t) - U(\{0,1\}^{l-1})\|_{\text{TV}} \le \frac{1}{2}\sqrt{(1+e^{-4\varphi t})^{l-1} - 1} \to 0$, *erasing the* $l-1$ *registry bits on timescale* $\tau_{\text{flip}} = O(\varphi^{-1} \log l)$ *at fixed total-variation tolerance.*

(iii) Chemical return relaxation ($r_n > 0, \varphi = 0$; $\neg H_{\text{ret}} \wedge H_{\text{lock}}$): *Every unit of degree* $n < 4$ *returns to* β *in mean time* $1/d_n + (1 + a_n/d_n)/r_n$ *(amplified by* $1 + a/d = 101$ *to* $101,020$ *at V11 rates) and advances irreversibly to* T, *driving the species-family projection* $\pi_{\text{family}}(\mu_t) \to \delta_{\{T\}^l}$ *with bulk residual* $\pi(4,\beta) = 9.9 \times 10^{-8}$ *(Y7).*

Proof. For (i), neither family labels nor registry bits change in the projected law. For (ii), each bit is an independent symmetric two-state chain. Its bias is $e^{-2\varphi t}$, so the product law has chi-square divergence $(1 + e^{-4\varphi t})^{l-1} - 1$ from uniform; Cauchy–Schwarz gives the displayed total-variation bound. For (iii), first-step equations from κ_n and σ_n give the stated mean time to β_n. Each lower-degree state has a positive-rate path to degree 4. Since degree never decreases and the state space is finite, no closed communicating class lies at degree below 4. All lower-degree states are transient, and absorption into the degree-4 family occurs almost surely. Phase exchange within that family can continue. Apply this argument to the finitely many slots and use a union bound. The numerical bulk residual is the stationary phase fraction for the selected V11 rates, not a universal constant. □

Corollary 7.2 (Cap conversion and conditional loss of frame-dependent readout)**. [D]** *Suppose $\varphi = 0$, $r_2 = 0$, and the only frame carrier is an initiator cap $I = \{3,6,12\}$ with $r_3 > 0$. Assume the six-state continuous-time chain on $\{\kappa_3, \sigma_3, \beta_3, \beta_4, \sigma_4, \kappa_4\}$ has transitions $\kappa_n \to \sigma_n$ at rate d_n, $\sigma_n \to \kappa_n$ at rate a_n, $\sigma_n \to \beta_n$ at rate r_n, $\beta_n \to \sigma_n$ at rate e_n, and $\beta_3 \to \beta_4$ at rate k_4; the target κ_4 is absorbing for the hitting-time calculation. For positive required rates, the mean time from κ_3 to κ_4 is*

$$\mathbb{E}[\tau_{\text{cap}}] = \frac{1}{d_3} + \frac{1 + a_3/d_3}{r_3}\left(1 + \frac{e_3}{k_4}\right) + \frac{1}{k_4} + \frac{1}{e_4} + \frac{1 + r_4/e_4}{a_4}. \tag{12}$$

The conversion $I \to T$ removes this frame carrier. If the observation rule H_{frame} identifies all registry fields after the loss of every frame carrier, then the resulting frame-dependent readout contains no registry information. This is a conditional statement about the observation quotient, not a claim that the physical relative bits η_i are destroyed by global-flip symmetry.

Proof. Let m_s be the mean hitting time of κ_4 from state s. The first-step equations $-1 = \sum_t q_{st}(m_t - m_s)$ with $m_{\kappa_4} = 0$ give (12). Substitution at position $9 = \iota(3)$ turns I into the inversion-invariant T. Under the specified observation quotient and absence of another reference, all frame-dependent outputs coincide. In contrast, the edge relation $\eta_i = w_i \oplus w_{i+1}$ is unchanged by a global flip; symmetry alone does not erase it. □

Proposition 7.3 (A finite-time chemical and registry retention window)**. [A]** *Let $l \geq 2$, $d_n > 0$, and $a_n, r_n, \varphi \geq 0$. Consider an l-site column initially in κ, with independent contact flips at rate φ and susceptible degree-2 or degree-3 units evolving by $\kappa_n \rightleftarrows \sigma_n \to \beta_n$ at rates d_n, a_n, r_n. For a period $\delta T > 0$, the probability that any susceptible unit returns to bulk is at most $\varepsilon_{\mathrm{chem}}$, and the registry distribution is within total variation $\varepsilon_{\mathrm{reg}}$ of its initial point mass, whenever*

$$\max_{n\in\{2,3\}} \bar{r}_n \leq \frac{\varepsilon_{\mathrm{chem}}}{l\,\delta T}, \qquad \bar{r}_n = \frac{r_n}{1 + a_n/d_n}, \qquad \varphi \leq \frac{\varepsilon_{\mathrm{reg}}}{(l-1)\delta T}. \tag{13}$$

The numerical ceilings at $l = 6$, $\delta T = 1$, and both tolerances 0.05 are $\bar{r}_n \leq 8.33 \times 10^{-3}$ and $\varphi \leq 1.00 \times 10^{-2}$ (block Y7). Degree-4 bulk returns are not included in this event; include $n = 4$ in the maximum if total monomer retention is required. Frame-dependent readout additionally requires survival of a reference carrier.

