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Quaternary Coding Theory of the Biological Ledger over GF(4)

Evaluation Codes, Plaquette Checks, and Conditional Syndrome Decoding

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Abstract

We construct explicit quaternary error-correcting codes for abstract locus, contact and lattice registers motivated by the Chrysene Formalism. Evaluation on the four affine points of GF(4) gives the Euclidean self-dual MDS code $[4, 2, 3]_4$. The five projective points give the perfect Hamming code $[5, 3, 3]_4$, whose 64 radius-one spheres partition $\mathrm{GF}(4)^5$; their tensor product is a $[20, 6, 9]_4$ array code correcting four arbitrary symbol errors. On the cubical box $B(3, 3, 6)$, the cycle and cut codes have parameters $[117, 64, 4]_4$ and $[117, 53, 3]_4$. Their full CSS check pair commutes and has zero logical dimension. An ideal 16-outcome syndrome instrument preserves each syndrome subspace and enables exact classical single-symbol correction. For independent symbol errors of probability p, decoded block failure is $10p^2 - 20p^3 + 15p^4 - 4p^5$. We compare uncoded blocks at both fixed coordinate count and equal information payload, distinguish binary-logistic and four-level Gibbs calibrations, and compute their conditional splitting thresholds. At complete quaternary randomization, $p = 3/4$, channel capacity is zero and decoder success is only $1/64$. Molecular encoders, Berry-phase readout and Stark control are explicit realization targets rather than consequences of code cardinalities. Executable finite verification and a scoped Lean companion accompany the constructions.

Keywords: Quaternary codes; evaluation codes; perfect Hamming code; tensor-product code; cubical homology; syndrome instrument; conditional molecular realization.

1 Introduction

The purpose of this paper is to give explicit error-correcting structures over GF(4) for the finite registers used in the Chrysene Formalism. Articles 4 and 9 motivate a quaternary alphabet and a proposed molecular readout architecture [Sch26d, Sch26a]; Articles 79 and 84 distinguish endogenous supply from placed preparation [Sch26h, Sch26g]. The corrected register arithmetic of Article 92 [Sch26f] supplies the field convention. Articles 96 and 97 separate finite channel models and conditional thermal preparation from their physical realization [Sch26c, Sch26b]. Here we construct the codes directly, so their algebraic conclusions do not depend on the molecular claims of those sources.

Three constructions organize the paper: a four-coordinate evaluation code, a five-coordinate perfect Hamming code with its tensor-product extension, and the cycle/cut pair of the cubical box $B(3, 3, 6)$. The first two are classical genus-zero evaluation constructions [TVN07, MS77]; their use with chrysene locant and contact labels is a proposed representation. We prove their parameters, give an exact classical syndrome instrument, and derive conditional block-error and thermal-splitting benchmarks. No molecular encoder, flux detector or feedback actuator is assumed to exist merely because these finite codes do.

2 Conventions and realization conditions

Convention 2.1 (Status)**.** **[A]** labels finite mathematical statements; **[D]** labels conclusions conditional on named statistical or implementation assumptions; **[E]** labels a proposed physical interpretation. Verification and Lean coverage are mapped in Appendix B. A successful finite test is not an experimental observation.

Notation 2.2 (Field and codes)**.** Write $\mathrm{GF}(4) = \mathbb{F}_2[\omega]/(\omega^2 + \omega + 1)$ and represent elements as $a + b\omega$. Hamming weight counts nonzero coordinates. A linear $[n, k, d]_4$ code has

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length n, dimension k over GF(4) and minimum nonzero weight d. Duality below uses the Euclidean bilinear form, not the Hermitian form. The phrase genus-zero geometric Goppa code means an algebraic-geometry evaluation code, not a binary subfield-subcode construction.

Hypothesis 2.3 (Molecular representation H_{enc})**.** A physical implementation provides distinguishable quaternary symbols at assigned coordinates, a specified encoder whose image is the stated code, calibrated storage/noise, and a recovery operation acting on those symbols. Locant names and equal alphabet cardinalities alone do not supply these operations. The primary and dual locant sets are alternative labels; their chemical behavior need not be identical.

Remark 2.4 (Source and placement)**.** An unconstrained quaternary register is $\mathrm{GF}(4)^n$ and has minimum distance one. Restricting allowed states to a code introduces redundancy. This is an explicit design restriction and not a theorem that every placed molecular assembly automatically satisfies parity checks.

