ANNALS OF THE CHRYSENE FORMALISM
RESEARCH ARTICLE | AI-AUGMENTED PEER-REVIEWED | VOL. 1, NO. 4 | NOVEMBER 2026

Topological Fault Tolerance, Fiedler Routability, and Partially Bridged Percolation of the Interior Chrysene Lattice

Homological Bypasses, Directional Attachment Constraints, and the Exclusion of Degenerate Star Graphs

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Abstract

We study directional port compatibility, finite bridge reliability and graph-theoretic fault tolerance in the interior chrysene lattice. In the fixed-port model, the disubstituted family $D = \{6, 12\}$ supports only the y-axis, while the admitted trisubstituted and tetrasubstituted families produce exact bridge–family counts. On a 3×3 layer, the fixed-port and orientation-admissible family sums are 4,410,944 and 10,648,000, compared with the corrected scalar-degree sum 60,454,593. Under independent lateral-edge installation on a compatible scaffold with intact vertical contacts, retention of at least one complete site in each prescribed daughter half has probability $[1 - (1 - p_b^4)^{h-2}]^2$. Within the $3 \times 3 \times 2h$ box family, the 95% retention thresholds are 90, 108 and 162 nodes at $p_b = 0.95$, 0.90 and 0.80; the stated containment model gives corresponding length parameters 239, 243 and 259 bp, with 233 bp at saturation. An edge lies on an elementary plaquette exactly when its endpoints admit a three-edge bypass. This yields a unit-metric distortion bound and preserves connectivity after deletion. Plaquette completeness directly excludes the seven- and fourteen-vertex crosses but has a sharp 19-vertex local minimum; the 27-vertex box requires an additional corner-closure hypothesis. We give the cycle ranks, boundary-map ranks, Laplacian gaps and single- and double-edge resilience of the two benchmark boxes, separating graph connectivity from mechanical stability and chemical repair. Reproducible verification and finite Lean certificates accompany the conditional molecular interpretation.

Keywords: Chrysene Formalism; interior chrysene lattice; Fiedler algebraic connectivity; homological scission; partially bridged percolation; directional attachment geometry; Axiom A_{12}; path metric.

1 Introduction

1.1 From saturated boxes to fault-tolerant networks

Article 80 (*The Completeness Condition*) [Sch26c] proves that the twelve-axiom framework of Article 40 [Sch26b] has a geometric realization at a site of the dividing protocell if and only if that site sits inside a laterally cross-linked three-dimensional network of extent at least three in each coordinate direction. Within the box configurations $B(n_x, n_y, n_z)$ of Article 78 [Sch26d] and under the saturated-bridging hypothesis H_{sat} ([Sch26c, Hyp. 3.3]), the minimal box carrying one complete site is $B(3, 3, 3)$ (27 nodes) and the minimal heritable box whose daughter halves after midplane cleavage each carry a complete site is $B(3, 3, 6)$ (54 nodes) [Sch26c, Thm. 5.1].

Four explicit open targets sit at the intersection of Article 80, Article 84 (*The Placed Source*) [Sch26e], Article 40 [Sch26b], and Article 15 [Sch26a]:

(i) *Attachment geometry of a two-armed unit ([Sch26e, Rem. 4.11, Target 14.8]).* Article 84 counts compatible configurations on a 3×3 layer by requiring only scalar degree compatibility $|S_{\mathbf{k}}| \geq d_{\text{lat}}(\mathbf{k})$ (obtaining $339,738,624$ layer states), while noting that an arm bridges in a definite lateral direction and leaving as Target 14.8 the question of whether $D = \{6, 12\}$ can carry connectors along two different lateral axes.

(ii) *Partial bridging and percolation of completeness ([Sch26c, Hyp. 3.3, Target 10.3], [Sch26e, Rem. 4.14, Target 14.13]).* When lateral bridging is incomplete ($\neg H_{\text{sat}}$), the installed connector set $E \subseteq E_{\text{lat}}(\Omega)$ becomes an in-

Citation: Schaper, C. D. (2026). Topological Fault Tolerance, Fiedler Routability, and Partially Bridged Percolation of the Interior Chrysene Lattice: Homological Bypasses, Directional Attachment Constraints, and the Exclusion of Degenerate Star Graphs. *Annals of the Chrysene Formalism*, 1(4):1866–1875.
https://chrysene.com/index.php/acf/article/view/94
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dependent datum and both the minimum within the prescribed box family and the conditional containment length L_{crit} rise.

(iii) *Direct exclusion of the degenerate cross ([Sch26c, Rem. 5.2, Target 10.1]).* Over arbitrary finite subsets of $\mathbb{Z}^3$, the least configuration with a six-neighbor site is the 7-node 3D cross X_7 (and X_{14} for the cleavable double cross). Article 80 asks in Target 10.1 for a strengthened predicate that excludes X_7 and X_{14} without appealing to the box restriction.

(iv) *Homological scission and Fiedler routability ([Sch26a, §2.3, §7.3], [Sch26b, Axiom A_{12}, Rem. 37]).* Article 15 models bond rupture via the Scission Projection $\hat{P}_{\text{sciss}}$, 2-cell boundaries $\partial_2 b$ in H_1, and the Fiedler eigenvalue $\lambda_2(L) > 0$ of the graph Laplacian $L = D - A$, while Article 40's Remark 37 notes that removing a constraint-violating edge leaves $d^*_C(x, y) < \infty$ whenever an admissible bypass exists.

1.2 What is established here, by kind

Recovered. Article 15's Scission Projection $\hat{P}_{\text{sciss}}$, homological boundary condition $\partial_2 b$, and Fiedler connectivity criterion $\lambda_2 > 0$ [Sch26a]; Article 40's directional attachment vertices $v_x^- = v_3, v_x^+ = v_9, v_y^- = v_{12}, v_y^+ = v_6$ ([Sch26b, eqs. (50)–(51)]), exact-closure connectors ([Sch26b, eq. (107)]), and Axiom A_{12} path metric d^*_C ([Sch26b, eq. (423), Rem. 37]); Article 80's site classes, completeness minima (27, 54), Minkowski-sum circumradius formula and the imported containment law (evaluated here as 233 bp with the printed constants), cross counts (7, 14), and Targets 10.1 and 10.3 [Sch26c]; Article 84's scalar layer count (339, 738, 624) and Targets 14.8 and 14.13 [Sch26e].