Proof. For one susceptible unit, let $S(t)$ be the probability of not yet hitting bulk, and $y(t)$ the conditional probability of being in σ given survival. The two-state transient equations imply

$$y' = d - (a + d + r)y + ry^2, \qquad y(0) = 0.$$

At $y = d/(a+d)$ the right-hand side is $-rad/(a+d)^2 \leq 0$, so $0 \leq y \leq d/(a+d)$. Since $S' = -ryS$, one has $S(t) \geq e^{-\bar{r}t}$ and hence $\Pr(\tau_\beta \leq t) \leq \bar{r}t$. A union bound over at most l susceptible units proves the chemical bound. For $m = l - 1$ independently flipping contact bits, each bit retains its initial value with probability $(1 + e^{-2\varphi t})/2$. Thus

$$\|\mu_{\mathrm{reg},t} - \delta_{\eta^0}\|_{\mathrm{TV}} = 1 - \left(\frac{1 + e^{-2\varphi t}}{2}\right)^m \leq m\varphi t.$$

This is a retention bound relative to the initial registry, distinct from the mixing-to-uniform estimate in Theorem 7.1(ii). □

8 Operator-algebraic and spectral embedding

Definition 8.1 (Explicit structural-state embedding j_{str})**. [D]** Let $\mathfrak{A}_{\mathfrak{C}} = \mathfrak{Q}_{\mathfrak{C}} \rtimes_\alpha \mathbb{Z}^3$ be the sequence algebra of [Sch26b, Axiom A_7] with conditional expectation $E : \mathfrak{A}_{\mathfrak{C}} \to \mathfrak{Q}_{\mathfrak{C}}$ and faithful trace τ ([Sch26b, eqs. (724)–(732)]), where $\mathcal{K}_k \cong V_{\mathrm{int}} \otimes K_{\mathrm{aux}} \otimes K_{\mathrm{dc}} \otimes K_z$ has $K_{\mathrm{aux}} \cong \mathbb{C}^{15} \otimes \mathbb{C}^{|\mathcal{A}_{\mathrm{arm}}|^4}$ indexing $(n, \phi) \in X = \{0, \ldots, 4\} \times \{\beta, \sigma, \kappa\}$ and the arm word. For $x \in X$, define $j_{\mathrm{str}}(x)(A) := (\mathrm{Tr}_{\mathcal{K}_0}(\rho_x \cdot) \otimes \tau_{\mathbb{Z}^3\setminus\{0\}})(E(A))$ where ρ_x is the rank-one projection onto the basis label x at site 0, tensored with fixed normalized reference vectors on every unused arm, V_{int}, K_{dc}, and K_z factor. For a fixed connected column of length $l \geq 2$ and $c = (\kappa, \epsilon, \eta) \in C_{\mathrm{src}}(\Omega)$ with site lift $w \in \mathrm{agr}^{-1}(\eta)$, let $\rho_c \in \mathfrak{Q}_\Omega$ use orthogonal basis labels for (κ_i, ϵ_i), fixed normalized vectors on unused factors, and the flip-symmetrized state $\rho_{z,\eta} := \frac{1}{2}(|w\rangle\langle w|_z + |\mathrm{flip}\, w\rangle\langle \mathrm{flip}\, w|_z)$ on $K_z^{\otimes l}$, and set $\varphi_c = j_{\mathrm{str}}(c) := (\mathrm{Tr}_{\mathcal{K}_\Omega}(\rho_c \cdot) \otimes \tau_{\mathbb{Z}^3\setminus\Omega}) \circ E \in S(\mathfrak{A}_{\mathfrak{C}})$.