3 The $[4, 2, 3]_4$ MDS locus evaluation code and dual-cipher linearity

We construct an evaluation code after assigning four abstract coordinates to four locant labels. This assignment is a coding convention, not a molecular encoding theorem.

Lemma 3.1 (Quaternary field arithmetic and the χ-register automorphism)**. [A]** *Under the $\mathbb{F}_2$-linear bijection $\phi : \mathbb{F}_2^2 \xrightarrow{\sim} \mathrm{GF}(4)$ defined by $\phi(a, b) := a + b\omega$ ($\omega^2 = \omega + 1$, $\omega^3 = 1$), the catalyst–substrate register shift $\chi(a, b) = (b, a + b)$ of [Sch26g, Sch26f] coincides with multiplication by ω in* GF(4) *($\phi(\chi(a, b)) = \omega\phi(a, b)$), and the four field elements $\mathcal{P}_4 := (0, 1, \omega, \omega^2)$ satisfy*

$$\sum_{x\in\mathrm{GF}(4)} x = 0, \qquad \sum_{x\in\mathrm{GF}(4)} x^2 = 0, \qquad \prod_{x\in\mathrm{GF}(4)^\times} x = 1 \tag{1}$$

(C1).

Proof. Since $\omega(a + b\omega) = a\omega + b(\omega + 1) = b + (a + b)\omega = \phi(b, a + b)$ and $0 + 1 + \omega + \omega^2 = 1 + (\omega^2 + \omega + 1) - 1 = 0$ in characteristic 2, (1) holds identically (C1). □

Theorem 3.2 (The $[4, 2, 3]_4$ MDS Locus Evaluation Code)**. [A]** *Identify the four substitution loci $(3, 6, 9, 12)$ of the primary C_{2h} chrysene tensor (or the four loci $(2, 5, 8, 11)$ of the dual cipher in [Sch26a, Rem. 1]) with the four affine points $\mathcal{P}_4 = (0, 1, \omega, \omega^2)$ of the genus-0 curve $\mathbb{P}^1(\mathrm{GF}(4))$. Let $\mathrm{GF}(4)[x]_{\le 1} = \{f(x) = c_0 + c_1 x : c_0, c_1 \in \mathrm{GF}(4)\}$ be the 2-dimensional space of linear polynomials, and define the evaluation code $C_{\mathrm{loc}} := \{\mathrm{ev}_{\mathcal{P}_4}(f) = (f(0), f(1), f(\omega), f(\omega^2)) : f \in \mathrm{GF}(4)[x]_{\le 1}\} \subset \mathrm{GF}(4)^4$ with generator matrix*

$$G_{\mathrm{loc}} := \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & \omega & \omega^2 \end{pmatrix} \in \mathrm{GF}(4)^{2\times 4}. \tag{2}$$

(a) Exact code parameters and weight enumerator: *C_{loc} is a $[4, 2, 3]_4$ linear code of cardinality $|C_{\mathrm{loc}}| = 4^2 = 16$ (with 16 abstract messages), with minimum Hamming distance $d_{\min}(C_{\mathrm{loc}}) = 3$ and exact weight enumerator*

$$W_{C_{\mathrm{loc}}}(z) = 1 + 12z^3 + 3z^4. \tag{3}$$

(b) Saturation of the Singleton and Riemann–Roch bounds ([Sch26d, App. A.3]) and Euclidean self-duality: *For the genus-0 curve $\mathbb{P}^1(\mathrm{GF}(4))$ ($g = 0$), the algebraic-geometry Goppa bound $d_{\min} \ge n - k + 1 - g$ and the Singleton bound $d_{\min} \le n - k + 1$ coincide at $d_{\min} = 4 - 2 + 1 = 3$, so C_{loc} is Maximum Distance Separable (MDS). Moreover, under the Euclidean bilinear form $\langle u, v\rangle = \sum_{i=1}^4 u_i v_i$, $G_{\mathrm{loc}} G_{\mathrm{loc}}^\top = 0_{2\times 2}$, so $C_{\mathrm{loc}} = C_{\mathrm{loc}}^\perp$ is strictly self-dual ($H_{\mathrm{loc}} = G_{\mathrm{loc}}$; C1, C2).*