Proved here. Axis-specificity of $D = \{6, 12\}$ ($\Lambda_x(D) = \emptyset$), proving that D cannot bridge along two different lateral axes and is excluded from all 9 sites of a saturated 3×3 layer, addressing [Sch26e, Target 14.8] (Lemma 3.1, Theorem 3.2); exact partially bridged partition functions on $B(3, 3, 1)$ and $B(3, 3, 6)$ ($2^6 \times 41^3 = 4,410,944$ fixed-locant and $2^6 \times 55^3 = 10,648,000$ orbit-flexible family-bridge pairs per layer), addressing [Sch26e, Target 14.13] (Theorem 4.2); exact completeness percolation law $P_{\text{herit}} = (1 - (1 - p_b^4)^{h-2})^2$, raised node minima $N_{\min}(p_b, \varepsilon)$ (90, 108, 162 nodes at $p_b = 0.95, 0.90, 0.80$), and percolation-coupled genome thresholds $L_{\text{crit}}(p_b, \varepsilon)$ (239, 243, 259 bp), addressing [Sch26c, Target 10.3] (Theorem 4.3, Corollary 4.4); equivalence of 2-cell plaquette boundaries ($\partial_2 p$), length-3 admissible bypasses ($d^*_{C,\Omega\setminus\{e\}} \le 3 d^*_{C,\Omega}$), which imply single-edge Fiedler survival (Theorem 5.1); direct exclusion of X_7 and X_{14} ($\beta_1^{(1)} = 0, |F_2| = 0, \kappa_{\text{edge}} = 1$) and, separately for full boxes, exact Euler–Poincaré, Fiedler, and Betti invariants of $B(3, 3, 3)$ and $B(3, 3, 6)$ ($\text{rk}(\partial_2) = |F_2| - |C_3| = \beta_1^{(1)} = 28, 64$; $\lambda_2 = 1.0000, 0.2679$; $\kappa_{\text{edge}} = 3$), addressing [Sch26c, Target 10.1] (Theorem 5.3).

2 Conventions, nomenclature, and non-implications

Convention 2.1 (Status labels)**.** Every numbered statement carries one of four labels: **[A]** finite, algebraic, or axiom-free; **[D]** derived under hypotheses named and registered in Appendix A; **[C]** imported from a cited source; **[E]** a molecular or physical reading, entering no proof. Scholia are unlabelled and enter no proof. Numerical macros are generated from the revised checks in `verify_a94.py` (blocks S1–S6, Appendix B).

Notation 2.2 (Network and homological objects)**.** $B(n_x, n_y, n_z) \subset \mathbb{Z}^3$ an occupied box; $E_x(\Omega), E_y(\Omega)$ the x- and y-lateral adjacencies, $E_{\text{lat}}(\Omega) = E_x(\Omega) \sqcup E_y(\Omega)$, and $E_z(\Omega)$ the vertical stacking adjacencies; $X_7 = \{\mathbf{0}, \pm e_x, \pm e_y, \pm e_z\}$ the 7-node 3D cross and X_{14} the 14-node cleavable double cross of [Sch26c, Rem. 5.2]; $\mathcal{K}(\Omega) = (\Omega, E(\Omega), F_2(\Omega), C_3(\Omega))$ the induced cubical complex with elementary 4-cycle plaquettes $F_2(\Omega)$, unit 3-cubes $C_3(\Omega)$, boundary maps $\partial_k : C_k \to C_{k-1}$, 1-skeleton cycle rank $\beta_1^{(1)}(\Omega) = |E(\Omega)| - |\Omega| + c(\Omega)$, graph Laplacian $L(\Omega) = \partial_1 \partial_1^T = D - A$ with spectrum $0 = \lambda_1 \le \lambda_2(\Omega) \le \cdots \le \lambda_{|\Omega|}$, and edge connectivity $\kappa_{\text{edge}}(\Omega)$.

Proposition 2.3 (Non-implications)**. [A]** *None of the following is asserted:*

(i) No autonomous scission from constraint violation alone. *In accordance with [Sch26b, Rem. 39, eq. (442)], $\hat{P}_{\text{sciss}}$ models the conditional graph restriction after a bond-rupture event rather than a spontaneous projection executed by the metric.*

(ii) No preservation of local site completeness when an incident lateral bridge ruptures. *A 2-cell plaquette bypass $\gamma_p = e - \partial_2 p$ preserves global routability ($\lambda_2 > 0$ and $d^*_C < \infty$) around a ruptured edge e, but the local completeness predicate at $\mathbf{k} \in K(\Omega)$ ([Sch26c, Def. 3.2]) requires all four incident lateral connectors at $\mathbf{k}$ to remain intact.*

(iii) No thermodynamic percolation limit applied to finite boxes. *On finite boxes $B(3, 3, 2h)$, completeness survival is governed exactly by the finite polynomial $P_{\text{herit}}(p_b)$ of Theorem 4.3 and $\kappa_{\text{edge}} = 3$, whereas the*

> *infinite-lattice cubic percolation threshold of [Sch26a, Fig. 3] applies only as $n_x, n_y, n_z \to \infty$.*

3 Directional attachment geometry of a two-armed unit (Target 14.8 of Article 84)

Article 84's Remark 4.11 and Target 14.8 ask whether a disubstituted unit $D = \{6, 12\}$, whose armed locants form one ι-orbit, can carry connectors along two different lateral axes (x and y) [Sch26e]. We resolve this directly from Axioms A_3, A_4, and A_6 of Article 40 [Sch26b].

Lemma 3.1 (Axis-specificity of primary locant pairs)**. [C], [A]** *By [Sch26b, Axiom A_3, eqs. (50)–(51)] and [Sch26b, Axiom A_6, eq. (107)], the ordered primary attachment vertices are*

$$v_x^- = v_3, \quad v_x^+ = v_9, \qquad v_y^- = v_{12}, \quad v_y^+ = v_6, \tag{1}$$

and the exact-closure connector in direction $\alpha \in \{x, y\}$ joins $(\mathbf{k}, v_\alpha^+)$ to $(\mathbf{k} + e_\alpha, v_\alpha^-)$ under the directional compatibility relation Comp_α of [Sch26b, Def. 14, eq. (85)]. For the three admitted families $D = \{6, 12\}$, $I = \{3, 6, 12\}$, and $T = \{3, 6, 9, 12\}$, the armed locant subsets on the two axes are:

$$\begin{aligned} \Lambda_x(D) &= \emptyset, & \Lambda_y(D) &= \{6, 12\}, \\ \Lambda_x(I) &= \{3\}, & \Lambda_y(I) &= \{6, 12\}, \\ \Lambda_x(\iota I) &= \{9\}, & \Lambda_y(\iota I) &= \{6, 12\}, \\ \Lambda_x(T) &= \{3, 9\}, & \Lambda_y(T) &= \{6, 12\}. \end{aligned} \tag{2}$$

Proof. Immediate from (1) and the definitions of D, I, T ([Sch26f, Def. 4.1]) together with $\iota(3) = 9, \iota(6) = 12$ (S1). □

Theorem 3.2 (Directional Compatibility Theorem; Model resolution of Article 84, Target 14.8)**. [A]** *Let $E = E_x \sqcup E_y \subseteq E_{\mathrm{lat}}(\Omega)$ be an installed lateral connector set on Ω, and at each site $\mathbf{k} \in \Omega$ let $d_x^-(\mathbf{k}), d_x^+(\mathbf{k}) \in \{0, 1\}$ and $d_y^-(\mathbf{k}), d_y^+(\mathbf{k}) \in \{0, 1\}$ denote the indicators of installed connectors incident to $\mathbf{k}$ in directions $-e_x, +e_x, -e_y, +e_y$.*

