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The Dual Cipher, Structurally Induced GF(4) *Multiplication, and Real Spectral Structure: Companion-Matrix Realization of the Register Automorphism, Frobenius Reversal, and the Order-One Commutant Condition*

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Declared review status

This is an AI-assisted simulated review record, not independently appointed human peer review. Separate mathematical and chemistry review passes informed the revision. Concordance checks compare the scope of statements, proofs, computations and companion files. Revision responses describe changes prepared for the author's consideration; they are not unprovided statements attributed to the author.

Scope of Review

The controlling source is the supplied Article 92 `main.tex`. The earlier `referee_report_a92.tex` and `verify_a92.py` are evidence to be audited, not substitutes for checking the manuscript. Finite-field identities, finite-dimensional real spectral data, infinite-dimensional analytic claims and proposed molecular realizations are assessed separately.

Combined recommendation

Retain the exact finite-field and finite internal spectral results after substantive revision. A counterexample excludes the asserted order-one completion by the symmetrized global filtration. The revised paper distinguishes the global obstruction, valid finite internal triple and conditional chemical interpretation.

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1 Source record and historical review

The supplied report is preserved unaltered in `provenance/Article92_Prior_Referee_Source.tex`. Its provisional Volume 1, Number 5, October 2026 metadata and recommendation "Accept with Minor Revisions (Macro/Table Adjustments & Three Algebraic Refinements)" belong to the historical record, not to this review's disposition.

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1.1 Earlier findings retained as provenance

The prior report identified the companion matrix of $x^2 + x + 1$, the trace and Frobenius realization of the frame action, a perimeter-register dictionary, and an internal bimodule-split operator. It proposed four groups of revisions: define the missing contraction macro and repair a table; distinguish the opposite gate orders CS and SC; add the field norm and RNA/DNA dictionary relation; and establish stability under internal real inner fluctuations. These refinements remain relevant.

The historical report also treated the global filtration and local matrix computations as a complete discharge of the real spectral-triple target. The new audit does not adopt that conclusion. Exact checks on $M_4(\mathbb{C})$ cannot establish the order-one property for a different infinite-dimensional operator, and invariance under a real structure is not equivalent to order one.

2 Review architecture and retained results

> *M* — **Mathematical validity**
>
> Audit the companion-matrix construction, field actions, crossed-product multiplication, global filtration, finite bimodule identities, grading signs and inner fluctuations. A valid local theorem is retained without extending it to a global claim it does not prove.

> *C* — **Concordance**
>
> Compare abstract, introduction, theorems, proof, synthesis table, precedence rulings, hypothesis register, code, Lean certificates and historical review. Resolve conflicting scope and provenance statements across all publication files.

H — **Chemistry and domain admissibility**

Separate formal locant shifts, state-label dictionaries and finite-field gates from constitutional isomerization, catalyst dynamics, hydrogen-bond measurements and coherent physical implementation. State what evidence would connect the mathematical model to a specified compound.

2.1 Core results retained

For $M = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 1 \end{smallmatrix}\right)$ over $\mathbb{F}_2$, the relation $M^2 + M + I = 0$ realizes $\mathbb{F}_2[M] \cong \mathrm{GF}(4)$. With the specified unit $(1, 0)$, the transported multiplication is

$$(e, h)(e', h') = (ee' + hh',\ eh' + he' + hh') \pmod 2.$$

The Frobenius and affine register actions recover the finite Burnside tuple $(64, 0, 8, 8)$ and twenty orbits. These finite identities do not require a physical quantum gate.

On the Hilbert–Schmidt space of matrices, $JX = X^*$ and $D = L(H) + \varepsilon' R(H)$, with $H = H^*$, satisfy $JD = \varepsilon' DJ$ and

$$[[D, L(a)], R(b)] = [L([H, a]), R(b)] = 0.$$

This proves the finite internal order-one identity for every pair of matrices, not merely the tested representatives. It does not remove the global filtration obstruction.