Proof. (a) A non-zero linear polynomial $f(x) = c_0 + c_1 x$ has at most $\deg f \le 1$ root in GF(4), so $\mathrm{wt}(\mathrm{ev}_{\mathcal{P}_4}(f)) \ge 4 - 1 = 3$. Specifically, the 3 non-zero constant polynomials ($c_1 = 0, c_0 \ne 0$) have weight 4, and the $4 \times 3 = 12$ polynomials with $c_1 \ne 0$ each have the unique root $x = c_0 c_1^{-1} \in \mathrm{GF}(4)$ and hence weight 3, giving (3) and $d_{\min} = 3$. (b) By Lemma 3.1, $\sum_{x\in\mathrm{GF}(4)} 1 = 0$, $\sum_{x\in\mathrm{GF}(4)} x = 0$, and $\sum_{x\in\mathrm{GF}(4)} x^2 = 0$, so both rows of G_{loc} are orthogonal to themselves and to each other; since $\dim C_{\mathrm{loc}} = 2 = 4 - 2 = \dim C_{\mathrm{loc}}^\perp$, $C_{\mathrm{loc}} = C_{\mathrm{loc}}^\perp$ (C2). □

4 The perfect $[5, 3, 3]_4$ quaternary Hamming code on the hexamer contact register

Across an $l = 6$ hexamer column ([Sch26e, Sch26g]), there are $n = l - 1 = 5$ inter-monomer contacts, matching the $q + 1 = 5$ projective points of $\mathbb{P}^1(\mathrm{GF}(4))$.

Definition 4.1 (Projective parity-check matrix H_5 on the hexamer contact register)**. [A]** Ordering the five projective

points of $\mathbb{P}^1(\mathrm{GF}(4))$ as ([1 : 0], [0 : 1], [1 : 1], [1 : ω], [1 : ω^2]), define the parity-check matrix $H_5 \in \mathrm{GF}(4)^{2\times 5}$ and systematic generator matrix $G_5 \in \mathrm{GF}(4)^{3\times 5}$ of the hexamer contact code $C_{\mathrm{hex}} := \ker H_5 \subset \mathrm{GF}(4)^5$ by

$$H_5 := \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & \omega & \omega^2 \end{pmatrix}, \qquad G_5 := \begin{pmatrix} 1 & 1 & 1 & 0 & 0 \\ 1 & \omega & 0 & 1 & 0 \\ 1 & \omega^2 & 0 & 0 & 1 \end{pmatrix}. \tag{4}$$

Theorem 4.2 (The Perfect MDS $[5, 3, 3]_4$ Hexamer Contact Code Theorem)**. [A]** *For the hexamer contact code $C_{\mathrm{hex}} = \ker H_5 \subset \mathrm{GF}(4)^5$:*

(a) MDS property, equidistant dual simplex code, and MacWilliams identity: *Because every pair of columns of H_5 represents two distinct points of $\mathbb{P}^1(\mathrm{GF}(4))$ and is linearly independent over* GF(4)*, C_{hex} has parameters $[5, 3, 3]_4$ ($|C_{\mathrm{hex}}| = 4^3 = 64$ codewords; cardinality equality with another 64-state alphabet alone supplies no physical encoder) and saturates the Singleton bound $d_{\min} = 5 - 3 + 1 = 3$. Its orthogonal dual $C_{\mathrm{hex}}^{\perp} = \mathrm{rowspan}_{\mathrm{GF}(4)}(H_5)$ is an equidistant $[5, 2, 4]_4$ simplex code with weight enumerator $W_{C_{\mathrm{hex}}^{\perp}}(z) = 1 + 15z^4$, which yields via the quaternary MacWilliams identity the exact weight enumerator of C_{hex}:*

$$W_{C_{\mathrm{hex}}}(z) = \frac{1}{16}(1+3z)^5\, W_{C_{\mathrm{hex}}^{\perp}}\!\left(\frac{1-z}{1+3z}\right) = 1 + 30z^3 + 15z^4 + 18z^5. \tag{5}$$

(b) Exact sphere-packing perfection and bijective syndrome decoding: *Each codeword $c \in C_{\mathrm{hex}}$ has a radius-1 Hamming sphere $B_1(c) = \{y \in \mathrm{GF}(4)^5 : d_H(y, c) \le 1\}$ of cardinality $1 + 5(4 - 1) = 16$. The 64 spheres $\{B_1(c)\}_{c\in C_{\mathrm{hex}}}$ are pairwise disjoint and partition the entire contact space $\mathrm{GF}(4)^5$ with zero leftover vectors:*

$$|C_{\mathrm{hex}}| \cdot \big(1 + 5(4-1)\big) = 64 \times 16 = 1,024 = 4^5. \tag{6}$$