Proposition 8.2 (Injectivity and pure-character contrast on the contact subalgebra)**. [A]** *On the labelled configuration space retaining η (not the post-cap-loss observation quotient), the map j_{str} of Definition 8.1 is injective on X and on $C_{\mathrm{src}}(\Omega)$, discharging item 1 of Axiom A_{12} in [Sch26i, Def. 9.1]. On the unobservable site algebra $B(K_z^{\otimes l})$, $\rho_{z,\eta}$ has rank 2, whereas restricted to the diagonal relative-registry subalgebra $\mathfrak{Z}_\Omega := C^*(\{Z_iZ_{i+1}\}_{i=0}^{l-2}) \cong \mathbb{C}^{2^{l-1}}$ ($Z_i = |0\rangle\langle 0|_{z,i} - |1\rangle\langle 1|_{z,i}$), $\varphi_{\mathrm{exo}}^{(c_0)}$ is a pure character (extremal state) and the endogenous state restricted to the blind-column registry factor is tracial. This does not assert a trace for an unrestricted chemical mixture:*

$$\varphi_{\mathrm{exo}}^{(c_0)}(Z_iZ_{i+1}) = (-1)^{\eta_i^0} \in \{-1, +1\}, \qquad \varphi_{\mathrm{endo}}(Z_iZ_{i+1})\big|_{blind} = 0. \tag{14}$$

Proof. Orthogonal projections in $K_{\mathrm{aux}} \otimes K_{\mathrm{dc}}$ separate distinct (n, ϕ) and (κ_i, ϵ_i), while $Z_iZ_{i+1}|w\rangle_z = (-1)^{w_i + w_{i+1}}|w\rangle_z = (-1)^{\eta_i}|w\rangle_z$ is invariant under $w \mapsto \mathrm{flip}\, w$ and separates distinct η. □

Theorem 8.3 (A conditional Connes spectral-distance lower bound)**. [D]** *Let $D_{\mathfrak{C}}$ be the Dirac operator of [Sch26b], let $l \geq 2$, and assume $H_{\mathrm{int\text{-}Dirac}}$: the target-dependent whole-sum observable*

$$B_{\eta^0} = \sum_{i=0}^{l-2} (-1)^{\eta_i^0} Z_iZ_{i+1}$$

lies in its commutator domain and satisfies

$$\|[D_{\mathfrak{C}}, \pi_\omega(B_{\eta^0})]\| \leq 2\gamma_z, \qquad \gamma_z > 0.$$

For a placed configuration c_0 on a blind column of length l,

$$d_D(\varphi_{\mathrm{endo}}, \varphi_{\mathrm{exo}}^{(c_0)}) \geq \frac{l-1}{2\gamma_z}. \tag{15}$$

This is 2.50 at $l = 6$ and 26.50 at $l = 54$ when $\gamma_z = 1$ (block Y8). No upper bound, finiteness, or bi-Lipschitz equivalence is asserted from this hypothesis alone.

Proof. The observable $A = B_{\eta^0}/(2\gamma_z)$ is admissible in the spectral-distance supremum. Proposition 8.2 gives $\varphi_{\mathrm{exo}}^{(c_0)}(A) = (l-1)/(2\gamma_z)$ and $\varphi_{\mathrm{endo}}(A) = 0$. Taking their absolute difference proves (15). The whole-sum commutator hypothesis is not inferred from the finite-state calculations, nor does it imply a bound for arbitrary partial sums. □

Table 1: Comparative synthesis within the stated source and contact models of the Endogenous Source [Sch26i], the Placed Source [Sch26h], and the Interior Lattice [Sch26d].

