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K-Theory of the Sequence Algebra, Spectral Dimension Three, and Path-Calibrated Spectral Metrics

The Pimsner–Voiculescu Sequence on $\mathfrak{Q}_{\mathfrak{C}} \rtimes_{\alpha} \mathbb{Z}^3$, Unconditional Morita Non-Equivalence, and Exact Metric Compatibility

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Abstract

We study three operator-algebraic constructions associated with the sequence framework: crossed-product K-theory, a confining spectral operator, and a calibrated finite-graph metric. For the quasi-local UHF algebra of type d^{∞}, the product trace proves that lattice shifts act trivially on $K_0 \cong \mathbb{Z}[1/d]$. The first crossed product has $K_0 \cong K_1 \cong \mathbb{Z}[1/d]$. For all three crossed-product stages, the inclusion-induced localization theorem for noncommutative Bernoulli shifts yields the exterior-power groups, trace decomposition and unconditional Morita obstructions. This replaces an insufficient elementary Pimsner–Voiculescu induction with an explicit analytic theorem dependency. Independently, a finite-fibre graded confining operator satisfies $N_D(\Lambda) = 32\pi d\Lambda^3/(3\kappa_x\kappa_y\kappa_z) + O(\Lambda^2)$ and has spectral dimension three. For every finite weighted graph, an edge representation augmented by isolated-vertex blocks is faithful and its spectral distance equals the extended shortest-path distance. The fifteen-state source example has 21 finite and 204 infinite ordered-pair distances. A graded product retains both this finite-graph metric and the confining counting exponent. Cut-set degeneration, rather than deletion of an arbitrary single edge, gives a sufficient condition for distance divergence. The mathematical constructions are separated from molecular realization, and companion verification and Lean dossiers certify finite consequences within their stated scope.

Keywords: Chrysene Formalism; operator K-theory; Pimsner–Voiculescu exact sequence; UHF algebra; Morita equivalence; Weyl asymptotics; spectral dimension; Connes spectral distance; path-metric calibration.

1 Introduction

The sequence algebra, confining spectral operator, and admissible path metric constitute distinct mathematical objects in the Chrysene Formalism [Sch26a, Sch26d, Sch26b, Sch26c]. This paper develops three corresponding calculations while keeping their hypotheses separate. The quasi-local algebra has an unconditional UHF K-theory calculation and a complete first crossed-product calculation. Higher-dimensional exterior-power formulas follow from an inclusion-induced localization theorem for the specified Bernoulli shift. A finite-fibre confining operator has spectral dimension three. A faithful finite-graph representation recovers the extended weighted path metric, including isolated states.

The resulting operator-theoretic statements provide a basis for further realization work. They do not identify a molecular assembly with an operator algebra, infer physical transport from a counting exponent, or identify an admissibility threshold with a chemical reaction. The finite constructions and analytic proofs below make those distinctions precise.

1.1 Contributions and status

We prove the trace-invariant action on the original UHF dimension group, explain why trace invariance alone does not justify iteration of the Pimsner–Voiculescu (PV) sequence, and obtain unconditional Morita obstructions with their analytic theorem dependency made explicit. We derive the Weyl coefficient and critical summability exponent for an explicit graded confining operator. We construct a faithful edge-and-isolated-vertex representation and prove exact metric calibration on every finite weighted graph. Finally, we combine this finite representation with a confining lattice factor without claiming a faithful representation of the full sequence crossed product.

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Convention 1.1 (Status and verification)**.** **[A]** denotes an explicit mathematical result; **[D]** a result under the named extra hypotheses; **[C]** an imported theorem; and **[E]** a physical interpretation. Verification code checks finite examples and arithmetic consequences. Neither finite enumeration nor Lean certificates establish the imported Bernoulli-localization theorem, infinite summability statements, or molecular realization.

2 The quasi-local algebra and crossed-product classification

Let $d \geq 2$, $R = \mathbb{Z}[1/d]$, and

$$\mathfrak{Q}_{\mathfrak{C}} = \varinjlim_{\Lambda \Subset \mathbb{Z}^3} M_{d^{|\Lambda|}}(\mathbb{C}).$$

The embeddings are $a \mapsto a \otimes I$, and α is the coordinate shift action. Put $\mathfrak{A}_r = \mathfrak{Q}_{\mathfrak{C}} \rtimes \mathbb{Z}^r$, for $0 \leq r \leq 3$.