Every non-zero syndrome $s = H_5 y \in \mathrm{GF}(4)^2 \setminus \{(0,0)\}$ ($4^2 - 1 = 15$ syndromes) factors uniquely as $s = \alpha\, h_r$ for a unique contact index $r \in \{0, 1, 2, 3, 4\}$ and error amplitude $\alpha \in \mathrm{GF}(4)^{\times} = \{1, \omega, \omega^2\}$, so $\widehat{c}(y) := y - \alpha e_r$ corrects 100% *of single-contact errors (C3).*

Proof. In characteristic 2, $H_5 G_5^{\top} = 0_{2\times 3}$ and $\mathrm{rk}\, G_5 = 3$. Every non-zero vector $u \in \mathrm{GF}(4)^2$ is orthogonal to a unique projective point $[h_r] \in \mathbb{P}^1(\mathrm{GF}(4))$, so $uH_5 \in C_{\mathrm{hex}}^{\perp}$ has exactly one zero coordinate and weight 4, giving $W_{C_{\mathrm{hex}}^{\perp}}(z) = 1 + 15z^4$ and (5) by MacWilliams' theorem. Since $d_{\min}(C_{\mathrm{hex}}) = 3$, the spheres $B_1(c)$ are disjoint and their union has size $64 \times 16 = 1,024 = |\mathrm{GF}(4)^5|$, proving (6) (C3). □

Corollary 4.3 (The Tensor-Product $[20, 6, 9]_4$ Monomer—Contact Product Code)**. [A]** *Taking two independent rows of hexamer-code messages and encoding each column with the two-dimensional locus code defines the tensor-product code $C_{\mathrm{prod}} := C_{\mathrm{loc}} \otimes_{\mathrm{GF}(4)} C_{\mathrm{hex}} \subset \mathrm{GF}(4)^{4\times 5} \cong \mathrm{GF}(4)^{20}$ with exact parameters*

$$C_{\mathrm{prod}} = [20,\ 6,\ 9]_4, \qquad |C_{\mathrm{prod}}| = 4^6 = 4,096, \tag{7}$$

whose minimum Hamming distance $d_{\min}(C_{\mathrm{prod}}) = d_{\min}(C_{\mathrm{loc}}) \cdot d_{\min}(C_{\mathrm{hex}}) = 3 \times 3 = 9$ corrects up to $t_{\mathrm{corr}} = \lfloor (9-1)/2 \rfloor = 4$ arbitrary locus errors (and detects up to 8 *errors) in a 4×5 abstract array (C3).*

Proof. Let $U \in \mathrm{GF}(4)^{2\times 3}$. The arrays $G_{\mathrm{loc}}^{\top} U G_5$ form the product code. Injectivity follows from full row rank of both generators, giving dimension six. Every nonzero array has a nonzero column of weight at least three, so at least three rows are nonzero; each such row has weight at least three. Thus its weight is at least nine. The outer product of minimum-weight words attains nine. Distance nine gives unique correction of four arbitrary symbols; a componentwise iterative decoder is not automatically guaranteed to attain that bound. □

5 Cycle and cut codes of the cubical box

Let $B(3, 3, 6)$ have vertices (i, j, k) with $0 \le i, j \le 2$ and $0 \le k \le 5$, nearest-neighbor edges and elementary square faces. There are 54 vertices, 117 edges and 84 faces. Let ∂_1 and ∂_2 be the vertex-edge and edge-face incidence matrices over GF(4). Characteristic two makes orientation signs unnecessary. Set $H_{\mathrm{plaq}} = \partial_2^{\top}$.

Theorem 5.1 (Cycle and cut parameters)**. [A]** *The box has*

$$C_{\mathrm{cyc}} = \ker \partial_1 = \mathrm{im}\, \partial_2 = [117, 64, 4]_4, \tag{8}$$
$$C_{\mathrm{cut}} = C_{\mathrm{cyc}}^{\perp} = \ker H_{\mathrm{plaq}} = [117, 53, 3]_4. \tag{9}$$

The graph cycle rank is 64, whereas the first homology of the filled box (and of its two-skeleton) is zero. The cut code corrects any single-edge symbol error globally and detects every error of weight one or two.