Axis	Endogenous Source (Article 79)	Exogenous Placed Source (Article 84)	Unified Comparative Theorem (Article 90)
1. Reachable set $R(\Omega)$	$\operatorname{supp}\mu_\Omega = C_{\mathrm{src}}(\Omega)$ ($\vartheta \in \Theta_+$)	$\operatorname{im}\mathrm{place} = C_{\mathrm{src}}(\Omega)$ (H_{place})	$R_{\mathrm{endo}}(\Omega) = R_{\mathrm{exo}}(\Omega) = C_{\mathrm{src}}(\Omega)$ across Θ_+; $\Pi(R_{\mathrm{endo}}) = \Pi(R_{\mathrm{exo}})$ (Lem. 3.6)
2. Species cost c_{spec}	Matched by $k_3 = e_2\rho^{*-1}(1+x)$; $\eta = 0.998437$	Set site-by-site by exogenous placer	20.0–23.0 bits on hexamer; $\partial c_{\mathrm{chem}}/\partial\vartheta_{\mathrm{cont}} \equiv 0$ (Thm. 4.1, Cor. 4.3)
3. Registry cost c_{reg}	Blind on $\{D,T\}$ ($v_0 = v_1$); I–I at 2.39×10^{-7}	Prescribes $\eta \in \{0,1\}^{l-1}$ (5 bits at $l=6$)	$\partial c_{\mathrm{reg}}/\partial\vartheta_{\mathrm{kin}} \equiv 0$ and $\partial c_{\mathrm{reg}}/\partial\Delta_{\mathrm{rgy}} \equiv 0$ on $\{D,T\}$-contact locus (Thm. 4.1, Rem. 4.2)
4. Mixed words ($\pi_h > 0$, Target 14.4)	Lowers $\eta_{\mathrm{arm}}(\pi_h)$ ([Sch26i, Prop. 10.11])	Open Target 14.4 of [Sch26h]	**Discharged**: $\Delta\nu = 2$ on $D(eh)^2$, but $\Delta\nu = 0$ on $D(eh)$–$T(eeee)$ and $I(eee)$–$D(eh)$; zero coding bias (Cor. 5.3)
5. 3D lattice $B(3,3,6)$ (Target 14.10)	Supplies $T2$ nodes (54 per heritable unit)	Open Target 14.10 of [Sch26h] (72 free terminals)	**Discharged**: Central column (4 complete sites) has $S_{\mathrm{free}} = \emptyset$ (blind); $45+8=53$ (Thm. 6.2, Rul. 9.2)
6. Persistence	H_{ret} ($r_n = 0$, no bulk return)	H_{place} prepares the state; H_{lock} ($\varphi = 0$) maintains registry	$r_3 > 0$ converts cap $I \to T$, removing its frame carrier even at $\varphi = 0$; readout loss requires the observation quotient (Thm. 7.1, Cor. 7.2)
7. Axiom A_{12} state	Partial instance (j_{str} omitted, [Sch26i, Def. 9.1])	Combinatorial quotient on $\Sigma^{\Omega}_{\mathfrak{C}}$	**Item 1 supplied**: Explicit injective $j_{\mathrm{str}} : X \to S(\mathfrak{A}_{\mathfrak{C}})$; pure character on $\mathfrak{Z}_\Omega$; $d_D \geq \frac{l-1}{2\gamma_z}$ (Def. 8.1, Thm. 8.3)

9 Precedence rulings and targets

Ruling 9.1 (Articles 40, 79, 80, and 84: Discharged reservations and targets)**. [D]** Parallel and consistent; three open items discharged:

(i) *Article 79, Definition 9.1:* Definition 8.1 and Proposition 8.2 supply the injective spectral realization $j_{\mathrm{str}} : X \to S(\mathfrak{A}_{\mathfrak{C}})$, supplying item 1 of Article 79's Axiom A_{12}; no spectral upper bound is inferred.

(ii) *Article 84, Target 14.4:* Proposition 5.1, Theorem 5.2, and Corollary 5.3 settle the mixed-arm cleavage problem both in the interior and at the cap.

(iii) *Article 84, Remark 9.4 and Target 14.10:* Proposition 6.1 and Theorem 6.2 settle the terminal blindness of $B(3,3,6)$ using Article 80's 72-bridge / 72-free-terminal count.

Ruling 9.2 (Reconciling the 45-contact and 53-bit figures at $|\Omega| = 54$)**. [A]** Article 84's Proposition 5.4 (and Article 80's Hypothesis 3.3) counts $|E_z(B(3,3,6))| = 3\times3\times(6-1) = 45$ vertical stacking contacts in the 54-node heritable minimum box, whereas Article 84's Abstract and Theorem 9.2 quote 53 bits at $l = 54$. Both figures are exact on their respective graphs: $B(3,3,6)$ consists of nine parallel six-site columns (45 vertical contacts), and adjoining the $9-1=8$ inter-column relative orientation agreements across a planar spanning tree of the nine columns gives $45+8=53$, the edge count of a spanning tree on 54 nodes (or of a single column $B(1,1,54)$). The 53-edge count is combinatorial. Its interpretation as 53 bits additionally requires eight independent unbiased inter-column observables, as stated in Corollary 6.3.

Target 9.3 (Homotypic $D(eh)$ vs. heterotypic $D(eh)$–$T(eeee)$ and $I(eee)$–$D(eh)$ dimer NMR)**.** Prepare and characterize $Dm = D(eh)$ and determine the two relative orientations structurally (for example, by resolved NMR constraints). Under H_v, equal orientation degeneracies, and the Boltzmann contact law, the unaligned fraction is

$$\theta^{(eh)}_{DD} = (1+e^{2\Delta_{\mathrm{rgy}}})^{-1},$$

whereas the unaligned-to-aligned population ratio is $e^{-2\Delta_{\mathrm{rgy}}}$. Compare these with the predicted equal populations for $D(eh)$–$T(eeee)$ and $I(eee)$–$D(eh)$ under the same effective model. Establish aggregation state, stoichiometry, exchange regime, and orientation degeneracy before inferring Δ_{rgy} from a spectrum. A measured bias in the heterodimers would test the coincidence hypothesis rather than refute the finite counting identity.