(a) D cannot bridge along two different lateral axes: *Because $\Lambda_x(D) = \emptyset$ and $\iota D = D$, in the fixed-port model a disubstituted unit $D = \{6, 12\}$ can support lateral edges exclusively along the y-axis ($v_6 \leftrightarrow v_{12}$) and never along the x-axis ($v_3 \leftrightarrow v_9$). A site $\mathbf{k}$ admits D if and only if $d_x^-(\mathbf{k}) = d_x^+(\mathbf{k}) = 0$.*

(b) Universal y-compatibility of all three families: *Because $\Lambda_y(D) = \Lambda_y(I) = \Lambda_y(T) = \{6, 12\}$, every admitted family $S \in \{D, I, T\}$ supports arbitrary y-axis bridging $(d_y^-(\mathbf{k}), d_y^+(\mathbf{k})) \in \{0, 1\}^2$. Directional compatibility at $\mathbf{k}$ therefore depends solely on the incident x-bridges:*

- *$T = \{3, 6, 9, 12\}$ is compatible with all $(d_x^-, d_x^+) \in \{0, 1\}^2$;*
- *$I = \{3, 6, 12\}$ is compatible iff $d_x^+ = 0$ (and $\iota I = \{6, 9, 12\}$ iff $d_x^- = 0$, i.e., $d_x^- + d_x^+ \leq 1$ when orientation-flexible);*
- *$D = \{6, 12\}$ is compatible iff $d_x^- = d_x^+ = 0$.*

(c) Reduction of port-compatible counts on a saturated 3×3 layer and on $B(3, 3, 6)$: *On a saturated 3×3 layer (H_{sat}), every site has $d_x^-(\mathbf{k}) + d_x^+(\mathbf{k}) \geq 1$, so D is excluded from all 9 sites (including the 4 corners!). The number of compatible family assignments per layer drops from $3^4 \cdot 2^4 \cdot 1^1 = 1,296$ (under scalar degree compatibility, [Sch26e, Rem. 4.11]) to $2^3 \cdot 1^6 = 8$ for fixed-locant $I = \{3, 6, 12\}$ (or $2^6 \cdot 1^3 = 64$ when ιI is admitted at $k_x = 0$), lowering the port-compatible configuration count on $B(3, 3, 6)$ by a factor of $(1296/8)^6 = 162^6 \approx 1.81 \times 10^{13}$ (or $(1296/64)^6 \approx 6.89525 \times 10^7$; S1).*

Proof. Parts (a) and (b) follow immediately from Lemma 3.1: $v_x^- = v_3$ and $v_x^+ = v_9$ are absent from D, while $\{v_y^-, v_y^+\} = \{12, 6\} \subseteq D \subset I \subset T$. For (c), in a saturated 3×3 layer $k_x \in \{0, 1, 2\}$, sites at $k_x = 0$ have $(d_x^-, d_x^+) = (0, 1)$, sites at $k_x = 1$ have $(1, 1)$, and sites at $k_x = 2$ have $(1, 0)$. Nowhere is $(d_x^-, d_x^+) = (0, 0)$, excluding D everywhere; $k_x = 1$ forces T (1 choice at 3 sites), $k_x = 2$ admits $\{I, T\}$ (2 choices at 3 sites), and $k_x = 0$ admits T (1 choice for fixed I, or 2 choices $\{\iota I, T\}$), giving $2^3 = 8$ and $2^6 = 64$ (S1). □

4 Partially bridged configurations and percolation of completeness (Target 10.3 of Article 80 and Target 14.13 of Article 84)

We now relax H_{sat} so that the installed lateral connector set $E \subseteq E_{\mathrm{lat}}(\Omega)$ is an independent datum, addressing [Sch26e, Target 14.13] and [Sch26c, Target 10.3].

Convention 4.1 (Projected and oriented family counts)**.** The orbit-flexible count records the three family labels D, I, T and asks whether an admissible orientation exists. In particular, an isolated I site is counted once even if

both I and ιI are permitted. It is a projected family count. A distinct orientation-resolved model counts $D, I, \iota I, T$ separately, giving local counts $4, 2, 2, 1$ for the four x-incidence patterns. Counts below are combinatorial generating sums, not thermodynamic partition functions.

Theorem 4.2 (Exact partially bridged configuration count; Model resolution of Article 84, Target 14.13)**. [A]** *Let $C_{\mathrm{part}}(\Omega)$ denote the set of compatible triples (κ, E, η) with decoration choices specified separately, as E ranges over all $2^{|E_{\mathrm{lat}}(\Omega)|}$ subsets of $E_{\mathrm{lat}}(\Omega)$. For each subset $E \subseteq E_{\mathrm{lat}}(\Omega)$, let $\Phi(\Omega, E) := \prod_{\mathbf{k}\in\Omega} N_{\mathrm{fam}}(E_{\mathbf{k}})$ count compatible family assignments $\Omega \to \{D, I, T\}$, and let $\Sigma(\Omega) := \sum_{E\subseteq E_{\mathrm{lat}}(\Omega)} \Phi(\Omega, E)$.*

(a) Exact factorization under directional compatibility: *On any box $B(n_x, n_y, n_z)$, because $N^{\mathrm{dir}}_{\mathrm{fam}}(E_{\mathbf{k}})$ depends only on E_x and not on E_y (Theorem 3.2(b)), the sum over all $2^{|E_{\mathrm{lat}}|}$ lateral bridge graphs factors into $2^{|E_y|}$ times the $n_y n_z$-th power of the single-row x-bridge partition function $Z_{\mathrm{row}}(n_x)$:*

$$\Sigma_{\mathrm{dir}}(B(n_x, n_y, n_z)) = 2^{n_x(n_y-1)n_z}\big(Z_{\mathrm{row}}(n_x)\big)^{n_y n_z}. \quad (3)$$

(b) Closed-form values on a 3×3 layer and on $B(3,3,6)$: *For $n_x = 3$ (a 3-site row with $2^2 = 4$ states of its two x-edges), the row sum is $Z^{\mathrm{fixed}}_{\mathrm{row}}(3) = 27+6+6+2 = 41$ for fixed $I = \{3, 6, 12\}$ and $Z^{\mathrm{orbit}}_{\mathrm{row}}(3) = 27+12+12+4 = 55$ under the projected orbit-flexible convention. Hence on a 3×3 layer ($|E_{\mathrm{lat}}| = 12$, $4,096$ bridge graphs):*

$$\begin{aligned}\Sigma^{\mathrm{fixed}}_{\mathrm{dir}}(B(3,3,1)) &= 2^6\times 41^3 = 4,410,944,\\ \Sigma^{\mathrm{orbit}}_{\mathrm{dir}}(B(3,3,1)) &= 2^6\times 55^3 = 10,648,000,\\ \Sigma_{\mathrm{scalar}}(B(3,3,1)) &= 60,454,593,\end{aligned} \quad (4)$$

and on the 6-layer box $B(3,3,6)$, $\Sigma(B(3,3,6)) = (\Sigma(B(3,3,1)))^6$, for family–bridge pairs (S2). A product with additional arm or registry multiplicities is valid only after an independent constant-size decoration fibre is defined; no such universal factor is asserted here.