Proof. The filled rectangular box is contractible. Its two-skeleton has the same first homology because three-cells do not change $\ker \partial_1 / \operatorname{im} \partial_2$. Connectivity gives $\operatorname{rk} \partial_1 = 53$, hence $\operatorname{rk} \partial_2 = 64$ and the stated spaces and dimensions. In the support graph of a nonzero circulation, no vertex has degree one. Its support therefore contains a cycle. The grid is simple and bipartite, so every such cycle has at least four edges; a square realizes weight four.
A cut-code word is $\partial_1^\top x$. For a nonconstant potential x, any nonempty proper level set has its entire edge cut in the support. Every nontrivial cut has at least three edges. To see this, slice the box into six 3×3 layers. Each layer is connected and has no bridge because every edge lies on a square, so a partly occupied layer contributes at least two horizontal cut edges. Two such layers contribute at least four. Exactly one partly occupied layer contributes at least two horizontal edges and at least one vertical edge to an adjacent wholly occupied or empty layer. If no layer is partial, a transition between full and empty layers contributes nine vertical edges. A corner singleton realizes three. Thus the minimum nonzero cut-word weight is three. The usual distance bound gives the stated correction and detection guarantees. Equivalently, no two distinct one-edge errors have the same syndrome, since their difference would be a nonzero codeword of weight at most two. □

Corollary 5.2 (CSS commutation and logical dimension)**.** **[A]** *The matrices $H_X = \partial_1$ and $H_Z = \partial_2^\top$ satisfy $H_X H_Z^\top = 0$. With generalized ququart Pauli operators defined through the field trace to $\mathbb{F}_2$, all their scalar-multiple checks commute. The complete pair has*

$$k_{\log} = 117 - 53 - 64 = 0$$

logical ququarts. If selected checks are removed and the total rank drops by r, commutation persists and $k_{\log} = r$. Removing redundant rows does not increase logical dimension; a nonzero distance must be computed for the resulting code and does not follow from commutation alone.

Proof. For $X(a)|x\rangle = |x + a\rangle$ and $Z(b)|x\rangle = (-1)^{\mathrm{Tr}_{4/2}(bx)}|x\rangle$, the commutation phase is the trace of the field inner product. A zero field inner product therefore gives commutation, including scalar multiples. The independent X and Z check ranks add. Each independent field check removes one ququart degree of freedom, giving the formula. Deleting constraints preserves pairwise commutation but can lower distance. □

6 An ideal syndrome instrument and decoder

Proposition 6.1 (Complete classical syndrome readout)**.** **[A]** *For the five-symbol computational basis, define*

$$P_s = \sum_{y : H_5 y = s} |y\rangle\langle y|, \qquad s \in \mathrm{GF}(4)^2.$$

These 16 orthogonal projectors sum to the identity. The instrument $\rho \mapsto P_s \rho P_s$ records the syndrome without disturbing any state supported within a single syndrome subspace. In particular it preserves arbitrary superpositions of error-free codewords. A complete outcome contains two quaternary symbols, equivalently four bits, not merely two binary parities.

Proof. The fibres of the surjective linear map H_5 partition all 4^5 basis states. Orthogonality and completeness follow. If $\rho = P_s \rho P_s$, its normalized conditional state is unchanged. A superposition across different syndrome subspaces is dephased by an unread measurement, as expected. For $s \neq 0$, the unique weight-one vector $e(s)$ with syndrome s defines recovery by $y \mapsto y - e(s)$. □

Hypothesis 6.2 (Physical syndrome extraction H_{synd})**.** A proposed device realizes a calibrated 16-outcome instrument equivalent to the projectors above, or four appropriate binary measurements, followed by symbol recovery. It must resolve syndrome values with quantified backaction, latency and errors. A Berry-phase or flux proposal from Article 9 is one candidate, not a consequence of this proposition. Spatial orthogonality of channels alone does not establish nondemolition readout.

Remark 6.3 (Classical versus quantum correction)**.** The perfect Hamming decoder corrects classical additive symbol errors. The syndrome instrument preserves code-space coherence but this fact alone does not prove correction of arbitrary phase errors or quantum fault tolerance. The separate CSS construction above has zero logical dimension before checks are removed.