3 Mathematical review and revision response

3.1 M1 The global order-one claim fails after symmetrization

Finding: critical. The finite internal triple does not repair the infinite network filtration. Let a be a trace-zero self-adjoint unitary at a site k of radius $n > 0$, in the faithful product-trace witness with translation index zero. Its trace-zero component is orthogonal to the preceding filtration level. Thus, for either the original or symmetrized filtration operator,

$$D1 = 0, \qquad Da = na, \qquad a^2 = 1.$$

The exact calculation is

$$\begin{aligned} [[D, L(a)], R(a)]1 &= D(a^2) - aD(a) - D(a)a + aD(1)a \\ &= -2n\, 1 \neq 0. \end{aligned}$$

Invariance under J is therefore insufficient for order one. Adding a finite order-one factor cannot cancel this obstruction in the usual separated construction.

Revision response. Accept. Replace the blanket discharge of the global real spectral target with two separate positive results: a J-compatible compact-resolvent filtration and a finite internal real triple. Add the counterexample and retain global order-one completion as an open target. This narrows a false claim while preserving the proved finite algebra.

3.2 M2 Repair the crossed-product argument

Finding. The original proof incorrectly leaves a local tensor at the same site under left multiplication by a translation. In fact,

$$u_{-e}\,\iota_e(\Delta)u_e = \iota_0(\Delta)u_0.$$

The statement and proof also use opposite translations, and membership in $\mathfrak{F}_n \setminus \mathfrak{F}_{n-1}$ alone does not establish a pure Q_n eigenvector.

Revision response. Accept. Replace that argument by the trace-zero local-unitary witness in M1, which supplies the required orthogonality. Retain the failure of the original J-sign relation only with its spectral-subspace justification; a nonzero positive operator cannot anticommute with an antiunitary preserving positivity.

3.3 M3 Correct the field dictionary

Finding. Direct substitution into χ contradicts the supplied RNA field table. With the stated bit convention, the corrected prime subfield is $\{0,1\} = \{A,U\}$, while purines occupy the scalar line $\omega\mathbb{F}_2 = \{0,\omega\}$, not a subfield. The additive coset $\omega + \mathbb{F}_2$ is distinct from that scalar line.

Revision response. Accept. Correct dictionary equations, fibres, complement maps and interpretations, and explicitly test the adopted convention. Retain the field multiplication, Frobenius and Burnside identities. See H1 for the corresponding chemical interpretation.

3.4 M4 State the contraction theorem conditionally

Finding. The four-label permutation U_χ is unitary, but a scalar interface contraction cannot by itself preserve a four-dimensional orientation space. A coarse-graining prescription must distinguish interface factors from retained orientation factors and intertwine the desired register map.

Revision response. Accept. State the representation-level isomorphism under an explicit retained-factor/coarse-graining hypothesis. Identifying its labels with covalent locants remains an additional physical realization premise. This avoids claiming a dimension-preserving mechanism from a scalar contraction alone.

3.5 M5 Supply the analytic filtration hypotheses and proof

Finding. The analytic result needs a faithful product trace, finite local matrix factors, a nested exhausting finite-support filtration and an algebraic crossed-product core. For the symmetrized orthogonal shells, the natural domain is

$$\mathrm{Dom}(D_{\mathrm{sym}}) = \left\{ \xi : \sum_{n\geq 0} n^2 \|Q_n^{\mathrm{sym}}\xi\|^2 < \infty \right\}.$$

Finite-rank nested exhaustion yields self-adjointness and compact resolvent. J preserves each shell and the domain. For bounded commutators, propagation for both A and A^* gives a two-sided band bound $Q_pAQ_q = 0$ when $|p-q| > r$. Summing the diagonal bands yields the coarse estimate

$$\|[D,A]\| \leq r(r+1)\|A\|.$$

Revision response. Accept. Add the witness assumptions, explicit domain and band argument. These analytic deductions are distinct from finite enumerations and do not establish order one.

3.6 M6 Make choices and notation explicit

Finding. The field construction is canonical relative to a declared coordinate identification and cyclic vector, not an empirically unique multiplication imposed by chemistry. The original expression applying Ψ_χ to an already vector-valued $\chi(a, b)$ is ill-typed. The alternative order-two generator produces a dual-number ring with a nonzero nilpotent element, not a ring all of whose elements are nilpotent.

Revision response. Accept. Use $x_r = \chi(a_r, b_r)$ interpreted in the transported field; state the coordinate and unit choices; use precise dual-number terminology. Retain the correct opposite gate-order identities.