Lemma 2.1 (UHF dimension group and shift invariance)**.** **[A]** $K_0(\mathfrak{Q}_{\mathfrak{C}}) = R$, $K_1(\mathfrak{Q}_{\mathfrak{C}}) = 0$. *Every shift acts as the identity on* $K_0(\mathfrak{Q}_{\mathfrak{C}})$.

Proof. At a finite matrix stage the groups are $\mathbb{Z}$ and 0. Normalizing rank by $d^{|\Lambda|}$ converts the connecting maps into inclusions in R, and continuity gives the asserted groups [Bla98]. The product trace induces an injective map $\tau_* : K_0(\mathfrak{Q}_{\mathfrak{C}}) \to \mathbb{R}$ with image R. Shift invariance gives $\tau_* \alpha_{m*} = \tau_*$, so injectivity implies $\alpha_{m*} = \mathrm{id}$. □

Proposition 2.2 (First crossed-product stage)**.** **[A]** $K_0(\mathfrak{A}_1) \cong R$ *and* $K_1(\mathfrak{A}_1) \cong R$ *as abelian groups.*

Proof. Apply the PV six-term exact sequence [PV80]. Both difference maps on the coefficient K-groups vanish, while $K_1(\mathfrak{Q}_{\mathfrak{C}}) = 0$. The inclusion identifies $K_0(\mathfrak{A}_1)$ with $K_0(\mathfrak{Q}_{\mathfrak{C}})$ and the boundary map identifies $K_1(\mathfrak{A}_1)$ with $K_0(\mathfrak{Q}_{\mathfrak{C}})$. No extension splitting is needed at this stage. □

Remark 2.3 (The limitation of an elementary PV induction)**.** Trace invariance on the coefficient algebra does not by itself establish the induced action on later crossed products. An automorphism can fix the subobject and quotient of an extension but act by a shear: $(x, y) \mapsto (x + y, y)$ on $R \oplus R$ fixes $R \oplus 0$ and the quotient. Naturality alone therefore does not produce the claimed identity or a cup-product splitting. We replace that argument with the inclusion theorem for noncommutative Bernoulli shifts.

Theorem 2.4 (Bernoulli-shift localization, imported)**.** **[C]** *The matrix-factor case of Chakraborty–Echterhoff–Kranz–Nishikawa, Theorem 3.5 [CEKN24], gives an inclusion-induced isomorphism*

$$K_*(C_r^* G)[1/d] \;\cong\; K_*\Big(\Big(\bigotimes_Z M_d\Big) \rtimes_r G\Big)$$

for the countably infinite G-set Z and groups satisfying the theorem's Baum–Connes-with-coefficients hypothesis. Here $G = \mathbb{Z}^r$ is amenable and satisfies that hypothesis, $Z = \mathbb{Z}^3$ with coordinate translations, and the tensor product is the quasi-local unital tensor product. Full and reduced crossed products coincide for these amenable groups.

Theorem 2.5 (Exact crossed-product K-groups)**.** **[A]** *For* $1 \leq r \leq 3$,

$$\begin{aligned} K_0(\mathfrak{A}_r) &\cong R \otimes \Lambda^{\mathrm{even}} \mathbb{Z}^r \cong R^{2^{r-1}}, \\ K_1(\mathfrak{A}_r) &\cong R \otimes \Lambda^{\mathrm{odd}} \mathbb{Z}^r \cong R^{2^{r-1}}. \end{aligned} \tag{1}$$

These are unconditional conclusions for the specified Bernoulli action, based on the imported localization theorem rather than the invalid elementary induction.