Proof. Since $E_{\mathrm{lat}}(\Omega) = E_x(\Omega) \sqcup E_y(\Omega)$ and $N^{\mathrm{dir}}_{\mathrm{fam}}(E_{\mathbf{k}})$ is independent of E_y, summing over $E_y \subseteq E_y(\Omega)$ contributes $2^{|E_y(\Omega)|} = 2^{n_x(n_y-1)n_z}$, while $E_x(\Omega)$ is the disjoint union of $n_y n_z$ rows of length n_x, proving (3). On a row of $n_x = 3$ sites with x-edges $(e_1, e_2) \in \{0,1\}^2$, the compatible family products (N_0, N_1, N_2) are $(3,3,3) \mapsto 27$ at $(0,0)$, $(1,2,3) \mapsto 6$ at $(1,0)$, $(3,1,2) \mapsto 6$ at $(0,1)$, and $(1,1,2) \mapsto 2$ at $(1,1)$ (sum 41), or $27+12+12+4 = 55$ with ιI. Confirmed by exhaustive enumeration over all $4,096$ layer graphs in block S2. For orientation-resolved types the four row terms are $64, 16, 16, 4$, summing to 100, and the layer count is $2^6 100^3 = 64{,}000{,}000$. This extra multiplicity is not counted again as a free orientation bit. □

Theorem 4.3 (Percolation of completeness and raised node minima; Model resolution of Article 80, Target 10.3)**. [D]** *Under Hypothesis H_{perc}, suppose each lateral bridge $e \in E_{\mathrm{lat}}(\Omega)$ is installed independently with bond yield $p_b \in (0,1]$ (H_{sat} being $p_b = 1$). All occupied sites are preassigned a port-compatible family (for example T), all vertical contacts and other prerequisites of the imported completeness predicate are held present, and cleavage preserves the retained edges. This is a Bernoulli edge law conditional on that scaffold; it is not the uniform law on all family–edge pairs counted above.*

(a) Single-site completeness yield: *An interior site $\mathbf{k}$ ($1 \le k_\alpha \le n_\alpha - 2$) is complete ([Sch26c, Def. 3.2]) if and only if all 4 of its incident lateral bridges are present, which occurs with probability*

$$p_{\mathrm{cpl}}(p_b) = p_b^4 \quad (5)$$

(0.8145 at $p_b = 0.95$, 0.6561 at $p_b = 0.90$, 0.4096 at $p_b = 0.80$).

(b) Heritable completeness probability on $B(3,3,2h)$: *Because lateral bridges in distinct z-layers $k_z \neq k'_z$ are disjoint, the events that the central sites $(1,1,k_z)$ ($1 \le k_z \le 2h-2$) are complete are mutually independent. Under the midplane cleavage of [Sch26d, Hyp. 6.2] into two daughter boxes $B(3,3,h)$ ($h \ge 3$), each half contains $h-2$ interior central sites, so the probability that* both *daughter halves contain at least one complete site is exactly*

$$P_{\mathrm{herit}}(B(3,3,2h);\, p_b) = \Big(1 - (1-p_b^4)^{h-2}\Big)^2. \quad (6)$$

(c) Raised completeness-retention minimum within the box family $N_{\min}(p_b, \varepsilon)$: *At $h = 3$ ($B(3,3,6)$, 54 nodes), $P_{\mathrm{herit}} = p_b^8$ falls to 0.6634 at $p_b = 0.95$ and 0.4305 at $p_b = 0.90$. Achieving $P_{\mathrm{herit}}(B(3,3,2h); p_b) \ge 1-\varepsilon$ for $\varepsilon \in (0,1)$ requires, for $0 < p_b < 1$, stack half-height and total node count*

$$\begin{aligned}h_{\min}(p_b,\varepsilon) &= 2 + \left\lceil \frac{\ln(1-\sqrt{1-\varepsilon})}{\ln(1-p_b^4)} \right\rceil,\\ N_{\min}(p_b,\varepsilon) &= 18\, h_{\min}(p_b,\varepsilon).\end{aligned} \quad (7)$$

At $p_b = 1$, define $h_{\min} = 3$ directly; the logarithmic formula is not evaluated at this endpoint. At $1-\varepsilon = 0.95$ (95% completeness-retention reliability), the minimum within this box family rises from 54 nodes ($p_b = 1$)

to 90 *nodes* ($h_{\min} = 5$, $B(3,3,10)$) *at* $p_b = 0.95$, 108 *nodes* ($h_{\min} = 6$, $B(3,3,12)$) *at* $p_b = 0.90$, *and* 162 *nodes* ($h_{\min} = 9$, $B(3,3,18)$) *at* $p_b = 0.80$ *(S3).*

Proof. In $B(3,3,2h)$, a site $\mathbf{k}$ has both x- and y-neighbors in Ω iff $(k_x, k_y) = (1,1)$, and has both z-neighbors in its cleaved daughter half $B(3,3,h)$ iff $1 \le k_z \le h-2$ (lower half) or $h+1 \le k_z \le 2h-2$ (upper half), giving $h-2$ candidate complete sites in each daughter half. Because the 4 lateral edges incident to $(1,1,k_z)$ lie entirely in plane k_z, the $2(h-2)$ candidate sites across the two halves depend on disjoint sets of 4 lateral edges. Thus each half fails to have a complete site with probability $(1-p_b^4)^{h-2}$, giving (6); inverting $(1-(1-p_b^4)^{h-2})^2 \ge 1-\varepsilon$ yields (7) (S3). □

Corollary 4.4 (Percolation-coupled critical genome length $L_{\mathrm{crit}}(p_b,\varepsilon)$)**. [D]** *Under the Minkowski-sum circumradius containment criterion and vessel-radius law* $R_6(L) = c\sqrt{L}$ *(*$c = 0.89923$ *nm·bp*$^{-1/2}$, $d_{\mathrm{bl}} = 4.0$ *nm) of [Sch26c, Props. 5.1–5.2, Thm. 5.3], a box* $B(3,3,2h)$ *with* $h = h_{\min}(p_b,\varepsilon)$ *and planar spans* $a_x = a_y = 7.90$ *nm and stacking span* $h_{\mathrm{span}}(2h) = 1.38 + 0.34(2h-3)$ *nm fits inside the lumen if and only if*

$$\begin{aligned} H &= 1.38 + 0.34(2h_{\min}(p_b,\varepsilon) - 3), \\ L &\ge L_{\mathrm{crit}}(p_b,\varepsilon) \\ &:= \left\lceil c^{-2}\left(\tfrac{1}{2}\sqrt{2a_x^2 + H^2} + 2d_{\mathrm{bl}}\right)^2 \right\rceil. \end{aligned} \tag{8}$$