Target 9.4 (Cap return $\bar{r}_3$ vs. stack flip φ)**.** Measure the desorption/return rate $\bar{r}_3$ of the trisubstituted cap I alongside the intra-stack flip rate φ against the window of Proposition 7.3 and Corollary 7.2.

Target 9.5 (Thermodynamic work ledger for H_{place})**.** Extend the material ledger of [Sch26c, Sch26a] with an explicit Hamiltonian, temperature, reservoirs, and preparation protocol. For a Gibbs reference μ at temperature T, with the same Hamiltonian before and after preparation, the nonequilibrium free-energy increase for a point target is

$$\Delta F = k_B T \ln 2\, D_{\mathrm{KL}}(\delta_{c_0}\|\mu) = k_B T \ln 2\, c(c_0;\vartheta).$$

The average work is bounded by the appropriate nonequilibrium free-energy change, with equality only in an ideal reversible limit [EVdB11]. An arbitrary kinetic source distribution need not be a Gibbs reference for the actual Hamiltonian. Therefore the information cost alone gives no universal physical work price; chemical reservoirs, feedback, and changes in interactions must be accounted for explicitly.

10 Conclusion

The endogenous self-separated source of Article 79 and the exogenous placed source of Article 84 operate on the same source-supported product-column space $C_{\mathrm{src}}(\Omega)$ under Convention 3.2. Within this model, positive source support and placed preparation have the same configuration and product images but control distinct factors. Bulk substitution and phase partition tune the vertex species distribution π_{chem} to match the stipulated material demand (giving $\eta = 0.998437$ for the selected model parameters), while the C_{2h} core symmetry that excludes four-ring competitors makes the stacking filter blind to the edge alignment field π_{reg} on the $\{D, T\}$-contact locus. Neither partial arm conversion ($\pi_h > 0$) nor lateral bridging in the $B(3,3,6)$ interior lattice breaks that blindness on the capped coding or complete-site locus: the accepted substrate $T2 = eeee$ blinds every $D(eh)$–$T(eeee)$ contact, symmetry on $\{6, 12\}$ blinds every $I(eee)$–$D(eh)$ cap contact, and the four complete sites of $B(3,3,6)$ have $S_{\mathrm{free}} = \emptyset$. Finally, the two persistence premises H_{ret} and H_{lock} meet at the trisubstituted initiator cap I: if the cap returns to the bulk and advances to T, it ceases to be a frame carrier even when every stacked unit is rotationally locked. Loss of frame-dependent readout follows only under the specified observation quotient and absence of another reference; global flip preserves physical relative alignment. The spectral comparison is a conditional lower bound, not a claim of finite distance or metric equivalence.

Chemical characterization and independence tests. A realization should establish identity, purity and arm connectivity for the h/e materials, and measure the $e \to h$ conversion time course and coproducts. Measured arm-count distributions can test the independent Bernoulli conversion premise; joint source and assembly compositions can test independent draws against correlated alternatives. These measurements would calibrate the finite model rather than being supplied results of this paper.

A Hypothesis register

Table 2 records the premises used in the conditional claims and the conclusions independent of each premise.

B Verification blocks

The accompanying `verify_a90.py` evaluates finite instances and numerical quantities using the Python standard library. Its 153 checks comprise 71 mathematical or sample checks and 82 checks that generated macro names are unique. The revised program passes both ordinary Python and optimized `python -O`; its guards are explicit runtime checks. Exact rational arithmetic uses `fractions.Fraction`; logarithms, exponentials, and displayed rounding use floating point. The separate Lean 4 file proves nine finite arithmetic certificates. Neither program proves the general theorems, physical hypotheses, source identifications, or operator-domain assumptions of this article.

`Y1` Baseline source arithmetic. Reproduces the selected exit distribution and matched design at $q = 10$, $k_1 = k_2 = k_4 = 100$, $R_D = 42,354$, $R_T = 208$, and $R_I = 1$, giving $k_3 \approx 5.40 \times 10^{-3}$, $\eta_{\mathrm{DT}} = 0.998413$, and $\eta = 0.998437$. Finite contact checks support the blindness premises; the Jacobian identity is proved analytically in Theorem 4.1, not by symbolic differentiation in the program.

`Y2` Arm classes and contact coincidences. Enumerates the 21 arm classes at $m = 2$ and 78 at $m = 3$, and checks selected blind/visible counts and the $D(eh)$–$D(eh)$, $D(eh)$–$T(eeee)$, and $I(eee)$–$D(eh)$ coincidences used in Proposition 5.1.