Proof. Theorem 2.4 identifies the groups with $K_*(C(\mathbb{T}^r))[1/d]$ by the actual group-algebra inclusion. The standard torus K-groups are the exterior algebra on the r coordinate unitary classes [Bla98]. Localization tensors their free abelian groups with R. The even and odd binomial sums both equal 2^{r-1} for $r \geq 1$. At $r = 0$ the groups remain $R, 0$. □

Corollary 2.6 (Trace decomposition)**.** **[A]** *For* $\omega = \tau \circ E$ *on* $\mathfrak{A}_3$*, the trace image is R, and*

$$K_0(\mathfrak{A}_3) = R[1] \oplus \ker \omega_* \cong R \oplus R^3.$$

Under the inclusion-induced localized torus basis, the three reduced two-torus Bott classes generate the kernel. The three coordinate unitaries and the degree-three exterior class generate K_1 over R.

Proof. On the group algebra, the canonical crossed-product trace restricts to the group trace, or Haar integration on $C(\mathbb{T}^3)$. A projection over the connected torus has constant rank, so the induced trace maps the unit to one and reduced Bott classes to zero. The inclusion-induced localized isomorphism makes every class a d^{-k} multiple of such a class. Unique d-divisibility and additivity determine the trace on these fractions, giving exactly the first-coordinate map and its rank-three kernel. □

Remark 2.7 (Relation to the PV sequence)**.** The localized exterior ranks $(1,0),(1,1),(2,2),(4,4)$ obey the split-PV numerical recursion. This is an arithmetic consistency check, not a substitute for the localization theorem. We do not claim that trace invariance alone constructs equivariant PV sections or selects canonical splittings at every intermediate stage.

Corollary 2.8 (Morita obstructions)**. [A]** *The three crossed products $\mathfrak{A}_1, \mathfrak{A}_2, \mathfrak{A}_3$ are pairwise not strongly Morita equivalent, and none is strongly Morita equivalent to an AF algebra.*

Proof. AF algebras have $K_1 = 0$. Strong Morita equivalence preserves K-groups [Bla98]. Rationalization gives K_1 dimensions $1, 2, 4$ for the three crossed products. A finite matrix algebra and the UHF algebra both have rational K_0 rank one, so this rank list does not distinguish every pair of coefficient algebras. □

3 Weyl asymptotics and analytic spectral dimension $d_{\mathrm{sp}} = 3$ for the explicit confining realization

We analyze an explicitly specified operator inspired by [Sch26a]. All lattice indices, operator coefficients and spectral cutoffs use a common dimensionless model-unit convention; interpreting D or w physically requires an independently specified scale. This operator calculation does not identify a faithful representation of the entire crossed-product algebra on the finite-fibre Hilbert space.

Lemma 3.1 (Exact spectrum of the graded confining Dirac operator)**. [A]** *Let $D_{\mathrm{lat}} = \sum_{j=1}^{3} X_j \otimes \gamma_j$ act on $\mathcal{H}_{\mathrm{lat}} = \ell^2(\mathbb{Z}^3)\otimes\mathbb{C}^4$ where the γ_j are self-adjoint, $(X_j\psi)(n) = n_j\psi(n)$, and $\{\gamma_j,\gamma_k\} = 2\delta_{jk}I_4$ ([Sch26a, Def. 44]), and let $D_{\mathrm{int}} = D_{\mathrm{int}}^*$ act on $\mathcal{K}_{\mathfrak{C}} \cong \mathbb{C}^d$ with eigenvalues $\mu_1,\dots,\mu_d \in \mathbb{R}$. For the graded product Dirac operator ([Sch26a, Rem. 22, eq. (316)])*

$$D_{\mathfrak{C}} := D_{\mathrm{int}} \otimes I_{\mathrm{lat}} \otimes \sigma_1 + I_{\mathrm{int}} \otimes D_{\mathrm{lat}} \otimes \sigma_2 \tag{2}$$

on $\mathcal{H}_{\mathrm{tot}} = \mathbb{C}^d \otimes \ell^2(\mathbb{Z}^3) \otimes \mathbb{C}^4 \otimes \mathbb{C}^2$ (fibre dimension $8d = 32$ at $d = 4$), its square is diagonal in the lattice-position and internal eigenbasis:

$$D_{\mathfrak{C}}^2 = \Big(D_{\mathrm{int}}^2 \otimes I_{\mathrm{lat}} + I_{\mathrm{int}} \otimes \sum_{j=1}^{3} X_j^2 \otimes I_4\Big) \otimes I_2, \qquad \sigma(D_{\mathfrak{C}}^2) = \{\mu_a^2 + \|n\|_2^2 : 1 \le a \le d,\ n \in \mathbb{Z}^3\}, \tag{3}$$

where each pair (a,n) has spinor-grading multiplicity $4 \times 2 = 8$ (R2).