At $1-\varepsilon = 0.95$, *the critical genome length rises from* $L_{\mathrm{crit}} = 233$ *bp at* $p_b = 1.00$ *(*$B(3,3,6)$*,* 54 *nodes; numerical evaluation here of the imported law) to* 239 *bp at* $p_b = 0.95$ *(*$B(3,3,10)$*,* 90 *nodes),* 243 *bp at* $p_b = 0.90$ *(*$B(3,3,12)$*,* 108 *nodes), and* 259 *bp at* $p_b = 0.80$ *(*$B(3,3,18)$*,* 162 *nodes; S3).*

Proof. Substitute $2h = 2h_{\min}(p_b,\varepsilon)$ from (7) into the circumradius formula $\frac{1}{2}\sqrt{a_x^2 + a_y^2 + h_{\mathrm{span}}^2} + d_{\mathrm{bl}} \le R_{\mathrm{lum}}(L) = c\sqrt{L} - d_{\mathrm{bl}}$ of [Sch26c, Prop. 5.1, Thm. 5.3] and take the ceiling (S3). □

5 Homological scission, Fiedler routability, and exclusion of the degenerate cross (Target 10.1 of Article 80)

We now unite Article 15's Theorem 2 (Homological Triviality of Bond Rupture via $\partial_2 b$ in H_1) and §7.3 (Fiedler algebraic connectivity $\lambda_2 > 0$) [Sch26a] with Article 40's Axiom A_{12} and Remark 37 [Sch26b] to discharge [Sch26c, Target 10.1].

Theorem 5.1 (Homological Bypass and Scission Resilience Theorem)**. [A]** *Let* $\Omega \subset \mathbb{Z}^3$ *be a connected finite network with cubical complex* $\mathcal{K}(\Omega) = (\Omega, E(\Omega), F_2(\Omega), C_3(\Omega))$ *and unit-weight shortest-path metric* $d^*_{C,\Omega}$ *in which every retained edge and every concatenation of retained edges is admissible ([Sch26b, Axiom* A_{12}*, eq. (423)]). For any edge* $e = (u,v) \in E(\Omega)$*, let* $\hat{P}^{(e)}_{\mathrm{sciss}}(\Omega) := \Omega \setminus \{e\}$ *be the network after scission of e ([Sch26a, Def. 4]). The deletion is a graph operation on the fixed vertex set. If a post-scission cubical subcomplex is desired, every higher-dimensional cell incident to the deleted edge must also be removed. Then the following are equivalent:*

(i) Homological 2-plaquette condition ([Sch26a, Thm. 2]): *e belongs to the boundary* $\partial_2 p$ *of at least one elementary 4-cycle plaquette* $p \in F_2(\Omega)$*.*

(ii) Minimal admissible bypass condition ([Sch26b, Rem. 37]): *In* $\Omega \setminus \{e\}$*, the endpoints* u, v *are joined by a simple 3-edge bypass path* $\gamma_p = e - \partial_2 p$ *with the same boundary as the oriented edge e in* $\mathcal{K}(\Omega)$ *(*$\gamma_p - e$ *is a 1-cycle bounding an oriented 2-chain), bounding the path-metric distortion everywhere on* Ω *by*

$$d^*_{C,\Omega\setminus\{e\}}(x,y) \;\le\; 3\, d^*_{C,\Omega}(x,y) < +\infty \qquad (\forall\, x,y \in \Omega). \tag{9}$$

Whenever (i)–(ii) hold, excising e preserves global Fiedler routability, $\lambda_2(L(\Omega\setminus\{e\})) > 0$*, and reduces the 1-skeleton cycle rank by* $\beta_1^{(1)}(\Omega\setminus\{e\}) = \beta_1^{(1)}(\Omega) - 1$ *(S4, S5).*

Proof. In the bipartite graph $\mathbb{Z}^3$, every cycle has even length ≥ 4, and a cycle of length 4 is precisely the boundary $\partial_2 p$ of an elementary unit square $p \in F_2(\Omega)$. Thus u and v have a path of length 3 in $\Omega\setminus\{e\}$ iff e and that simple 3-path form a 4-cycle $\partial_2 p$ in Ω, proving (i) $\Longleftrightarrow$ (ii). Replacing any occurrence of e in a geodesic of Ω by the 3-path $\gamma_p = e - \partial_2 p$ multiplies its length by at most 3, giving (9). Since $d^*_{C,\Omega\setminus\{e\}} < \infty$, $\Omega\setminus\{e\}$ has $c = 1$ connected component, whence $\lambda_2(L(\Omega\setminus\{e\})) > 0$ [Fie73] and $\beta_1^{(1)}(\Omega\setminus\{e\}) = (|E|-1) - |\Omega| + 1 = \beta_1^{(1)}(\Omega) - 1$. □

Definition 5.2 (Plaquette-complete site)**. [A]** A site $\mathbf{k} \in \Omega$ is *plaquette-complete* (or *scission-resiliently complete*) if (i) $\mathbf{k} \in K(\Omega)$ is complete in the sense of [Sch26c, Def. 3.2], and (ii) for every pair of orthogonal axes $\alpha \ne \beta$ in $\{x,y,z\}$ and signs $s_\alpha, s_\beta \in \{-1,+1\}$, the two incident edges $(\mathbf{k}, \mathbf{k} + s_\alpha e_\alpha)$ and $(\mathbf{k}, \mathbf{k} + s_\beta e_\beta)$ bound an elementary 2-plaquette in $F_2(\Omega)$ (equivalently, $\mathbf{k} + s_\alpha e_\alpha + s_\beta e_\beta \in \Omega$).

Theorem 5.3 (Direct Exclusion of the Star Graphs and Spectral/Homological Invariants of the Complete Boxes; Model resolution of Article 80, Target 10.1)**. [A]** *(a)* Exclusion of the 7-node and 14-node crosses: *The unrestricted 7-node cross X_7 and 14-node double cross X_{14} of [Sch26c, Rem. 5.2] are trees:*

$$\beta_1^{(1)}(X_7) = \beta_1^{(1)}(X_{14}) = 0, \quad |F_2(X_7)| = |F_2(X_{14})| = 0, \\ \kappa_{\text{edge}}(X_7) = \kappa_{\text{edge}}(X_{14}) = 1. \tag{10}$$

For every *edge e of X_7 (6 edges) or X_{14} (13 edges), single-bond scission $\hat{P}^{(e)}_{\text{sciss}}$ disconnects the assembly: $\lambda_2(L(X_7 \setminus \{e\})) = \lambda_2(L(X_{14} \setminus \{e\})) = 0$ and $\operatorname{diam}_{d_C^*}(X_7 \setminus \{e\}) = +\infty$. Neither X_7 nor X_{14} contains a plaquette-complete site (S4, S5).*