`Y3` Homotypic orientation bias. Evaluates the selected Boltzmann weights and binary information gains at $\Delta_{\mathrm{rgy}} = 1, 2$, including $(0.1192, 0.4729)$ and $(0.0180, 0.8700)$.

`Y4` Cleavage efficiency samples. Evaluates η_{arm} at $\pi_e \in \{0.99, 0.95, 0.90, 0.75, 0.50\}$ and the homotypic visible-contact frequency. Zero coding bias follows from the analytic equal-contact argument in Corollary 5.3, not from a symbolic proof by the script.

`Y5` Initiator-doping samples. Evaluates the selected cooperation ratios and initiator-pair frequencies at $x \in \{0.1, 1, 10\}$. These numerical samples do not establish the general inequality of Theorem 5.2.

`Y6` Hexamer costs. Evaluates the cost formula at $\rho^* = 100, 200$ and $x = 0.1$, giving species costs 20.0 and 23.0 bits and registry cost 5 bits.

`Y7` Persistence samples. Checks the selected buffering ratio 101, stack-start mean $101,000$, column-start mean $101,020$, residual bulk weight, and sufficient rate ceilings 8.33×10^{-3} and 1.00×10^{-2} at $l = 6$ and tolerances 0.05. Additional selected checks distinguish registry retention from mixing and test the cap hitting-time formula. The general semigroup conclusions and survival bound rely on the stated analytic proofs and hypotheses.

Table 2: Hypotheses of this paper and imported premises, where each is used, and what survives its failure.

Tag	Statement	Used in	What survives its failure
H_{draw}, H_{v}, H_{rgy} [Sch26i, Sch26h]	source-supported product column; independent draws; $v_\eta = \lvert \mathrm{pat}_1 \cap \mathrm{pat}_\eta \rvert$; Boltzmann registry law	§§3–5	Finite arm-word counts survive; full source support requires the stated positive product law or a separately verified compatible support.
H_{acc} [Sch26i]	T accepts $T2(eeee)$; D accepts $\{D2, Dm\}$; I accepts $I2(eee)$	Thm. 5.2(b), Cor. 5.3	Proposition 5.1(c) on homotypic $D(eh)$–$D(eh)$ contacts
H_{sat} [Sch26d], [Sch26f, Hyp. 7.1]	saturated lateral bridging; accretion precedes linker closure	Prop. 6.1, Thm. 6.2	Given the same saturated central-site pattern, central blindness is independent of accretion order. Free-terminal counts require saturated bridging.
H_{frame} [Sch26h], H_{cov} [Sch26i]	specified frame-dependent observation quotient; no other reference for cap-loss conclusion; irreversible substitution	Thm. 7.1, Cor. 7.2	Sections 3–6 and Theorem 8.3
H_{joint} (this paper)	return $\mathcal{L}_{\mathrm{ret}}$ and flip $\mathcal{L}_{\mathrm{flip}}$ superpose independently	Thm. 7.1, Prop. 7.3	Absorption persists only if every lower-degree state still has a positive-rate path to degree 4; cap loss also requires advance and re-export.
$H_{\text{int-Dirac}}$ (this paper)	whole-sum observable B_{η^0} belongs to the commutator domain and has seminorm at most $2\gamma_z$; $\gamma_z > 0$	Thm. 8.3	Injectivity of j_{str} (Definition 8.1, Proposition 8.2)

Y8 Finite dimensions and witness arithmetic. Evaluates the site-code dimension $4^6 = 4,096$ and the conditional witness values $(l-1)/(2\gamma_z)$ at $l = 6, 54$ with $\gamma_z = 1$. The program does not construct density matrices, verify Pauli expectations, prove injectivity, or establish the whole-sum commutator hypothesis in Theorem 8.3.

Y9 Finite lattice counts and terminal symmetry. Enumerates the 54 nodes, 72 lateral bridges, 45 vertical contacts, 72 free terminals, central-column pattern, and the count $45 + 8 = 53$. It tests finite terminal-pattern symmetries; it does not verify the physical accretion order or supply the probability law needed to interpret 53 relations as 53 bits.

C Molecular commentary, isolated

Remark C.1 (Status)**. [A]** Nothing in this appendix enters a numbered statement or proof of the main text.

Remark C.2 (The filing's two routes and the origin of the duality)**. [E]** The Background of US 12,195,423 [Sch25] contrasts two synthetic regimes: de novo synthesis "processed through directed placement of monomers to build a single strand of DNA", which faces a "limitation as to the number of bases synthesized", versus schemes in which "molecules randomly self-organize", where "these random events provide unknown variability and ambiguity in DNA and cellular production". Theorem 4.1 and Corollary 4.3 place that contrast on an exact mathematical footing on a common material stock: the selected bulk-source model supplies the chemical species at model material efficiency $\eta = 0.998437$ while leaving the 5 binary agreements of a hexamer maximally ambiguous (1/32 per field), whereas directed placement resolves the 5 bits of alignment without enlarging the reachable chemical or product set.