Proof. All Clifford matrices are self-adjoint. The multiplication operator is taken on its maximal square-summable domain, where the sum of squared fibre norms is finite; the bounded finite internal term preserves self-adjointness. Since $\sigma_1^2 = \sigma_2^2 = I_2$ and $\{\sigma_1,\sigma_2\} = 0$, the cross-terms between $D_{\mathrm{int}} \otimes I_{\mathrm{lat}}$ and $I_{\mathrm{int}} \otimes D_{\mathrm{lat}}$ vanish identically upon squaring (2), while $D_{\mathrm{lat}}^2 = \sum_{j=1}^{3} X_j^2 \otimes I_4$ by [Sch26a, eq. (306)], yielding (3). □

Theorem 3.2 (Spectral dimension of the confining realization)**. [A]** *For the Dirac operator $D_{\mathfrak{C}}$ of Lemma 3.1:*

(a) Weyl asymptotic law ([Sch26a, eq. (555)]): *The eigenvalue counting function $N_D(\Lambda) := \#\{\lambda \in \sigma(|D_{\mathfrak{C}}|) : \lambda \le \Lambda\}$ (counted with multiplicity) satisfies as $\Lambda \to \infty$:*

$$N_D(\Lambda) = \frac{32\pi d}{3}\,\Lambda^3 + O(\Lambda^2), \tag{4}$$

with explicit positive Weyl constant $C = \frac{32\pi d}{3} > 0$ ($C = 128\pi/3 \approx 134.0413$ at $d = 4$; for the zero-internal isotropic witness $\mu_a = 0$, at $\Lambda = 30$, $N_D(30)/(C\cdot 30^3) = 0.9999$; R2).

(b) Analytic spectral dimension $d_{\mathrm{sp}} = 3$ ([Sch26a, Def. 52, eq. (556)]): *For $p > 0$, the spectral zeta trace*

$$\zeta_D(p) := \mathrm{Tr}\big((I + D_{\mathfrak{C}}^2)^{-p/2}\big) = 8 \sum_{a=1}^{d} \sum_{n\in\mathbb{Z}^3} \big(1 + \mu_a^2 + \|n\|_2^2\big)^{-p/2} \tag{5}$$

converges if and only if $p > 3$, with critical residue $\lim_{p\downarrow 3}(p-3)\zeta_D(p) = 32\pi d$. Consequently,

$$d_{\mathrm{sp}} = \inf\{p > 0 : \mathrm{Tr}\big((I + D_{\mathfrak{C}}^2)^{-p/2}\big) < \infty\} = 3, \tag{6}$$

for this realization (R2).

Proof. (a) By Lemma 3.1, $N_D(\Lambda) = 8\sum_{a=1}^{d} \#\{n \in \mathbb{Z}^3 : \|n\|_2^2 \le \Lambda^2 - \mu_a^2\}$. By Gauss's lattice-point volume bound for the 3-ball in $\mathbb{R}^3$, $\#\{n \in \mathbb{Z}^3 : \|n\|_2 \le R\} = \frac{4\pi}{3}R^3 + O(R^2)$ as $R \to \infty$. Since $(\Lambda^2 - \mu_a^2)^{3/2} = \Lambda^3 + O(\Lambda)$ for each fixed μ_a, summing over $a = 1,\dots,d$ and multiplying by 8 gives $8d \cdot \frac{4\pi}{3}\Lambda^3 + O(\Lambda^2) = \frac{32\pi d}{3}\Lambda^3 + O(\Lambda^2)$. (b) By integral comparison with $\int_0^\infty (1+r^2)^{-p/2} 4\pi r^2\,dr$, the series (5) converges iff $\int_1^\infty r^{2-p}\,dr < \infty \iff p > 3$, and the residue follows rigorously from part (a): writing $N(t) = Ct^3 + O(t^2)$ and integrating the Stieltjes trace by parts gives a leading term $pC\int_1^\infty t^{2-p}dt$ plus a term bounded as $p \downarrow 3$. Thus $(p-3)\zeta_D(p) \to 3C = 32\pi d$ (R2). □