7 Independent-error performance and thermal models

Hypothesis 7.1 (Independent symbol channel H_{ind})**.** Each of five stored symbols is changed with total probability p, independently across positions. In the quaternary symmetric channel each of the three nonzero additive changes has conditional probability $1/3$. The syndrome instrument and recovery are ideal for the following benchmarks. Correlated failures, erasures, readout faults and actuator faults require separate models.

Theorem 7.2 (Exact block failure and fair comparisons)**. [D]** *Under H_{ind}, nearest-codeword Hamming decoding succeeds exactly for error weight zero or one. Thus*

$$P_{\mathrm{cod}}(p) = 1 - (1-p)^5 - 5p(1-p)^4 \tag{10}$$

$$= 10p^2 - 20p^3 + 15p^4 - 4p^5. \tag{11}$$

An uncoded five-symbol register has failure $P_{\mathrm{unc},5} = 1-(1-p)^5$, but stores five rather than three information symbols. The same-payload uncoded comparator is $P_{\mathrm{unc},3} = 1-(1-p)^3$. The code has rate $3/5$; its small-p failure is quadratic, while the two uncoded comparators have leading terms $5p$ and $3p$.

Proof. The decoder subtracts either zero or a weight-one syndrome leader. It returns the originally transmitted word if and only if the actual error is that leader. Hence every weight-two-or-more pattern fails for this deterministic decoder, even when its syndrome is zero. Independence gives the binomial weight probabilities. The same counting with three or five unprotected coordinates gives the comparators. □

Proposition 7.3 (Maximum-entropy channel limit)**. [A]** *At $p = 3/4$ the quaternary symmetric channel output is uniform and independent of its input. Its capacity is zero. The perfect-code decoder succeeds with probability $1/64$, so $P_{\mathrm{cod}} = 63/64$. Redundancy cannot recover information from that fully randomized channel without additional side information or a changed channel.*

Proof. Each output symbol has probability $1/4$ for every input. The input and output are independent, giving zero mutual information for every input law. Among 4^5 equally likely error vectors, exactly $1 + 5 \cdot 3 = 16$ decode to the transmitted word, giving $16/1024 = 1/64$. □

Table 1: Splittings in meV at 300 K for 99% block success; $k_BT = 25.8519998$ meV.

Model/block	$g = 1$	$g = 3$
Uncoded five symbols	160.50	188.91
Uncoded three symbols	147.28	175.68
Coded three in five	87.58	115.98

Hypothesis 7.4 (Calibrated Gibbs surrogate H_{Gibbs})**.** An intended state has free energy zero and each error level has the same free-energy difference Δ, with relative degeneracy g not already included in that difference. At equilibrium,

$$p_g(\Delta) = \frac{g e^{-\Delta/(k_BT)}}{1 + g e^{-\Delta/(k_BT)}}, \quad \Delta_g(p) = k_BT \log \frac{g(1-p)}{p}. \tag{12}$$

The aggregate error-sector free-energy difference is $\Delta - k_BT \log g$. The binary-logistic surrogate is $g = 1$. Four equiprobable levels comprising one intended and three equal-error levels give $g = 3$. A molecular channel need not satisfy either surrogate; its state-dependent free energies and transition/readout statistics must be measured.

Proposition 7.5 (Conditional splitting benchmarks)**. [D]** *Let $p_* \in (0,1)$ be the unique solution $P_{\mathrm{cod}}(p_*) = 0.01$, and set $p_n = 1 - 0.99^{1/n}$ for $n = 3, 5$. Then the required splittings in the calibrated surrogate are $\Delta_g(p_*)$ and $\Delta_g(p_n)$. Switching from $g = 1$ to $g = 3$ adds $k_BT \log 3$ to each splitting. The difference between coded and uncoded splittings is unchanged, but the percentage reduction changes. These are equilibrium splitting comparisons, not total work or electric-field savings.*

Proof. On $(0,1)$, $P'_{\mathrm{cod}}(p) = 20p(1-p)^3 > 0$, giving a unique root. The uncoded equation is immediate; inversion of (12) proves the remaining claims. Electric-field conversion additionally requires a calibrated difference in dipoles and polarizabilities, including orientation; thermodynamic work includes preparation, memory, measurement and reset costs. □

The computed thresholds are $p_* = 0.0326821122$, $p_5 = 0.00200804834$ and $p_3 = 0.00334450659$. Relative to five uncoded coordinates, the splitting reductions are 45.44% ($g = 1$) and 38.60% ($g = 3$); the same-payload reductions are 40.54% and 33.98%. At fixed noise giving 5% or 1% failure in five uncoded coordinates, coded failure is respectively 0.001021 or 0.0000402, giving factors 49.0 and 249.0. These factors compare different information payloads; Table 1 includes the equal-payload alternative.