3.7 M7 Preserve the finite real structure at its proper scope

Finding. The finite split operator, real-sign relation, order-one identity and self-adjoint internal fluctuations are valid. The doubled even sign triples use $\varepsilon' = +1$ under the stated convention. They do not establish every possible geometric reconstruction axiom or the global network result.

Revision response. Accept. State finite real even spectral triples/sign data at the proved scope and test genuine complex adjoints, explicit doubled matrices and nontrivial self-adjoint fluctuations. The symbolic proof covers arbitrary complex self-adjoint H; selected matrix checks illustrate and audit that proof rather than replace it.

4 Concordance review and revision response

4.1 C1 Correct the inherited cross-term count

Finding: critical concordance defect. Independent execution of the supplied verifier stops at its assertion that the selected cross-term has 192 nonzero double commutators. Exact enumeration gives 256: all 16×16 matrix-unit pairs fail for that example.

Revision response. Accept. Replace the incorrect number in generated values, manuscript and report. Preserve the failed source as provenance; do not repeat its blanket success claim.

4.2 C2 Match verification claims to implemented checks

Finding. The supplied U4 block does not implement the claimed order-one witness; U7 emits dimension and KO constants without constructing the doubled matrices. The original fluctuation check adds only a scalar multiple of the identity, leaving commutators unchanged.

Revision response. Accept. Implement the explicit global obstruction, doubled grading/sign checks, a nontrivial self-adjoint one-form fluctuation, complex-adjoint cases and negative controls. Use checks that remain active in optimized Python and compare both execution modes. Give each Lean certificate an explicit finite coverage description.

4.3 C3 Synchronize sources and conclusions

Finding. Imported theorem numbering from Articles 40 and 84 is source context, not independently certified concordance with unavailable full source documents. Abstracts, summaries, tables, hypothesis registers and companions must consistently distinguish finite algebra, analytic witness results and physical realization.

Revision response. Accept. Use the supplied bibliography export for verified metadata, propagate the narrowed global and physical scope throughout the package, and distinguish reviewed local statements from imported cross-reference claims. No finite certificate is presented as a proof of chemistry or an infinite analytic theorem.

5 Chemistry review and revision response

5.1 H1 Correct the register dictionary and its chemical classes

Finding. The stated orientation dictionary FF $= A$, FB $= C$, BF $= G$, BB $= U$, with $F = 0$, $B = 1$, and $\chi(a, b) = (b, a + b)$ gives

$$A = 0, \qquad C = \omega^2, \qquad G = \omega, \qquad U = 1.$$

The supplied field table disagrees with direct substitution. Consequently its identification of purines with the prime subfield and its complement formula cannot both be retained. The correct coordinate fibres are $\epsilon = 0 : \{A, G\}$ and $\epsilon = 1 : \{U, C\}$, while trace $\eta = 0$ selects $\{A, U\}$ and $\eta = 1$ selects $\{C, G\}$. Complement is translation by 1; Frobenius exchanges C, G and fixes A, U. The norm's nonzero class is encoding-dependent, not a chemical activity measure.

Revision response. Accept. Use the displayed orientation dictionary as the controlling convention and propagate its direct field images through equations, chemical classes, summaries and tests. This corrects the interpretation without changing the valid field multiplication.

5.2 H2 Separate coordinate recoding from molecular conversion

Finding. A unitary permutation of four basis labels is an exact coordinate map. A scalar bond contraction does not itself establish a retained four-dimensional physical subsystem or convert one substitution pattern into another. Primary and dual substituted compounds can be constitutional isomers; relabeling is not an interconversion mechanism.

Revision response. Accept. Separate the exact label isomorphism, the coarse-graining convention that retains both bits, and the physical-realization premise requiring a state/observable correspondence. Resolve the earlier target at that representational level rather than claim a demonstrated molecular transducer.

5.3 H3 Qualify decorated flip-blindness

Finding. Invariant locant sets are insufficient for an invariant decorated state or measured interaction. Substituent identities, conformations, external fields, partners and the readout kernel may break the relevant symmetry.

Revision response. Accept. Retain the locant-set theorem and state physical blindness only under matched decorations, an invariant state or ensemble and a symmetry-covariant interaction/readout. This is consistent with the revised Articles 90 and 91.