Corollary 3.3 (Anisotropic confining Dirac operator and directional speeds)**.** **[A]** *For the anisotropically weighted confining Dirac operator* $D_{\mathrm{lat}}^{(\kappa)} := \sum_{\alpha\in\{x,y,z\}} \kappa_\alpha X_\alpha \otimes \gamma_\alpha$ *with directional translation seminorms* $l_D(e_\alpha) = \|[D_{\mathfrak{C}}^{(\kappa)}, U_{e_\alpha}]\| = \kappa_\alpha > 0$ *([Sch26a, Def. 46, Rem. 23]), the eigenvalue counting function and spectral dimension satisfy*

$$N_{D^{(\kappa)}}(\Lambda) = \frac{32\pi d}{3\,\kappa_x\kappa_y\kappa_z}\,\Lambda^3 + O(\Lambda^2), \qquad d_{\mathrm{sp}}(D_{\mathfrak{C}}^{(\kappa)}) = 3 \tag{7}$$

(R2).

Proof. The ellipsoid $\{x \in \mathbb{R}^3 : \sum_\alpha \kappa_\alpha^2 x_\alpha^2 \le \Lambda^2\}$ has semi-axes Λ/κ_α and Euclidean volume $\frac{4\pi}{3\kappa_x\kappa_y\kappa_z}\Lambda^3$; multiplying by the fibre multiplicity $8d$ yields (7). □

4 Faithful finite-graph spectral metric calibration

Definition 4.1 (Finite admissibility graph and isolated vertices)**.** Let X be a nonempty finite set, E a set of unordered pairs of distinct vertices, and $w : E \to (0,\infty)$. In an admissibility model, E contains only allowed edges; all excluded states remain vertices with no incident edge. Let I be the set of isolated vertices. Define the extended path metric d_C^* with $d_C^*(x,x) = 0$ for every x, and $d_C^*(x,y) = \infty$ for distinct components. Set $\mathcal{A}_X = C(X)$ and

$$\mathcal{H}_{\mathrm{cal}} = \bigoplus_{e\in E} \mathbb{C}_e^2 \oplus \bigoplus_{x\in I} \mathbb{C}_x.$$

For an arbitrarily chosen ordering $e = (u,v)$, define

$$\begin{aligned} \pi_{\mathrm{cal}}(f) &= \bigoplus_e \mathrm{diag}(f(u), f(v)) \;\oplus \bigoplus_{x\in I} f(x), \\ D_{\mathrm{cal}} &= \bigoplus_e w(e)^{-1}\sigma_1 \;\oplus \bigoplus_{x\in I} 0. \end{aligned} \tag{8}$$

Proposition 4.2 (Faithfulness and finite spectral realization)**.** **[A]** *The representation is faithful,* D_{cal} *is self-adjoint, and its commutators are bounded. Its resolvent is compact.*

Proof. Every vertex occurs either in an edge block or an isolated block. Hence $\pi(f) = 0$ forces every value of f to vanish. The remaining assertions follow from finite dimension and Hermitian blocks. □

Lemma 4.3 (Lipschitz calibration)**.** **[A]** *For real-valued* f*,*

$$L(f) = \|[D_{\mathrm{cal}}, \pi_{\mathrm{cal}}(f)]\| = \max_{\{u,v\}\in E} \frac{|f(u) - f(v)|}{w(u,v)}, \tag{9}$$

with empty maximum zero.

Proof. The commutator on an edge is $(f(v) - f(u))w(e)^{-1}\left(\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right)$, whose norm is the displayed quotient. Isolated blocks contribute zero, and the norm of a finite direct sum is the maximum of the block norms. □

Theorem 4.4 (Exact extended metric equality)**.** **[A]** *For all* $x, y \in X$*,*

$$\sup_{f=f^*,\, L(f)\le 1} |f(x) - f(y)| = d_C^*(x,y).$$

This includes zero self-distance at isolated or rejected states.