Remark 7.6 (Operational accounting)**.** The ideal block-error benefit pays two extra quaternary storage symbols for the same three-symbol payload. No experimental fault-tolerance threshold is claimed. If q is the probability of any implementation fault in a block, a conservative union bound is $P_{\text{actual}} \leq P_{\text{cod}} + q$ when the ideal model applies conditional on no implementation fault. Common-mode errors can produce linear rather than quadratic scaling.

8 Molecular realization and experimental closure

[E] The four locant labels and five-contact labels are an abstract coordinate assignment. To instantiate the code, a preparation process must enforce the codeword constraints, not merely prepare a stock with the same number of species. Likewise, a graph cycle is not itself a molecular parity measurement. Different substitution patterns may have different energies, mobilities and spectroscopy; exchanging coordinate labels is not a chemical equivalence theorem.

Aromatic magnetic response does not by itself supply a stable binary flux memory, a quantized Berry phase, or a nondemolition 16-outcome syndrome detector. A Berry phase depends on a specified adiabatic loop and Hamiltonian; quantization needs protecting conditions [Ber84]. Magnetic sensing can be tested using an independently characterized probe, but sensitivity to a local field is not evidence of syndrome selectivity or negligible backaction [L+16]. A Stark splitting depends on differences of permanent dipole and polarizability between actual states [BB97]. The coding calculation specifies a target error probability, not the field required to obtain it.

A staged experiment would (i) identify stable symbols and characterize structure; (ii) prepare all 64 contact-code words and measure code-space occupancy; (iii) deliberately introduce each of the 15 single-symbol errors and determine the complete 16-outcome confusion matrix; (iv) test deliberate multi-error miscorrections, leakage and preservation of code states, and, if quantum nondemolition behavior is intended, code-space coherence; (v) measure conditional cross-contact error correlations and decoder latency; and (vi) account for encoder, detector, feedback and reset resources. Only then can a measured physical block-error reduction be compared with the ideal curve. The rate and energetic costs of the full cycle remain separate from a reduced equilibrium splitting [Lan61].

9 Conclusion and further targets

The explicit codes have parameters $[4, 2, 3]_4$, $[5, 3, 3]_4$ and $[20, 6, 9]_4$. The cubical box yields dual classical cycle and cut codes $[117, 64, 4]_4$ and $[117, 53, 3]_4$. Their full CSS check pair commutes but encodes no logical ququarts. These algebraic constructions address the coding-design target motivating Article 97; molecular realization and readout remain separate targets.

Ideal independent symbol errors give exact quadratic failure suppression for the perfect Hamming decoder. The uniform maximum-entropy channel still has zero capacity. The splitting benchmarks are conditional on a calibrated Gibbs model, and fairness requires stating whether the comparator fixes storage coordinates or information payload. Future work should examine correlated noise, faulty syndrome extraction, nonzero-logical CSS variants and higher-genus evaluation codes [TVN07].

A Hypothesis and dependency register

H_{enc}
: A physical encoder maps distinguishable symbols into the stipulated code. If absent, all abstract code results survive but no molecular protection is inferred.

H_{synd}
: A complete calibrated syndrome instrument and recovery are realized. If absent, deterministic algebraic decoding remains valid but physical correction is unestablished.

H_{ind}
: Independent symbol errors and ideal operations. If absent, distance-based worst-case guarantees survive, but binomial failure and quadratic scaling need not.

H_{Gibbs}
: Equilibrium populations with specified error degeneracy and free-energy difference. If absent, coding guarantees survive but thermal-splitting benchmarks do not.

B Verification and formal-proof scope

The verification companion constructs field tables, both small codes and all 4096 product words; enumerates decoder responses; constructs the actual cubical boundary matrices; checks their ranks, distance witnesses and error syndromes; and independently evaluates performance and Gibbs benchmarks. Negative controls reject deliberately corrupted claims. Its emitted numerical macros are used here.

The Lean companion states its exact finite coverage and dependency audit separately. Neither enumeration nor a finite certificate proves chemical realization, arbitrary quantum-error correction, or the operation of a proposed flux detector. Historical supplied files are retained for provenance, with their claim/coverage differences documented in the referee report.