5.4 H4 Use hydrogen-bond-count terminology

Finding. Canonical Watson–Crick hydrogen-bond count is not a universal measure of pairing strength. Duplex free energy also depends on stacking, solvent, salt, sequence, temperature and conformation. Purine/pyrimidine denotes a fused-ring structural class. The orientation dictionary selects the coordinate map as a model premise; chemistry alone does not force that labeling.

Revision response. Accept. Use "canonical Watson–Crick hydrogen-bond-count class" and distinguish a dictionary bit from a thermodynamic prediction. The primary experimental study of Yakovchuk, Protozanova and Frank-Kamenetskii supports this separation [1].

5.5 H5 Distinguish perimeter combinatorics from site chemistry

Finding. A cyclic perimeter step preserves specified combinatorial inversion orbits, not molecular energies, aromatic bond orders, reaction rates or three-dimensional exclusion volumes. K-region bonds should not be described as permanently localized double bonds. Substitution can change bay-region geometry [2].

Revision response. Accept. Preserve the valid no-Euclidean-symmetry and no-steric-disjointness qualifications, use K-region bond/site terminology, and tie any steric test to actual structures, protonation, environment and conformer coverage.

5.6 H6 Specify the meaning of gate and spectral language

Finding. A CNOT–swap permutation represents a mathematical gate but does not demonstrate coherent molecular control, catalysis, selectivity or kinetics. An abstract Dirac operator or KO sign classification is not automatically an electronic Hamiltonian or a measured spectrum.

Revision response. Accept. Keep the exact algebra while describing the required state preparation, independent readout, control implementation and calibration for a molecular realization. "Structurally induced" remains relative to the selected operator and distinguished unit.

5.7 H7 Provide empirical closure targets

Finding. The formal paper benefits from a concise test plan, not invented experiments or a demand that all chemistry be completed before publishing mathematics.

Revision response. Accept. Specify isolation and characterization of substitution patterns, locant assignment, conformer and steric assessment under stated conditions, independent preparation/readout of register bits, complete truth-table testing for a claimed gate, and calibration of proposed operators against observables. These are future tests of realization premises, not completed results.

6 Integrated disposition and verification evidence

Scope-preserving resolution

The revised paper retains the field law, frame counts and finite bimodule real structure. It withdraws the global order-one completion and makes molecular realization conditional.

Empirical tests concern realization premises, not the exact field identities.

6.1 Execution and formal coverage

The replacement checker completes **6,325 explicit checks in each of normal and optimized Python modes**, with identical results and **ten negative controls**. The checks include the corrected RNA dictionary, field and orbit identities, the $-2n$ global obstruction for both filtrations, all 256 cross-term violations, genuine Gaussian-integer complex-adjoint cases, explicit doubled grading/sign identities, and a non-scalar represented self-adjoint one-form.

Lean 4.24.0 freshly compiles 53 named theorems. These cover field axioms, the RNA mapping, codon counts, finite support/cross-position/doubling certificates, and arbitrary-entry integer 2×2 associativity and split-order-one identities. The dependency audit reports 46 without axioms, one using propositional extensionality, and six using propositional extensionality and quotient soundness. The formal scope is partial: the Lean file does not formalize the full complex M_4 theorem, the infinite analytic filtration theorem, or molecular realization. The manuscript's general algebraic arguments and analytic witness proofs remain separate evidence.

The general finite complex theorem follows from left/right multiplication identities. Finite tests do not establish the global order-one property, which the counterexample disproves.

6.2 Focused revision closure

Focused chemistry and mathematical rechecks support the revision. Final corrections to the global-scope wording, J_- tensor factor, local M_4 witness and hypothesis register are incorporated. The review is closed for this package, subject to the stated scope and author review.

References

[1] P. Yakovchuk, E. Protozanova and M. D. Frank-Kamenetskii. Base-stacking and base-pairing contributions into thermal stability of the DNA double helix. *Nucleic Acids Research* **34** (2006), 564–574. https://doi.org/10.1093/nar/gkj454.

[2] D. E. Zacharias et al. The bay-region geometry of some 5-methylchrysenes: steric effects in 5,6- and 5,12-dimethylchrysenes. *Carcinogenesis* **5** (1984), 1421–1430. https://doi.org/10.1093/carcin/5.11.1421.