Proof. If $x = y$, both sides vanish. For distinct vertices in the same component, telescoping along any path bounds every admissible difference by its length, hence by the shortest length. Conversely, set $f(z) = d_C^*(x,z)$ on the component of x and zero on other components. This is a finite real-valued function in $C(X)$; the reverse triangle inequality gives $L(f) \le 1$, and its difference at x, y attains the path distance. If x, y lie in different components, $f = M\mathbf{1}_{C_x}$ has $L(f) = 0$ and difference M. Letting M grow proves infinite distance. □

Example 4.5 (Fifteen-state source graph)**.** Take $X = \{0,1,2,3,4\} \times \{\beta,\sigma,\kappa\}$, with three unit edges joining (n,σ) and (n,κ) for $n = 2,3,4$. There are nine isolated vertices and three two-vertex components. Therefore the number of finite ordered pairs is $3 \cdot 2^2 + 9 \cdot 1^2 = 21$; the other $225 - 21 = 204$ pairs have infinite distance. The faithful representation has dimension $2 \cdot 3 + 9 = 15$.

Remark 4.6 (Scope of the state graph)**.** This graph encodes a chosen adjacency convention for the source labels [Sch26d, Sch26c]; it is not the entire reaction or persistence generator. In particular, omitting transitions between degree slices is a modeling choice rather than a derived kinetic prohibition. Infinite graphs require a specified observable algebra, domains, and compactness conditions; the finite theorem is not asserted for arbitrary $C_0(X)$ graphs.

Corollary 4.7 (Unified finite-graph and confining product)**.** **[A]** *Let* $m = 2|E| + |I|$*. On* $\mathcal{H}_{\mathrm{cal}} \otimes \ell^2(\mathbb{Z}^3) \otimes \mathbb{C}^4 \otimes \mathbb{C}^2$*, put*

$$D_{\mathrm{unif}} = D_{\mathrm{cal}} \otimes I \otimes \sigma_1 + I \otimes D_{\mathrm{lat}} \otimes \sigma_2.$$

Its counting coefficient is $32\pi m/3$ *and spectral dimension is three. On* $C(X) \otimes I$*, its spectral metric is* d_C^**. For the source example* $m = 15$*, the fibre dimension is* 120 *and the coefficient is* 160π*.*

Proof. The spectral assertion is Theorem 3.2 with $d = m$. The lattice term commutes with $\pi_{\mathrm{cal}}(f) \otimes I$, leaving $[D_{\mathrm{cal}}, \pi_{\mathrm{cal}}(f)] \otimes I \otimes \sigma_1$, with the same norm. Apply Theorem 4.4. □

Remark 4.8 (Algebra represented by the product)**.** The finite graph algebra is represented faithfully in this construction. No faithful representation of the infinite UHF crossed product $\mathfrak{A}_{\mathfrak{C}}$ on this finite internal fibre has been constructed here. The product metric and counting results therefore do not complete a spectral triple for the full sequence algebra.

5 Cut-set degeneration and calibrated weights

Proposition 5.1 (When metric divergence follows)**.** **[A]** *Let* $S \subset X$ *separate* $x \in S$ *from* $y \notin S$ *in a fixed finite graph. Suppose every edge in the cut* δS *has weight tending to infinity. Then* $d_t(x, y) \to \infty$*. Conversely, if an* x*–*y *path has uniformly bounded total weight, the distance stays bounded even if another edge diverges.*

Proof. Every x–y path crosses δS, so $d_t(x, y) \geq \min_{e \in \delta S} w_t(e)$. A finite cut whose individual weights diverge has diverging minimum. A surviving path supplies the opposite upper bound by definition of shortest distance. If the cut is empty, the vertices are disconnected already. □

This proposition permits a calibrated model of loss of accessibility. A physical use requires measuring or justifying the map from chemical state to w_t. Edge weight is a metric cost; $w^{-1}\sigma_1$ is an operator coefficient, not automatically a rate or hopping Hamiltonian. Deleting an individual bond need not destroy connectivity, as bypass paths can remain [Sch26e].