(b) Exact plaquette minimum and conditional box closure: *A plaquette-complete site forces precisely its center, six axial neighbors and twelve planar diagonal neighbors. Thus the smallest induced configuration with such a site has* 19 *vertices,*

$$Q_{19} = \{v \in \{-1, 0, 1\}^3 : |\operatorname{supp} v| \le 2\}.$$

It has 30 *edges and* 12 *elementary plaquettes, and is not a* 27*-vertex box. If the independent corner-closure hypothesis H_{corner} additionally requires all eight triple-diagonal neighbors, the minimal occupied set is uniquely a translate of $B(3, 3, 3)$. Under the same closure in each of two disjoint daughter halves and the prescribed planar cleavage, the* 54*-vertex $B(3, 3, 6)$ is a realizing minimum for this strengthened predicate. These are conditional box minima, not consequences of plaquette completeness alone.*

On any box $B(n_x, n_y, n_z)$, the contractible cubical complex $\mathcal{K}(\Omega)$ satisfies the Euler–Poincaré and 2-boundary surjectivity identities

$$\operatorname{rk}(\partial_2) = |F_2(\Omega)| - |C_3(\Omega)| = \beta_1^{(1)}(\Omega), \\ H_1(\mathcal{K}(\Omega); \mathbb{Z}) = \ker \partial_1 / \operatorname{im} \partial_2 = 0, \\ \lambda_2(L(B(n_x, n_y, n_z))) = 2 - 2\cos\left(\frac{\pi}{\max(n_x, n_y, n_z)}\right), \tag{11}$$

for $\max(n_x, n_y, n_z) \ge 2$, giving the following invariants:

- *On $B(3, 3, 3)$ (27 nodes, 54 edges): $|F_2| - |C_3| = 36 - 8 = 28 = \beta_1^{(1)}$, $\kappa_{\text{edge}} = 3$, and $\lambda_2 = 1.0000$. All 54 single-edge ruptures and all $\binom{54}{2} = 1,431$ double-edge ruptures preserve $\lambda_2 > 0$ and $d_C^* < \infty$.*
- *On $B(3, 3, 6)$ (54 nodes, 117 edges): $|F_2| - |C_3| = 84 - 20 = 64 = \beta_1^{(1)}$, $\kappa_{\text{edge}} = 3$, and $\lambda_2 = 2 - \sqrt{3} \approx 0.2679$. All 117 single-edge ruptures and all $\binom{117}{2} = 6,786$ double-edge ruptures preserve $\lambda_2 > 0$ and $d_C^* < \infty$ (S4–S6).*

Proof. (a) X_7 (7 vertices, 6 edges) and X_{14} (14 vertices, 13 edges) are trees ($\beta_1^{(1)} = 0, |F_2| = 0$). Removing any edge of a tree produces 2 components ($\lambda_2 = 0, d_C^* = +\infty$). (b) A plaquette-complete site $\mathbf{k}$ contains $\mathbf{k}$ (1 node), its 6 axial neighbors, its 12 planar diagonal neighbors, giving 19 distinct vertices. The induced set Q_{19} realizes all required plaquettes, proving sharpness. Under H_{corner}, adjoining the 8 corner completions forms $\mathbf{k} + \{-1, 0, 1\}^3 \cong B(3, 3, 3)$. Because the solid cubical complex $\mathcal{K}(\Omega)$ is contractible, $H_k(\mathcal{K}(\Omega); \mathbb{Z}) = 0$ for $k \ge 1$ and $\chi = |V| - |E| + |F_2| - |C_3| = 1$, whence $\operatorname{rk}(\partial_2) = |F_2| - |C_3| = |E| - |V| + 1 = \beta_1^{(1)}$ and $\operatorname{im} \partial_2 = \ker \partial_1$. The Cartesian product spectrum of $P_{n_x} \square P_{n_y} \square P_{n_z}$ gives (11) [Fie73]. For a full box with each side length at least two, every edge belongs to at least two elementary squares whose alternative three-edge paths are edge-disjoint. If at most two edges are removed, each removed edge therefore retains a three-edge bypass avoiding the other deleted edge. Substitute such bypasses along an original shortest path. The resulting surviving walk proves connectedness and the factor-three distance bound. For the displayed three-dimensional boxes, a three-edge corner cut disconnects a vertex; hence $\kappa_{\text{edge}} = 3$. The exhaustive sweeps independently check these finite instances (S4–S6). □

6 Precedence rulings and targets

Ruling 6.1 (Articles 15, 40, 80, and 84)**. [A], [D]** Consistent; the cited targets are addressed within the fixed-port, independent-edge and corner-closure models:

(i) *Article 84, Target 14.8 (Attachment geometry of D):* Discharged by Lemma 3.1 and Theorem 3.2.

(ii) *Article 84, Target 14.13 (Partial bridging count):* Discharged by Theorem 4.2.

(iii) *Article 80, Target 10.3 (Saturated vs. partial bridging minima):* Discharged by Theorem 4.3 and Corollary 4.4.

(iv) *Article 80, Target 10.1 (Excluding the degenerate cross directly):* Discharged by Theorem 5.1, Definition 5.2, and Theorem 5.3, which connect Article 15's 2-cell boundary condition $\partial_2 p$ and Fiedler connectivity $\lambda_2 > 0$ directly to Article 40's Remark 37.

Target 6.2 (Bridge identity, geometry and edge occupancy)**.** Authenticate the proposed phosphorus-containing bridge using appropriate standards, high-resolution mass spectrometry where soluble, and multinuclear NMR including ^{1}H–^{31}P correlations. Distinguish phosphodiester, monoester

Table 1: Synthesis of directional attachment compatibility, partially bridged percolation, and homological fault tolerance across X_7, X_{14}, $B(3,3,3)$, and $B(3,3,6)$.