Earlier article references provide context; this paper's finite statements are proved from the displayed definitions.

C1 Field arithmetic and evaluation words.

C2 Euclidean duality and exact finite code properties.

C3 Complete Hamming decoding and tensor-product enumeration. The full verification companion also supplies lattice, channel and energy checks.

References

[BB97] Gerold U. Bublitz and Steven G. Boxer. Stark spectroscopy: applications in chemistry, biology, and materials science. *Annual Review of Physical Chemistry*, 48:213–242, 1997. https://doi.org/10.1146/annurev.physchem.48.1.213.

[Ber84] Michael V. Berry. Quantal phase factors accompanying adiabatic changes. *Proceedings of the Royal Society A*, 392:45–57, 1984. https://doi.org/10.1098/rspa.1984.0023.

[L+16] Igor Lovchinsky et al. Nuclear magnetic resonance detection and spectroscopy of single proteins using quantum logic. *Science*, 351:836–841, 2016. https://doi.org/10.1126/science.aad8022.

[Lan61] Rolf Landauer. Irreversibility and heat generation in the computing process. *IBM Journal of Research and Development*, 5:183–191, 1961. https://doi.org/10.1147/rd.53.0183.

[MS77] Florence Jessie MacWilliams and Neil James Alexander Sloane. *The Theory of Error-Correcting Codes*. North-Holland, 1977.

[Sch26a] Charles D. Schaper. Geometric Quantization of Molecular Quantum Transport: The CohBit Chrysene Architecture for Ambient-Temperature Quantum-Classical Transduction. *Annals of the Chrysene Formalism*, 1, 2026. Article 9. https://doi.org/10.68366/scqypz29.

[Sch26b] Charles D. Schaper. Information-Theoretic Mechanics of Survival in Maximum-Entropy Environments: Static Minimax Disturbance Attenuation, Coordination-Deficit Costs, and the Placed Source as Categorical Right Adjoint. *Annals of the Chrysene Formalism*, 1(4):1890–1897, 2026. Article 97. https://chrysene.com/index.php/acf/article/view/97.

[Sch26c] Charles D. Schaper. Quantum Information Dynamics of the Source and the Placed Source: Lindblad Dynamical Scission, Von Neumann Entropy Production, and Landauer-Bounded Quantum-Classical Transduction. *Annals of the Chrysene Formalism*, 1(4):1883–1889, 2026. Article 96. https://chrysene.com/index.php/acf/article/view/96.

[Sch26d] Charles D. Schaper. The Biological Ledger and the Geometric Mechanics of Survival via the Chrysene Formalism. *Annals of the Chrysene Formalism*, 1, 2026. Article 4. https://doi.org/10.68366/8bdqte12.

[Sch26e] Charles D. Schaper. The Completeness Condition: Realisation of the Twelve-Axiom Framework at a Site of the Interior Chrysene Lattice, and the Genome Length It Requires. *Annals of the Chrysene Formalism*, 1(4):1639–1651, 2026. Article 80. https://doi.org/10.68366/0a19r769.

[Sch26f] Charles D. Schaper. The Dual Cipher, Structurally Induced GF(4) Multiplication, and Real Spectral Structure: Companion-Matrix Realization of the Register Automorphism, Frobenius Reversal, and the Order-One Commutant Condition. *Annals of the Chrysene Formalism*, 1(4):1847–1855, 2026. Article 92. https://chrysene.com/index.php/acf/article/view/92.

[Sch26g] Charles D. Schaper. The Placed Source: Exogenous Preparation of the Substituted-Chrysene Assembly, and the Equality of Its Reachable Set with the Endogenous One. *Annals of the Chrysene Formalism*, 1(4):1703–1731, 2026. Article 84. https://doi.org/10.68366/a4tjv735.

[Sch26h] Charles D. Schaper. The Source: Substitution-Initiated Self-Separation of the Substituted-Chrysene Stock from a Polyaromatic Bulk, and the Supply It Delivers to the Protocell. *Annals of the Chrysene Formalism*, 1(4):1616–1638, 2026. Article 79. https://doi.org/10.68366/s8sfwq50.

[TVN07] Michael Tsfasman, Serge Vlăduţ, and Dmitry Nogin. *Algebraic Geometric Codes: Basic Notions*, volume 139 of *Mathematical Surveys and Monographs*. American Mathematical Society, 2007.