6 Conclusions and remaining obligations

The UHF and all three crossed-product K-groups are established for the specified Bernoulli shift. The higher-stage classification uses the noncommutative Bernoulli-shift localization theorem, not trace invariance alone. The inclusion-induced localized torus basis also determines the trace kernel and proves the Morita obstructions.

For the specified finite-fibre confining operator, the Weyl law and summability threshold give spectral dimension three. For every finite weighted admissibility graph, the faithful edge-and-isolated-state construction gives exact path/spectral metric equality. These two realizations combine in one graded product for the finite graph algebra. A faithful crossed-product spectral realization and experimental interpretation of its weights remain separate targets.

A Hypothesis and correspondence register

Bernoulli localization
: Countably infinite tensor-index set, matrix factors M_d, and amenable coordinate-translation group $\mathbb{Z}^r$. Theorem 2.4 supplies the inclusion-induced localized isomorphism; numerical rank recursion does not prove it.

Trace
: Canonical conditional-expectation trace restricts to Haar trace on the included group algebra; its extension through localization gives the trace decomposition.

Confining operator
: Finite self-adjoint internal operator, positive fixed κ_α, four-dimensional Clifford matrices and an additional grading factor. These determine the counting exponent and multiplicity.

Finite graph
: Positive symmetric weights, isolated-state blocks, and zero self-distance for every state. Infinite-graph extension is not included.

Physical correspondence
: A specified compound, assembly geometry, state dictionary, measured coupling or kinetics, and a justified metric calibration. None is inferred from abstract K-theory.

B Computational and formal verification

The companion Python dossier evaluates finite arithmetic, explicit matrix blocks and graph examples, including negative controls. Its output generates the numerical macros. The Lean dossier proves selected finite arithmetic and graph certificates. The analytic localization theorem and Weyl asymptotic are cited or proved in the text, not by finite computation.

R1 Exterior-rank arithmetic and extension-shear counterexample.

R2 Lattice enumeration and finite product multiplicities; finite counts illustrate rather than prove an asymptotic.

R3 Edge-block commutator identities and faithfulness at isolated states.

R4 All ordered source pairs, including the nine isolated self-pairs.

R5 Weighted path potentials, grid distances and cut/bypass examples.

R6 Scope and negative controls. Historical finite tests are not treated as proofs of the original universal claims.

C Molecular interpretation and empirical tests

Remark C.1 (Conditional modeling dictionary)**. [E]** The lattice axes may be assigned to candidate lateral covalent connectivity and stacking directions after the assembly has been characterized. Abstract translation classes are not bond counts, and Bott labels do not identify molecular bundles or demonstrate a particular phosphate linkage. The unit in K_0 is the algebraic identity class; normalized projection trace is not a measured monomer count. The exterior-power pattern expresses formal dimensional organization rather than evidence that a molecule realizes all axes.

Remark C.2 (Spectral and transport dimensions)**. [E]** The exponent three is a summability exponent of the chosen confining operator. Its value follows from the three integer coordinates and finite multiplicity. It is neither an experimental diffusion dimension nor an electronic density-of-states measurement. Inferring transport would require a specified Hamiltonian or stochastic generator, disorder and contact models, and comparison to data.

Remark C.3 (Empirical closure)**. [E]** A molecular test program should establish exact compound identity and purity, specify substituents and linkage products, verify connectivity and stacking by appropriate structural measurements, measure assembly survival and exchange as well as cleavage and repair kinetics under stated conditions, and choose observables that calibrate edge weights. Single-edge deletion and complete cut-set deletion should be compared separately. Measured rates or conductances need not obey the exact metric calibration unless that relation is independently demonstrated.

D Relation to preceding papers

The source manuscript cited theorem numbers and targets in Articles 40, 80 and 90 as motivations. This revision makes only the scoped statements proved above; it does not certify every earlier theorem-number attribution or declare all earlier obligations discharged. In particular, the revised Article 90 uses a corrected spectral-separation argument rather than the source manuscript's blanket assertion of a universal distance bound of two. That unsupported bound and the associated alleged filtration commutator formula are not used here. A filtration with superpolynomial cumulative eigenspace growth cannot be finitely summable, but this requires a correctly specified filtration and does not identify its metric with the finite graph metric constructed here.

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