Remark C.3 (Why the initiator cap is both the kinetic pinch point and the informational anchor)**. [E]** Across the filing and Articles 79 and 84, the 3,6,12-trisubstituted chrysene I plays two seemingly opposite roles. In the four-day bulk bromination of Figure 12 (clause (i) of [Sch26i, Rul. 13.6]), positions 6 and 12 react first and together and position 3 reacts next; to obtain the 3,6,9,12-tetrasubstituted substrate T without flooding the stock with I, the fourth substitution at position 9 must outrun export of I ($k_4 \geq e_3$, Hypothesis (K3) of [Sch26i]). In the coding stacks of Figure 4(g)–(j) (clause (j) of [Sch26h, Rul. 13.7]), that same trisubstituted unit I caps the stack, establishes the directional reference frame for the base-four code, and terminates polymerization "due to a lack of a sidechain at the 9 position in the compound". Theorem 5.2(a) and Corollary 7.2 show that these two roles are one geometric fact seen from two sides: position 9 is $\iota(3)$ under the C_{2h} inversion. Leaving position 9 empty breaks ι-symmetry and creates the unique frame carrier I; filling position 9 in the bulk restores ι-symmetry, turning the cap into a blind substrate T and removing this frame carrier. Loss of the reading frame additionally assumes no other reference survives.

Remark C.4 (Why mixed $Dm(eh)$ cannot orient an accepted coding stack or its cap)**. [E]** In an aqueous stack, a mixed-arm catalyst Dm = 6-hydroxyethoxy-12-hydroxy chrysene is chemically asymmetric along its y-axis: one end carries a short phenolic −OH (h) and the other a longer glycol ether $-\mathrm{OCH_2CH_2OH}$

(*e*). When two *Dm* units stack face to face, aligning *e*-over-*e* and *h*-over-*h* gives two matched label-coincidence scores under H_v ($v_0 = 2$), whereas anti-alignment places *e*-over-*h* at both ends ($v_1 = 0$, Proposition 5.1(c)). Why does that asymmetry fail to orient a coding cluster? Because the stipulated contact model pairs *Dm* with *T*2 = 3,6,9,12-tetrahydroxyethoxy chrysene (*eeee*) in the interior and against *I*2 = 3,6,12-trihydroxyethoxy chrysene (*eee*) at the cap. Both *T*2 and *I*2 carry *e*-arms at *both* position 6 and position 12! Flipping *Dm* merely moves its single *e*-arm from matching position 6 to matching position 12 ($v_0 = v_1 = 1$, Proposition 5.1(d)). The *y*-axis symmetry of both partners neutralizes the *y*-axis asymmetry of *Dm*.

Remark C.5 (Accretion before linker closure in the interior lattice)**. [E]** Article 78's Hypothesis 7.1 models the assembly of the interior chrysene lattice in two chemical steps: first, monomeric units are assumed to associate face to face by π-stacking; second, lateral phosphodiester bridges condense between adjacent primary locants ($3 \leftrightarrow 9$ along x, $12 \leftrightarrow 6$ along y). Theorem 6.2 shows why that order matters for the orientation field. During the π-stacking step, all four arms of each T monomer are unbridged and chemically identical, so every vertical contact is flip-blind. Once the 72 phosphodiester bridges close, the 72 remaining free terminals sit strictly on the exterior faces of the $B(3,3,6)$ box, while the interior column $(1,1,k_z)$—where all 4 complete sites of Article 80 reside—has all four arms covalently locked into the planar network and zero free terminals.

Remark C.6 (Chromonic residence vs. intra-stack rotational barriers)**. [E]** Proposition 7.3 compares two physical relaxation pathways of a stacked assembly: monomer desorption back into a hydrophobic phase ($\bar{r}_n$, governed by the solvation transfer barrier $\Gamma_{\mathrm{ret},n}$ and buffered by the column association ratio $1 + a_n/d_n = 101$) versus in-plane 180° rotation or facial turnover of a unit inside an aqueous column (φ, governed by the steric/π-overlap barrier Γ_{flip}). These are distinct testable kinetic premises. All $k_n, e_n, r_n, a_n, d_n, \varphi$ are effective inverse-time rates; numerical times use the reciprocal of the selected rate unit. Association, dissociation, and phase transfer require separate measurements at specified solvent, temperature, concentration, and ionic conditions. Additional polar arms alone do not determine activation barriers, and no compound-specific rotational rate is established here. Placed preparation does not imply locking: H_{place} concerns the initial state, whereas H_{lock} or a finite flip-rate bound concerns its retention.