Structural Problem	Open Target / Baseline	Resolved Mathematical Invariant (Article 94)	Physical / Corpus Consequence
1. Two-armed D bridging ([Sch26e, Target 14.8])	Scalar degree rule $\lvert S_{\mathbf{k}}\rvert \geq d(\mathbf{k})$ gave 1,296 family maps (339,738,624 states) on $B(3,3,1)$	$\Lambda_x(D) = \emptyset$: $D = \{6,12\}$ bridges along y only ($d_x^{\pm} = 0$); excluded from all 9 sites of a saturated 3×3 layer	**Addresses [Sch26e, Target 14.8]**: layer family maps drop from 1,296 to 8 (or 64 with ιI) (Thm. 3.2)
2. Partially bridged space ([Sch26e, Target 14.13])	H_{sat} fixed $E = E_{\text{lat}}(\Omega)$; partial bridging left open	$\Sigma_{\text{dir}}(B(n_x,n_y,n_z)) = 2^{\lvert E_y\rvert} Z_{\text{row}}(n_x)^{n_y n_z}$; on $B(3,3,1)$, $2^6 \times 41^3 = 4,410,944$ (or 10,648,000)	**Addresses [Sch26e, Target 14.13]** in closed form across all 4,096 layer bridge graphs (Thm. 4.2)
3. Completeness percolation ([Sch26c, Target 10.3])	Saturated minima 27 and 54 nodes ($L_{\text{crit}} = 233$ bp) at $p_b = 1$	$p_{\text{cpl}} = p_b^4$; $P_{\text{herit}}(B(3,3,2h); p_b) = (1-(1-p_b^4)^{h-2})^2$; $h_{\min}(p_b,\varepsilon)$ and $L_{\text{crit}}(p_b,\varepsilon)$	**Addresses [Sch26c, Target 10.3]**: 95% minimum is 90 (239 bp at $p_b = 0.95$), 108 (243 bp at $p_b = 0.90$) (Thm. 4.3, Cor. 4.4)
4. Exclusion of star graphs ([Sch26c, Target 10.1])	Unrestricted local completeness admitted X_7 (7 nodes) and X_{14} (14 nodes)	X_7, X_{14} are trees ($\beta_1^{(1)} = 0, \lvert F_2\rvert = 0, \kappa_{\text{edge}} = 1$); any 1-bond scission gives $\lambda_2 = 0, d_C^* = \infty$	**Addresses [Sch26c, Target 10.1]**: plaquette completeness ($\partial_2 p \ni e$) excludes X_7, X_{14}; corner closure additionally gives boxes (Thms. 5.1, 5.3)
5. Fiedler & Betti invariants ([Sch26a, §2.3, §7.3])	Homological scission $\hat{P}_{\text{sciss}}$ and $\lambda_2 > 0$ stated on abstract $\mathbb{Z}^3$	$\text{rk}(\partial_2) = \lvert F_2\rvert - \lvert C_3\rvert = \beta_1^{(1)}$ ($36-8=28$ on $B(3,3,3)$; $84-20=64$ on $B(3,3,6)$)	100% of 1-edge and 2-edge ruptures (6,786 pairs on $B(3,3,6)$) preserve $\lambda_2 > 0$ and $d_C^* \leq 3d_{C,0}^*$ (Thms. 5.1, 5.3)

and phosphoanhydride species and validate quantitative calibration. For insoluble material, use appropriate solid-state methods. Aggregate conversion alone does not identify a particular graph edge or prove independent occupation. Structural scattering, diffraction or microscopy should test the registered port geometry, stacking and finite-network morphology.

Target 6.3 (Kinetics and functional survival)**.** Measure installation, cleavage, dissociation and activated religation under stated pH, temperature and ionic conditions, and test correlations among neighboring edge events. Compare topology-preserving deletion with separate mechanical and transport assays. Redundant adjacency supplies a possible route, but persistence, registration and useful function require additional evidence.

7 Conclusion

The fixed-port attachment rule makes directional compatibility depend on the installed x-edges and yields an exact row-factorization of the family–bridge sum. Projected family counts and resolved molecular orientations are distinct accounting conventions. Conditional on a compatible scaffold, intact vertical contacts and independent lateral-edge installation, the completeness-retention law on $B(3,3,2h)$ is $[1-(1-p_b^4)^{h-2}]^2$. The resulting node minima and containment lengths apply to this prescribed family and its imported geometric parameters.

Plaquette completeness excludes the tree crosses directly and has a sharp 19-vertex local minimum. The additional corner-closure hypothesis recovers the 27-vertex box and a 54-vertex two-daughter realization. Elementary plaquettes give three-edge bypasses, while full three-dimensional boxes retain graph connectedness and a factor-three metric bound after any two edge deletions. These graph conclusions do not require a measured molecular repair mechanism; realizing them chemically calls for the linker, stacking and kinetic tests described below.

A Hypothesis register

Hypothesis A.1 (Corner closure H_{corner})**.** Every complete site considered for the strengthened box-minimum assertion also has all eight triple-diagonal neighbors occupied. This is independent of plaquette completeness and is used only for the 27- and 54-vertex conditional minima.

B Verification blocks

The accompanying verification dossier records 29,784 checks per normal and optimized run, with byte-identical generated reports: 29,776 exact predicates and eight numerical spectral comparisons, including ten negative controls. Boundary ranks are computed over $\mathbb{F}_2$; integral homology of the boxes follows from the manuscript's contractibility argument, not from those finite-field ranks alone. The Lean companion compiles 42 axiom-free finite certificates and states its limited scope explicitly. Neither

Table 2: Hypotheses of this paper and imported premises, where each is used, and what survives its failure.

Tag	Statement	Used in	What survives its failure
H_{cov} ([Sch26c, Hyp. 3.5], imported)	exact-closure connectors require covalent linkers on $\{3,9\}$ (x) and $\{6,12\}$ (y)	Lem. 3.1, Thm. 3.2, Thm. 4.2	Scalar degree counts ($1,296$ and $60,454,593$) and all graph/Laplacian theorems of Section 5
H_{perc} (Thm. 4.3)	a preassigned compatible scaffold has fixed vertical edges; lateral edges are independent Bernoulli variables with $p_b \in (0,1]$ at a specified observation time	Thm. 4.3, Cor. 4.4	If the compatible scaffold and vertical contacts are retained, the deterministic case $p_b = 1$ (H_{sat}) gives $h_{\min} = 3$, $N_{\min} = 54$ nodes, and $L_{\mathrm{crit}} = 233$ bp from the printed constants here; Theorem 4.2 is a non-probabilistic exact sum
[Sch26d, Hyp. 6.2] (imported)	midplane cleavage perpendicular to the stacking axis separates $B(3,3,2h)$ into two $B(3,3,h)$ halves	Thm. 4.3(b)–(c), Cor. 4.4, Thm. 5.3(b)	Single-box completeness percolation $1-(1-p_b^4)^{n_z-2}$ and all invariants of $B(3,3,3)$

execution certifies the imported molecular realization or every earlier theorem-number attribution.

Every quantitative macro in the body is emitted by `verify_a94.py` (Python with NumPy for numerical spectra; exact integer and rational arithmetic for all configuration sums, Betti numbers, plaquette and cube counts, and edge-cut sweeps, rational threshold certificates for containment ceilings, and numerical spectral checks against analytic formulas):

S1 Directional attachment compatibility on a saturated 3×3 layer.
Verifies $\Lambda_x(D) = \emptyset$, reproduces Article 84's scalar count $3^4\cdot2^4\cdot1 = 1,296$ ($1,296\times4^9 = 339,738,624$), and verifies the directional counts 8 (fixed $I = \{3,6,12\}$) and 64 (with $\iota I = \{6,9,12\}$; Theorem 3.2). Exact.

S2 Partially bridged partition function over all $4,096$ layer graphs.
Enumerates all $2^{12} = 4,096$ subsets $E \subseteq E_{\mathrm{lat}}(B(3,3,1))$ and verifies that the brute-force sum matches the factored closed form $2^6\times41^3 = 4,410,944$ (fixed I) and $2^6\times55^3 = 10,648,000$ (orbit-flexible I), alongside the scalar sum $60,454,593$ (Theorem 4.2). Exact.

S3 Completeness percolation, raised node minima, and $L_{\mathrm{crit}}(p_b,\varepsilon)$.
Evaluates p_b^4 $(0.8145, 0.6561, 0.4096)$, $P_{\mathrm{herit}}(B(3,3,6);p_b)$ $(0.6634, 0.4305)$, solves (7) at $1-\varepsilon = 0.95$ to confirm $h_{\min} = 5,6,9$ and $N_{\min} = 90,108,162$, and evaluates (8) to confirm $L_{\mathrm{crit}} = 233,239,243,259$ bp at $p_b = 1.00, 0.95, 0.90, 0.80$ (Theorem 4.3, Corollary 4.4).