Remark C.7 (Connectivity, phosphorylation, and physical locking)**. [E]** Saturated bridging fixes an endpoint-labelled connectivity graph. A forbidden single-node flip is a relabelling that violates this fixed-connector compatibility condition; its exclusion does not establish a molecular rotational barrier for flexible linkers. Physical phosphodiester formation requires specified phosphorylation states, activation chemistry, and reaction conditions. The exact terminal count is conditional on the assumed completed bridge graph. Arm conversion, lateral bridge formation, source transfer, and rotational relaxation are chemically distinct processes and require separate measurements.

D What was set aside in preparing this paper

Remark D.1 (A claimed cleavage bias at the initiator cap)**. [A]** A first reading of Proposition 5.1 suggested that while interior *D*(*eh*)–*T*(*eeee*) contacts are blind ($\Delta v = 0$), the terminal contact between a trisubstituted initiator cap *I*(*eee*) and a cleaved catalyst *D*(*eh*) would be orientation-selective, since neither *I*(*eee*) nor *D*(*eh*) is a blind unit. Direct evaluation in block `Y2` refuted that expectation: $\mathrm{pat}(I, eee) = \{(3,e),(6,e),(12,e)\}$ is symmetric on the $\{6,12\}$ orbit of D, so $v_0(I(eee), D(eh)) = |\{(6,e)\}| = 1$ and $v_1(I(eee), D(eh)) = |\{(12,e)\}| = 1$, giving $\Delta v = 0$. The contact is an explicit chemical instance of a blind contact between two flip-visible units ([Sch26h, Thm. 7.4]), and Corollary 5.3 holds at the cap contact as well as in the interior.

Remark D.2 (The 45-contact and 53-bit figures at 54 nodes)**. [A]** Article 84's Proposition 5.4 counts $|E_z(B(3,3,6))| = 45$ vertical stacking relations in the 54-node heritable minimum box, whereas its Abstract and Theorem 9.2 quote 53 bits at 54 sites. Both figures are exact on their respective graphs, as Ruling 9.2 records: $B(3,3,6)$ consists of nine parallel six-site columns (45 vertical contacts), and adjoining the $9 - 1 = 8$ inter-column relative orientation agreements across a planar spanning tree of the nine columns gives $45 + 8 = 53$, the edge count of a spanning tree on 54 nodes. A 53-bit selection cost further requires the independent unbiased law specified in Corollary 6.3.

Remark D.3 (A pure-state embedding on the site lift)**. [A]** An early form of Definition 8.1 embedded a configuration $c = (\kappa, \epsilon, \eta)$ into $\mathfrak{Q}_\Omega$ by choosing one site lift $w \in \mathrm{agr}^{-1}(\eta)$ and setting $\rho_z = |w\rangle\langle w|_z$. That choice made $j_{\mathrm{str}}(c)$ depend on a convention—which of the two lifts w, flip w is chosen—and so reintroduced the global flip bit that [Sch26h, Rul. 13.3] and [Sch26g, Prop. 4.7] exclude. Replacing $|w\rangle\langle w|_z$ by the flip-symmetrized density operator $\frac{1}{2}(|w\rangle\langle w|_z + |\mathrm{flip}\, w\rangle\langle \mathrm{flip}\, w|_z)$ makes $j_{\mathrm{str}}(c)$ a canonical function of the edge field η alone: rank-2 on the unobservable site algebra $B(K_z^{\otimes l})$ and a pure character on the observable contact subalgebra $\mathfrak{Z}_\Omega$ (Proposition 8.2).

Remark D.4 (Retention is not mixing, and a mean delay is not stochastic domination)**. [A]** The original metastability argument incorrectly used a mixing-to-uniform estimate to justify retention near a prepared point mass. The corrected Proposition 7.3 uses the exact independent-bit retention probability and a union

bound. Its chemical estimate follows from the two-transient-state equations, not from the positivity of a mean desorption delay. The stack-start mean is $(1 + a_n/d_n)/r_n$; the column-start mean adds $1/d_n$.

Remark D.5 (Spectral and frame claims not established by the finite calculations)**. [A]** The former spectral upper bound and bi-Lipschitz assertion have been withdrawn. A whole-sum commutator bound alone does not justify partial-sum bounds, finite spectral distance, or comparison with a path metric. Likewise, cap conversion removes a reference carrier but does not erase relative bits by global symmetry; any all-output identification is a separate observation assumption. The cap hitting time is now derived from its explicit six-state generator.

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