S4 Cubical complexes, Euler–Poincaré identity, and Fiedler spectra.
Constructs $X_7, X_{14}, B(3,3,3)$, and $B(3,3,6)$, verifying $(|V|,|E|,\beta_1^{(1)},|F_2|,|C_3|) = (7,6,0,0,0)$ on X_7, $(14,13,0,0,0)$ on X_{14}, $(27,54,28,36,8)$ with $|F_2|-|C_3| = 28$ and $\lambda_2 = 1.0000$ on $B(3,3,3)$, and $(54,117,64,84,20)$ with $|F_2|-|C_3| = 64$ and $\lambda_2 = 2-\sqrt{3} \approx 0.2679$ on $B(3,3,6)$, plus translation overlaps (2 vs. 18) and midplane cuts (1 vs. 9; Theorem 5.3). Cell counts and finite-field boundary ranks are exact; numerical eigenvalues supplement the analytic spectrum proof.

S5 Single-edge scission sweep on $X_7, X_{14}, B(3,3,3)$, and $B(3,3,6)$.
Applies $\hat{P}^{(e)}_{\mathrm{sciss}}$ to every edge of X_7 (6 edges) and X_{14} (13 edges), confirming 100% disconnection ($\lambda_2 = 0$), and to every edge of $B(3,3,3)$ (54 edges) and $B(3,3,6)$ (117 edges), confirming that every edge bounds ≥ 2 elementary plaquettes and has $d^*_{C,\Omega\setminus\{e\}}(u,v) = 3$ (Theorem 5.1). Exact.

S6 Exhaustive double-edge scission sweep ($\kappa_{\mathrm{edge}} = 3$).
Tests all $\binom{54}{2} = 1,431$ edge pairs of $B(3,3,3)$ and all $\binom{117}{2} = 6,786$ edge pairs of $B(3,3,6)$ by breadth-first reachability, confirming that 100% of 2-edge ruptures leave Ω connected ($\lambda_2 > 0$) whereas a 3-edge corner cut disconnects the corner vertex ($\kappa_{\mathrm{edge}} = 3$; Theorem 5.3(b)). Exact.

C Molecular commentary, isolated

Remark C.1 (Status)**. [A]** Nothing in this appendix enters a numbered statement or proof of the main text.

Remark C.2 (Conditional chemical interpretation of redundant connectivity)**. [E]** A proposed one-phosphorus lateral bridge has the connectivity R–O–P(=O)(O$^-$)–O–R', where R, R' denote the arm-bearing units. A molecular implementation must specify a phosphorus-balanced synthesis, protonation state, activation chemistry and reaction conditions. Condensing two phosphate-bearing arms into a one-phosphorus product plus only water is not balanced; no such reaction is assumed here. An activated phosphate

monoester and an alcohol-bearing partner are a different possible substrate design, to be established experimentally. A two-phosphorus phosphoanhydride would instead be a different bridge species.
Covalent lateral links and retained noncovalent stacking contacts are treated as graph adjacencies. A bypass preserves graph connectedness under this model, not automatically rigidity, mechanical strength, charge transport or long-lived spatial registration. A tree-edge deletion disconnects the retained graph but does not exclude later rebinding or new contacts.
Phosphodiester hydrolysis rates are substrate- and condition-dependent. Simple alkyl phosphate diester monoanions can be highly resistant to uncatalyzed phosphorus attack [SLW+06]; this precedent supplies no rate for the proposed chrysene linker. A twelve-hour cycle therefore does not itself imply appreciable cleavage. Catalyst-dependent scission and activated repair must be measured. Proximity does not provide activation or free energy for religation in water; reagent-dependent phosphorylation precedents on other substrates [G+18] do not validate this bridge.
The fixed-frame port model excludes alternative linker conformations and orientations by hypothesis. Molecular locants alone do not forbid flexible arms reaching other directions. The counting sums are not equilibrium weights, and the 95% threshold denotes model reliability of a specified completeness-retention event, not a statistical confidence interval or biological heredity. Likewise, the imported vessel law and dimensions define a conditional containment length, not a functional-genome or viability threshold.

D What was set aside in preparing this paper

Remark D.1 (Scalar degree compatibility on the corners of a 3×3 layer)**. [A]** Article 84's Remark 4.11 noted that in a saturated 3×3 layer the four corners have lateral degree 2 and counted them as admitting $D = \{6, 12\}$, while registering Target 14.8 to check whether D's two locants can bridge along orthogonal axes [Sch26e]. Because a corner of a 3×3 layer has one x-bridge and one y-bridge, Theorem 3.2(a) shows that D cannot occupy a saturated corner: D can have degree 2 only when both bridges lie along the y-axis (at $(k_x, 1)$ when both x-bridges are absent under partial bridging).

Remark D.2 (Independence of candidate complete sites in $B(3, 3, 2h)$)**. [A]** In computing the percolation probability P_{herit} on a general box $B(n_x, n_y, 2h)$ with $n_x \geq 4$ or $n_y \geq 4$, adjacent candidate complete sites within the same horizontal plane k_z share an interior lateral bridge, making their completeness indicators positively correlated (by Harris's FKG inequality). On the minimal cross-section $n_x = n_y = 3$ ($B(3, 3, 2h)$), there is only *one* interior column $(1, 1, k_z)$, and its candidate sites lie in distinct horizontal planes $k_z \neq k_z'$ whose lateral edge sets are completely disjoint. Thus the product formula (6) on $B(3, 3, 2h)$ is exact with no correlation correction required.

Remark D.3 (Reconciling $|F_2|$ and $\beta_1^{(1)}$ via the unit 3-cubes $|C_3|$)**. [A]** On $B(3, 3, 3)$ and $B(3, 3, 6)$, the elementary 4-cycle plaquettes number $|F_2| = 36$ and 84, exceeding the 1-skeleton cycle ranks $\beta_1^{(1)} = 28$ and 64 by 8 and 20, respectively. Those differences are the elementary unit 3-cubes $|C_3| = (n_x - 1)(n_y - 1)(n_z - 1)$: the six faces of each unit 3-cube satisfy one linear dependency $\partial_2(\partial_3 c) = 0$, so $\operatorname{rk}(\partial_2) = |F_2| - |C_3| = \beta_1^{(1)}$ identically (Theorem 5.3(b)).

Remark D.4 (Containment thresholds under the stated size law)**. [D]** At $p_b = 0.90$, the model doubles the required node count from 54 to 108. Applying the printed circumradius formula and its stated constants gives $L_{\text{crit}} = 233$ and 243 bp respectively, a difference of 10 bp. The length is a conditional size parameter in the imported vessel law, not a demonstrated functional-genome requirement. All four thresholds are recalculated from that formula rather than inherited numerical labels.

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